Effects of Polarity Randomization in Impulse Radio UWB Systems

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Signal Model

- Transmitted signal from user k in an N_u-user environment

$$s_{\mathrm{tx}}^{(k)}(t) = \sqrt{\frac{E_k}{N_f}} \sum_{j=-\infty}^{\infty} d_j^{(k)} \, b_{\lfloor j/N_f \rfloor}^{(k)} w_{\mathrm{tx}}(t - jT_f - c_j^{(k)} T_c - \epsilon_j^{(k)}), \qquad (1)$$

$w_{\mathrm{tx}}(t)$: Transmitted UWB pulse

E_k : Energy coefficient for user k

$b_{\lfloor j/N_f \rfloor}^{(k)}$: Equiprobable binary data, $\{-1, +1\}$

$\epsilon_j^{(k)}$: Timing jitter, i.i.d. $\forall j$, smaller than the pulse width

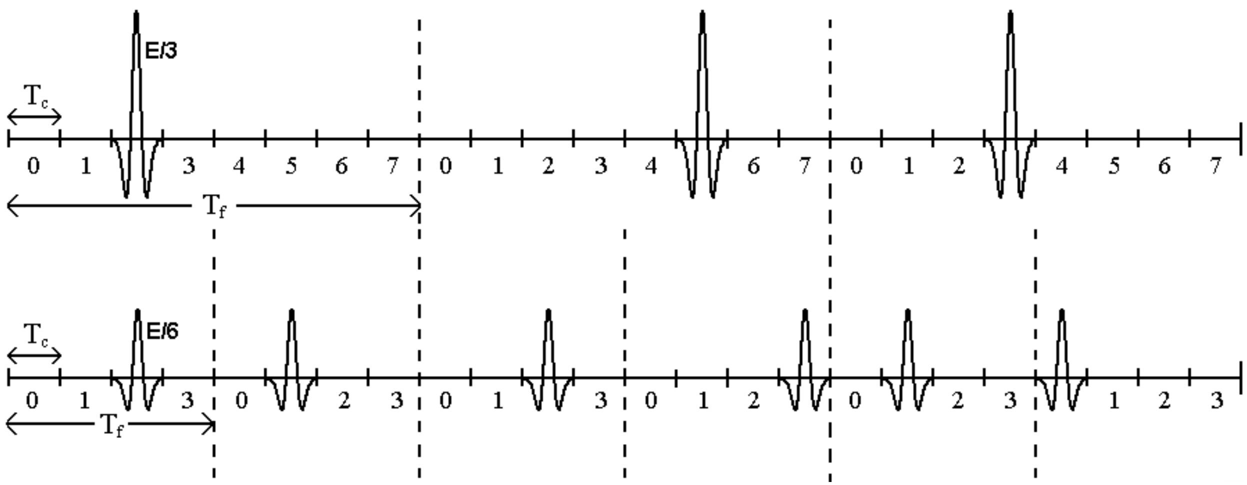
$c_j^{(k)}$: Time hopping code, $\in \{0, 1, \ldots, N_c - 1\}$

$d_j^{(k)}$: Polarity randomization code

- $d_j^{(k)}$ are i.i.d., ± 1 with equal probability $\longrightarrow$ "Coded" system
  $d_j^{(k)} = 1 \ \forall j, k \longrightarrow$ "Uncoded" system

Problem Formulation

- Two types of processing gain:
  - Pulse combining gain, N_f
  - Pulse spreading gain, $N_c = T_f / T_c$
- For a fixed symbol interval and pulse width, the total processing gain, $N = N_c N_f$, is fixed. Two extreme cases:
  - $N_f = 1 \longrightarrow$ Reduce probability of overlaps (TDMA)
  - $N_f = N \longrightarrow$ Orthogonalize signals of different users (CDMA)



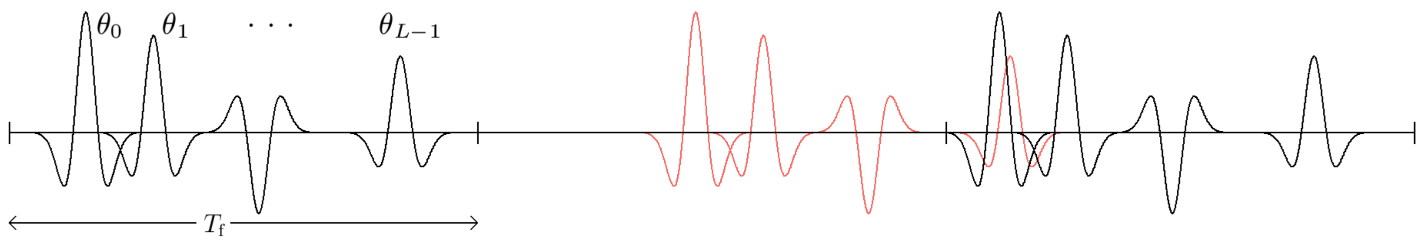
Receiver Model

- Rake receiver with various combining schemes

$$s_{\text{temp}}^{(1)}(t) = \sum_{j=iN_f}^{(i+1)N_f-1} d_j^{(1)} v_j(t - jT_f - c_j^{(1)}T_c), \tag{2}$$

$$v_j(t) := \sum_{l=0}^{L-1} \theta_l w_{\text{rx}}(t - \tau_l^{(1)} - \hat{\epsilon}_{j,l}), \tag{3}$$

