A Theoretical Model of a Voltage Controlled Oscillator

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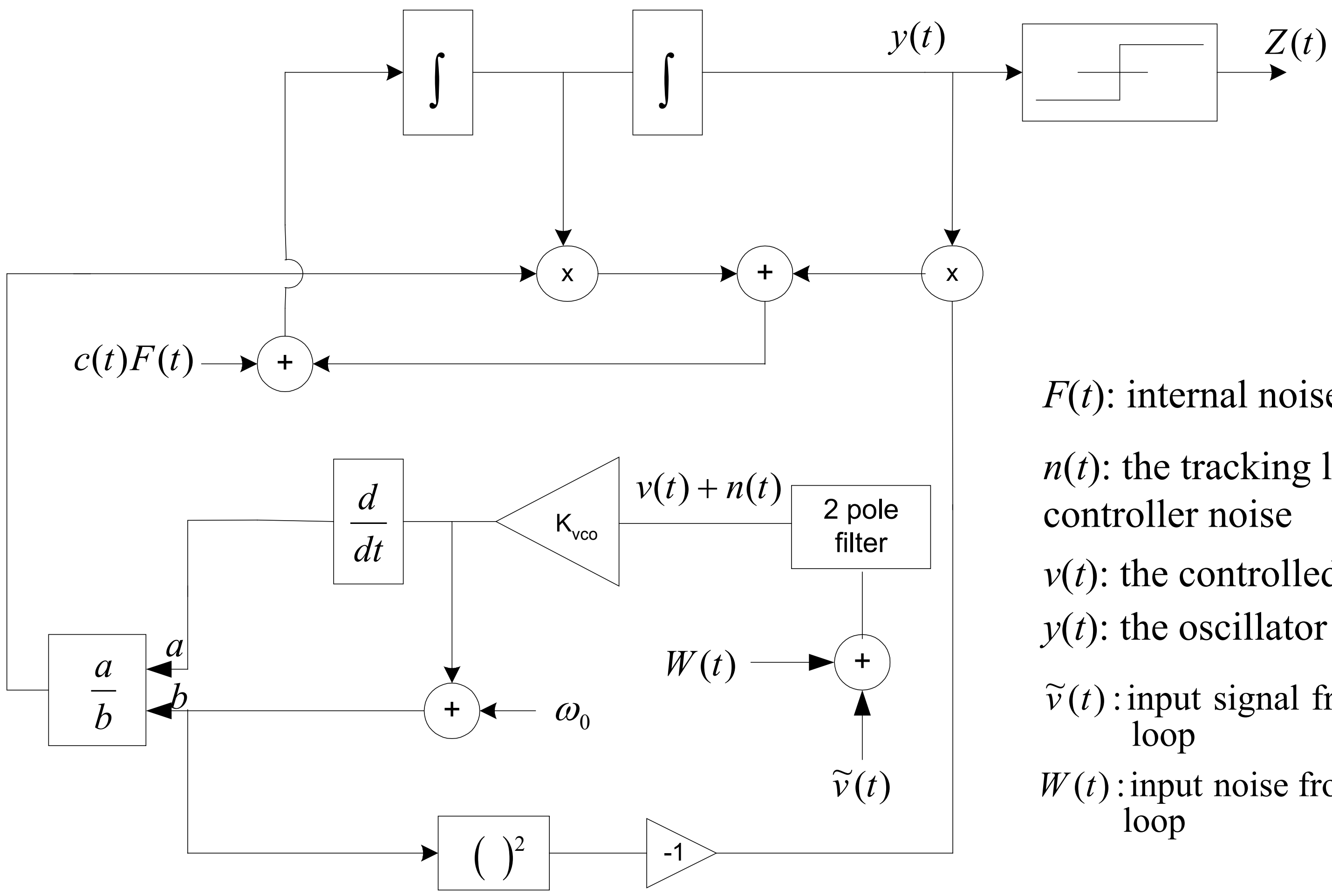
Introduction

- **Motivation**
  - The voltage controlled oscillator of the PLL has been modeled as a simple integrator, it does not capture the essence of dynamics of oscillation
  - Oscillator instability issue critical for communication systems
  - Need a high speed clock for ultra-wideband (UWB) communication systems
  - Synchronization analysis for UWB systems
- **Problem statement**
  - Need a mathematical model of VCO that fully incorporates a means of non-linearity for stabilizing oscillators

