A DIFFUSION MODEL FOR UWB INDOOR PROPAGATION

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ABSTRACT

This paper proposes a diffusion model of the spatially averaged energy profile in time for indoor ultra-wideband links. The consecutive indoor reflections cause the energy to have a diffusion-like behavior that might be modeled by diffusion equations. Basic relations between model parameters i.e., room dimensions, diffusion coefficients, average reflection and absorption coefficients are hypothesized when no exact information about the rooms and objects inside them are available. The validity of this analytical model is verified with different propagation measurement sets. Also further modifications to improve the model accuracy have been proposed.

1. INTRODUCTION

Although Ultra-wide band communications (UWB) with its many inherent capabilities has gained a lot of attentions, yet no perfect model for UWB indoor channel has been offered. Most proposed stochastic models and ray tracing methods need more or less precise information about the building as well as information about the objects inside the building. Typically there are many reflections in an indoor UWB environment which depend on objects' positions in the building and different reflection, transmission and absorption coefficients that walls and objects inside the building have. Providing these pieces of information for the receiver is quite difficult, if not impossible, even for strongest multipath components.

Looking at some recent works in UWB ranging applications [1] and UWB Transmitted Reference (TR) [2][3], we need to know the length of the window in which received signal exists efficiently. Considering that the received UWB signal is constructed of many multipath signals, which although typically decay in time, but may last from tens to several thousands of nanoseconds, the selection of the optimum window size becomes more critical.

This work was supported by the Army Research Office under MURI grant No. DAAD19-01-1-0477.

In this paper we attempt to obtain a profile for the energy envelope at each room, averaged spatially and averaged on several received signals, with simple information about building dimensions. From that we can estimate the average received temporal voltage/power/energy profile at the receiver. This energy profile provides important information to many design efforts, including UWB ranging and transmitted reference receivers. This energy profile contains approximate characteristics of UWB energy growth, decay, and delay spread in different locations, so that the envelope-like parameters of the UWB impulse response at various locations in the structure are known.

We start the paper with a theoretical derivation of diffusion equations for the simple case where we have two rooms in the system. Then we generalize the equations to more complex structures. Some modifications to compensate for the wave limited speed of propagation will be our next focus. We conclude the paper with fitting curves to the real measured data and estimating the basic parameters of the model.

2. UWB DIFFUSION MODEL
2.1. TWO ROOM CASE

Consider a simplified one floor model of two rooms with a common wall and enough objects in them, so that the high number of diffused and specular reflections produces a diffusion-like propagation characteristic. This is true practically for most buildings with objects in them. The walls of these rooms are made of materials that will let electromagnetic energy pass trough them, as well as reflect from them. Assume that ρ_{ij}'s are the average transmission coefficients equal to the fraction of the incident energy that passes through the wall from room i to room j. The energies corresponding to ρ_{10} and ρ_{20} leave the system and will never come back, e.g., there are no significant reflectors around the building. We call the portion of the signal which diffuses outside the system, the escaped energy. Figure 1 shows this scenario together with transmission coefficients for two room model. The energy corresponding to reflected waves remains in the room and does not change the average energy of the room. Each room also has an average loss which includes the floor and ceiling signal dissipation and transmission.

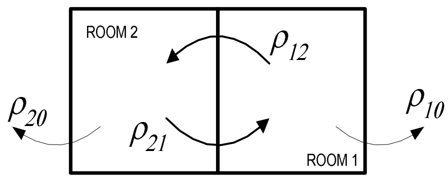

Figure 1. Two room model with transmission coefficients shown.

We assume that the diffusion phenomena expands with velocity D , i.e., the edge of the area which has the diffused electromagnetic energy has the speed of D m/sec. Our parameter of interest in this system is the energy density envelope in the rooms averaged on the whole space for all possible transmitter and receiver positions. We denote this quantity, which is diffusing with diffusion coefficient of D, by $E_i(t)$ for room i. Knowing $E_i(t)$, our purpose is to calculate $E_i(t+\Delta t)$, the average energy density envelope in room i at time $t+\Delta t$, where Δt is a very small time duration (Mathematically, $dt=\Delta t \to 0$).

Conservation of energy implies that the electromagnetic energy in the whole system at time t is equal to the electromagnetic energy at time $t+dt$ plus the absorbed and escaped energy during time dt. Electromagnetic energy diffuses (expands with velocity D from each side) through the walls in all the directions. Although the energy will diffuses spherically, but since $\Delta t \to 0$, we can approximate the area which diffused energy is in at time $t+dt$ with an area similar to the room, with larger dimensions. Figure 2 shows room 1 dimensions and the approximate area that energy diffuses at time $t+dt$. We denote this bigger area $S_1^{'}$.