$\hat{\epsilon}_{j,l} \longrightarrow$ Timing jitter at lth finger, jth frame of template (i.i.d.)
$\theta_0, \theta_1 \ldots, \theta_{L-1} \longrightarrow$ Rake combining coefficients
$\tau_0^{(1)}, \tau_1^{(1)}, \ldots, \tau_{L-1}^{(1)} \longrightarrow$ Delays of multipath components for user 1



- Assume negligible inter-frame interference (IFI)

Coded Systems

- As $N \longrightarrow \infty$ and $\frac{N_{\rm f}}{N_{\rm c}} \longrightarrow c > 0$, the MAI from user k

$$a^{(k)} \sim \mathcal{N}\left(0\,,\ \frac{N_f}{N_c}\sum_{j=-\tilde{L}}^{\tilde{L}} \mathrm{E}\{[\phi_{uv}^{(m)}(jT_{\rm c}+\epsilon^{(k)})]^2\}\right) \tag{4}$$

where $u(t) = \sum_{l=0}^{L-1} \alpha_l w_{\rm rx}(t-\tau_l)$ and $\phi_{uv}^{(m)}(x) = \int u(t-x)v_m(t)dt$.

- Bit error probability (BEP)

$$P_{\rm e} \approx Q\left(\frac{\sqrt{E_1}\,\mathrm{E}\{\phi_{uv}^{(m)}(\epsilon^{(1)})\}}{\sqrt{\frac{E_1}{N_f}\mathrm{Var}\{\phi_{uv}^{(m)}(\epsilon^{(1)})\} + \frac{1}{N}\sum_{k=2}^{N_{\rm u}} E_k\hat{\gamma}_2^{(k)} + \bar{E}_v\sigma_n^2}}\right) \tag{5}$$

$$\hat{\gamma}_2^{(k)} = \sum_{j=-\tilde{L}}^{\tilde{L}} \mathrm{E}\{[\phi_{uv}^{(m)}(jT_{\rm c}+\epsilon^{(k)})]^2\} \tag{6}$$

- Effects of MAI depend only on the total processing gain.

Uncoded Systems

- Approximate BEP for large number of equal energy interferers

$$Q\left(\frac{\sqrt{E_1}\,\mathrm{E}\{\phi_{uv}^{(m)}(\epsilon^{(1)})\}}{\sqrt{\frac{E_1}{N_\mathrm{f}}\mathrm{Var}\{\phi_{uv}^{(m)}(\epsilon^{(1)})\}+(N_\mathrm{u}-1)E\left(\frac{\hat{\gamma}_2^{(k)}}{N}+\frac{(\hat{\gamma}_1^{(k)})^2}{N_\mathrm{c}^2}-\frac{(\hat{\gamma}_1^{(k)})^2}{N_\mathrm{c}N}\right)+\bar{E}_v\sigma_n^2}}\right)$$

$$\hat{\gamma}_n^{(k)}=\sum_{j=-\tilde{L}}^{\tilde{L}}\mathrm{E}\{[\phi_{uv}^{(m)}(jT_\mathrm{c}+\epsilon^{(k)})]^n\} \tag{7}$$

for $n = 1, 2$.

- Effects of MAI increase with N_f.
- MAI is more effective than that in the coded case for $N_\mathrm{f} > 1$.