Schematic of the proposed VCO with noise



$F(t)$: internal noise of the clock

$n(t)$: the tracking loop plus the controller noise

$v(t)$: the controlled voltage

$y(t)$: the oscillator waveform

$\tilde{v}(t)$: input signal from the tracking loop

$W(t)$: input noise from the tracking loop

Theoretical model

- The voltage controlled oscillator can be described by the following stochastic differential equation which includes the internal noise, $F(t)$, of the clock and the tracking loop plus the controller noise, $n(t)$.

$$\ddot{y} - \frac{K_{\mathrm{vco}}(\dot{v} + \dot{n}(t))}{\omega_0 + K_{\mathrm{vco}}(v + n(t))}\dot{y} + \left[\omega_0 + K_{\mathrm{vco}}(v + n(t))\right]^2 y = c(t)F(t)$$

where $F(t)$ is assumed to be a white Gaussian noise, independent of $n(t)$. In addition, $n(t) \in C^2([0,\infty))$, $y(t)$ is the oscillator waveform, $v(t)$ is the controlled voltage, $c(t)$ is a scaled factor, ω_0 is the clock rest frequency $(\omega_0 > 0)$, and K_{vco} is the VCO gain.

- The output after the hard-limiter is $Z(t) = \mathrm{sgn}(y(t))$

Analytic solutions of the model

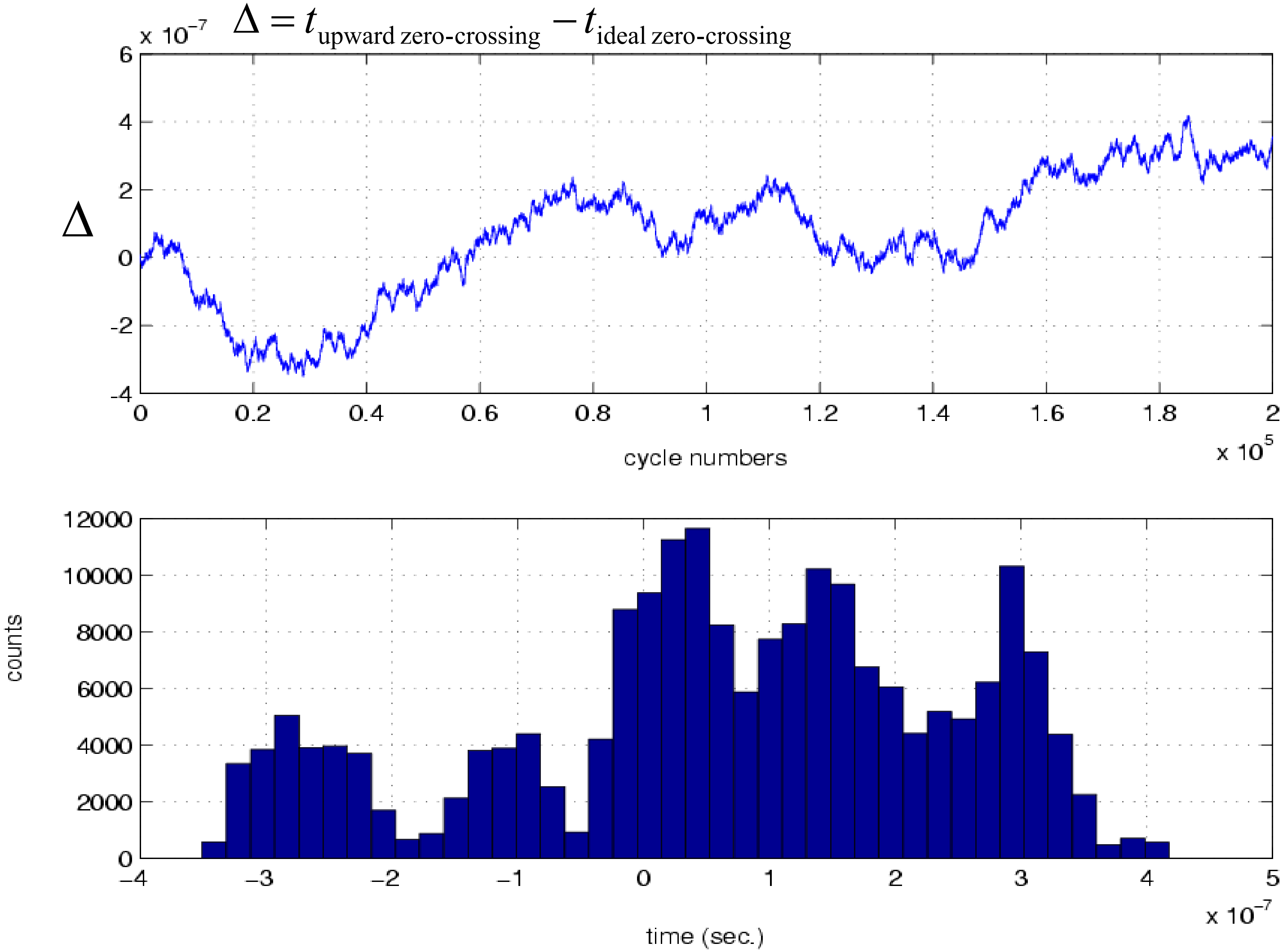
- Initial conditions: $y(0) = 0$, $v(0) = 0$, $y_1(0) = 0$, $y_2(0) = \hat{a}\omega_{\text{new}}$, $\hat{a} \in R$
- When $v(t) = \begin{cases} at & \text{if} \quad t \le t_s \\ C & \text{if} \quad t > t_s \end{cases}$ where a, C are constants, and t_s is the time of steady state, unless otherwise specified.

	Analytic solution	Phase noise	Statistical properties
No noise present	$y_1(t) = \sin(\omega_0 t + \int_0^t K_{\text{vco}} a\tau \ d\tau)$	N/A	N/A
Internal noise $\tilde{F}(t)$ present	$y_1(t) = \hat{a}\sin(\omega_{\text{new}} t) + \int_0^t c(s)\frac{\sin(\omega_{\text{new}}(t-s))}{\omega_{\text{new}}} dB_s$ where $c(s)$ is the scaling factor for the noise $F(t)$, $v(t) = a \ \forall\, t$, $\omega_{\text{new}} = \omega_0 + K_{\text{vco}}a$, and B_s is a 1-dimensional Brownian motion	$\varphi(t) = \tan^{-1}\left(\frac{n_2(t)}{1+n_1(t)}\right)$ $n_1(t) = K\int_0^t \cos\omega_{\text{new}} s \ dB_s$ $n_2(t) = K\int_0^t \sin\omega_{\text{new}} s \ dB_s$, $K = \frac{c}{\omega_{\text{new}}}$	$E[\varphi(t)] \approx \frac{c^2}{4\omega_{\text{new}}^3}[\cos(2\omega_{\text{new}} t) - 1]$ $R_\varphi(t,s) \approx R_{n_2}(t,s)$ $= \frac{c^2}{2\omega_{\text{new}}^2}\left[\min(s,t) - \frac{\sin(2\omega_{\text{new}}\min(s,t))}{2\omega_{\text{new}}}\right]$
Internal noise and external noise present	$y_1(t) = \hat{a}\sin\Phi_{\text{new}}(t) + \int_0^t c(s)\frac{\sin\hat{b}(s,t)}{\omega_0 + K_{\text{vco}}(as+n(s))} dB_s$ where $\Phi_{\text{new}}(t) = \omega_0 t + \int_0^t K_{\text{vco}} a\tau\, d\tau + \int_0^t K_{\text{vco}} n(\tau) d\tau$, $\hat{b}(s,t) = \omega_0(t-s) + \int_s^t K_{\text{vco}} a\tau\, d\tau + \int_s^t K_{\text{vco}} n(\tau) d\tau$	$\tilde{\varphi}(t) = \tan^{-1}\left(\frac{\tilde{n}_2(t)}{1+\tilde{n}_1(t)}\right)$ $\tilde{n}_1(t) = \int_0^t \tilde{K}(s,n(s))\cos\Phi_{\text{new}}(s) dB_s$ $\tilde{n}_2(t) = \int_0^t \tilde{K}(s,n(s))\sin\Phi_{\text{new}}(s) dB_s$ $\tilde{K}(s,n(s)) = \frac{c}{\omega_0 + K_{\text{vco}}(as+n(s))}$ $\varphi_n(t) = \int_0^t K_{\text{vco}} n(z) dz$	$E[\tilde{\varphi}(t)]$ and $R_{\tilde{\varphi}}(t,s)$ are both time-varying and no closed form solution can be obtained. $R_{\varphi_n}(t,s) = \int_0^t\int_0^s K_{\text{vco}}^2 R_n(\tau,z) dz d\tau$