Simulations - AWGN Case

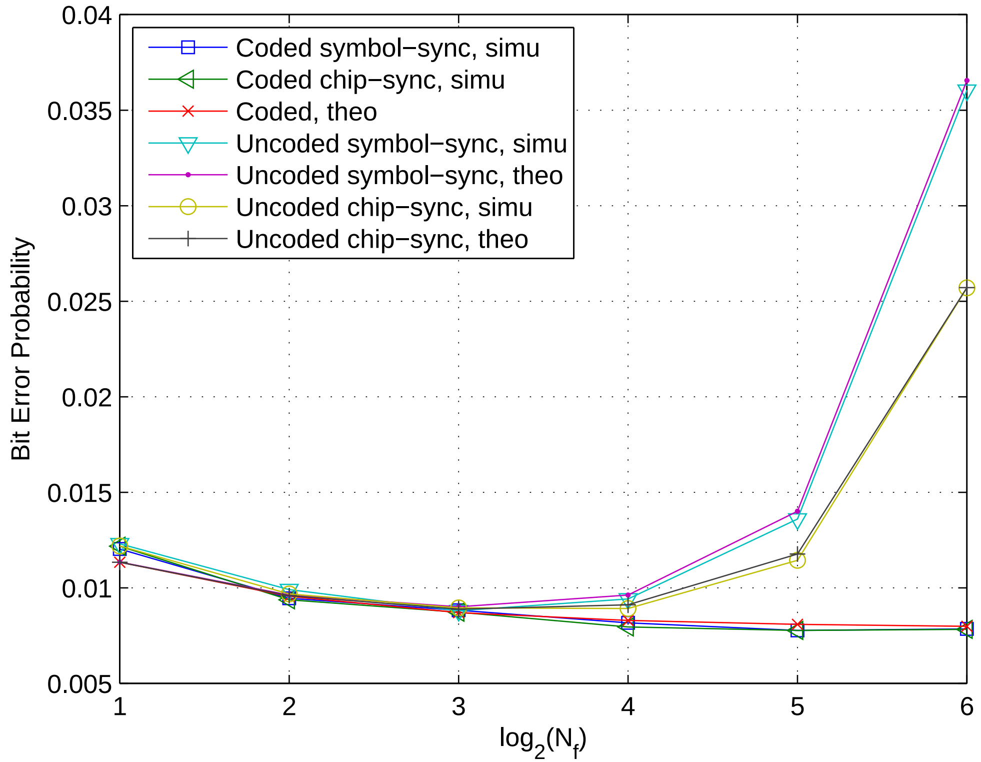


Figure 1: BEP vs $\log_2 N_\mathrm{f}$ for coded and uncoded systems for the AWGN channel case. The timing jitter is modelled by $\mathcal{U}[-25\,\mathrm{ps}, 25\,\mathrm{ps}]$, $T_c = 0.25\,\mathrm{ns}$, and $N = N_c N_f = 512$. All 10 users are assumed to be sending unit-energy bits ($E_k = 1\ \forall k$) and $\sigma_n^2 = 0.1$.

Simulations - Multipath Case

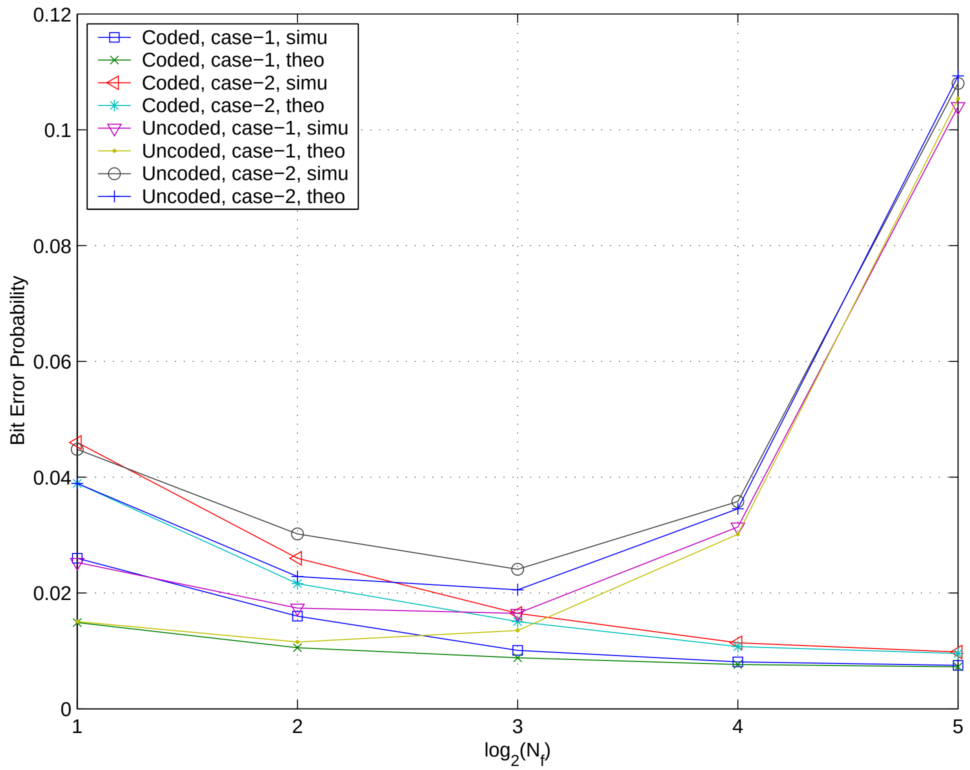


Figure 2: BEP versus $\log_2 N_f$ for coded and uncoded systems over the multipath channel $[0.4653\ 0.5817\ 0.2327 - 0.4536\ 0.3490\ 0.2217 - 0.1163\ 0.0233 - 0.0116 - 0.0023]$. Case-1: I.i.d. jitter for all different finger and frame indices. Case-2: Same jitter value for all fingers of a given frame, and i.i.d. jitter values are for different frames.

Concluding Remarks

- Better MAI suppression using random polarity codes
- More gain of coded systems compared to uncoded ones as N_f gets larger. Constraints on peak-to-average power ratio $\longrightarrow$ N_f cannot be very small
- By central-limit argument for dependent sequences, approximate BEP expressions without need for large number of or equal power interferers for the coded case
- Polarity randomization also helps optimize the spectral shape according to the FCC specifications by eliminating the spectral lines inherent in "classical" IR-UWB systems [Nakache and Molisch, 2003]