Results to date: simulation results internal noise only

Averaged upward zero-crossing jitter

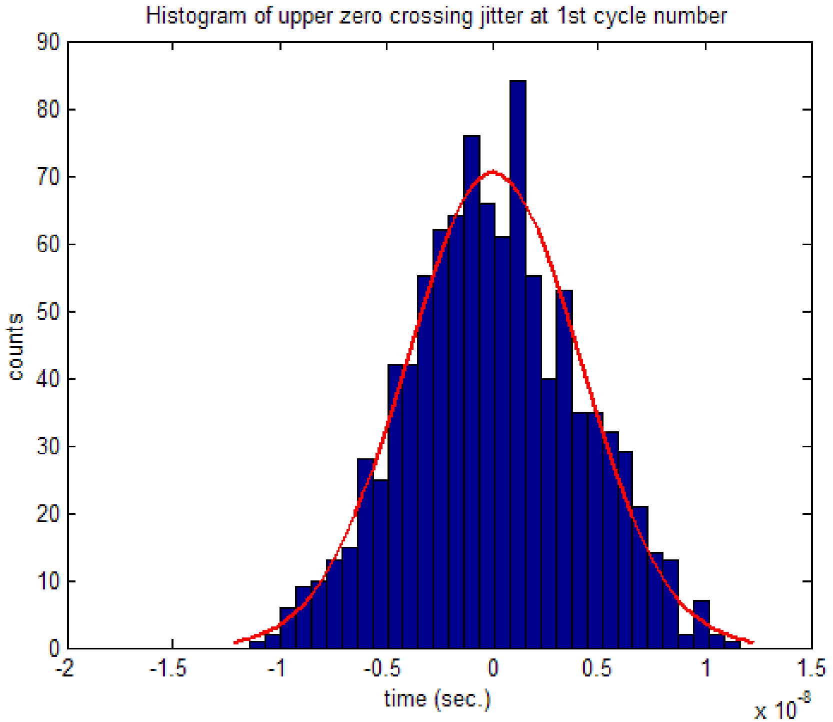
$$\Delta = t_{\text{upward zero-crossing}} - t_{\text{ideal zero-crossing}}$$

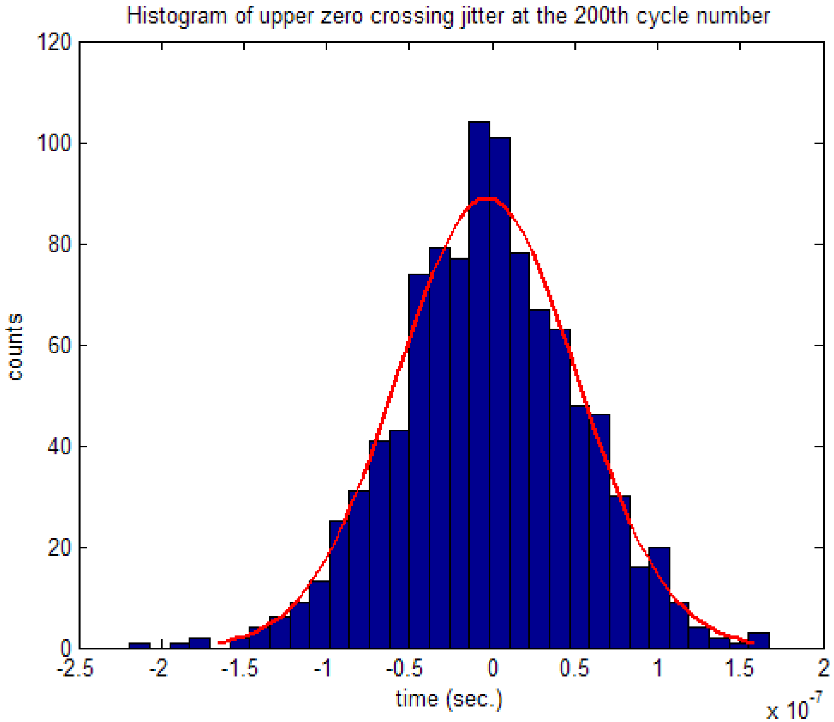


Simulation done at VCO frequency of 2 kHz and $\widetilde{F}(t)$ is on.

Simulation results

- Histograms of upward zero crossing jitter at different time instants, 1000 runs.



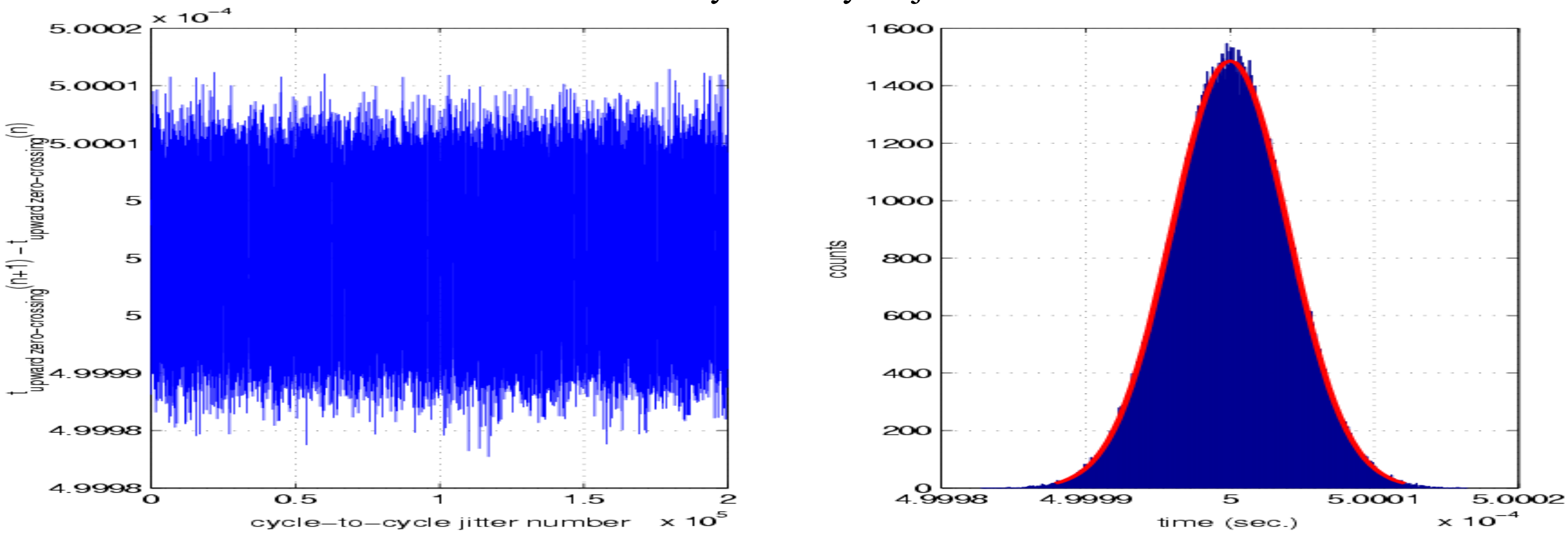


Note: Diffusion occurs

Simulation results

- Cycle to cycle jitter simulated for 100 sec.

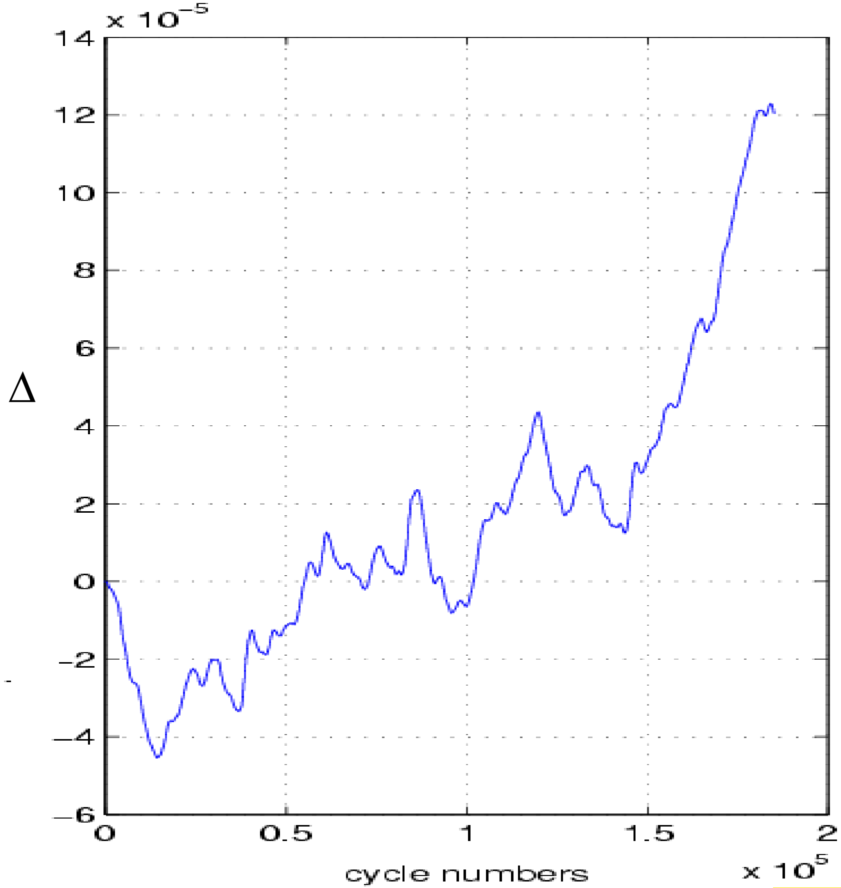
Cycle-to-cycle jitter

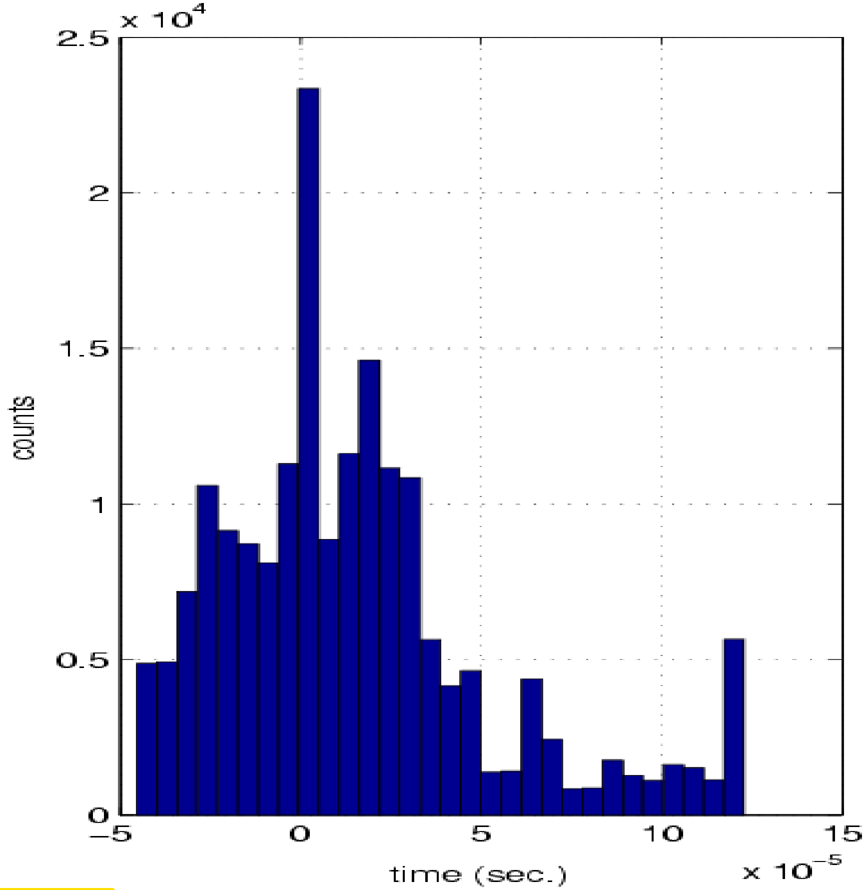


- Chi-squared goodness-of-fit test is used to test the normality of the test statistics
- The parametric Pearson correlation test is performed to see that the cycle to cycle jitters are independent.

Simulation results for internal + external noises

timing jitter when both $F(t)$ and $n(t)$ are present





Summary of results

- A theoretical model of a voltage controlled oscillator is proposed based on physical dynamics with noise.
- Analysis of the resulting phase noise along with the simulation suggest that the timing jitter process has a random walk behavior (with restoring force) and the upward zero crossing jitter is normal distributed.
- The increment of the timing jitter process, cycle-to-cycle jitter, is shown to behave as normal distributed and independent.
- Results suggest that the tracking loop plus the controller noise $n(t)$ cause the oscillator phase to drift while the internal noise $\widetilde{F}(t)$ tends to cause diffusion on the oscillator phase.