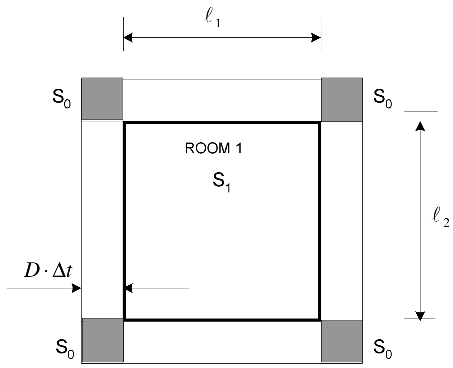

Figure 2. Room one with approximate diffused energy area.

$$S_1^{'} = S_1 + 2(\ell_1 D \cdot \Delta t) + 2(\ell_2 D \cdot \Delta t) + 4S_0 \tag{1}$$

where

$$S_0 = D^2 (\Delta t)^2 \tag{2}$$

and ℓ_1 and ℓ_2 are the dimensions of the room.

Considering that $dt \to 0$ and D is a finite number close to the electromagnetic wave propagation speed, in (1) we can neglect the S_0 as the second order infinitesimal compared to the first order, so (1) will change to:

$$S_1^{'} \cong S_1 + 2(\ell_1 D \cdot \Delta t) + 2(\ell_2 D \cdot \Delta t) \tag{3}$$

Now from calculation of the whole energy in the system at times t and $t+\Delta t$ we have:

$$\begin{aligned} & S_1 h E_1(t) - \bar{a}_1 E_1(t) dt \cong S_1 h E_1(t+dt) + \\ & (\rho_{12}\ell_2 + \rho_{10}\ell_2 + \rho_{10}\ell_1 + \rho_{10}\ell_1) h D E_1(t+dt) dt \end{aligned} \tag{4}$$

where $\bar{a}_1$ is the average loss coefficient per unit of time in the whole room 1, and corresponds to objects and walls losses, and floor and ceiling losses and transmissions. h is the height of the building. In a homogeneous environment we can assume that $\rho_{12} = \rho_{21} = \rho$. For simplicity purposes at this step we also assume that all the walls are made of almost the same materials e.g., big windows are similar in all the rooms, therefore $\rho_{i0} = \rho_0$ for all i. Also since $h\ell_i D dt \to 0$, and $E_i(t)$ represents a continuous (averaged physical) phenomena, with minor effect, we can substitute $h\ell_i \rho_{ij} D dt E_1(t+dt)$ with $h\ell_i \rho_{ij} D dt E_1(t)$, and hence

$$\begin{aligned} & S_1 h [E_1(t+dt) - E_1(t)] = -\bar{a}_1 E_1(t) dt - \\ & E_1(t)\rho h D \ell_2 dt - E_1(t)\rho_0 h D(\ell_2 + 2\ell_1) dt \end{aligned} \tag{5}$$

We emphasize that in (5), the first term in the right hand side corresponds to losses, the second term corresponds to the energy which is diffused to room 2, and the third term corresponds to the escaped energy. Rewriting (5) gives

$$dE_1(t) = -(a_1 + X + X_0) E_1(t) dt \tag{6}$$

where

$$\begin{aligned} X &= (\ell_2 D\rho)/S_1 \\ X_0 &= (2\ell_1 + \ell_2) D\rho_0 / S_1 \end{aligned} \tag{7}$$

and

$$a_1 = \bar{a}_1 / (S_1 h)$$

Now assume that at time t, $E_2(t)$ is the energy that already exists at room 2. Similar to previous discussion, we can derive the following equation for room 2

$$dE_2(t) = -(a_2 + X + X_0) E_2(t) dt \tag{8}$$

From (5) we also deduce that $E_2(t)\rho hD\ell_2 dt$ is the amount of diffused energy from room 2 to room 1. Since we are interested in the average energy density in each room, we need to average this diffused energy over the whole room volume. Therefore the change of the average energy density at room 1 due to diffusion from room 2 will be

$$\frac{hE_2(t)\ell_2 D\rho dt}{hS_1} = \frac{\ell_2 D\rho}{S_1} E_2(t)dt = XE_2(t)dt \tag{9}$$

Assuming linearity, the total energy in room 1 can be calculated as the superposition of the two responses, i.e.,

$$\frac{dE_1(t)}{dt} = -(a_1 + X + X_0)E_1(t) + XE_2(t) \tag{10}$$

similarly we have

$$\frac{dE_2(t)}{dt} = -(a_2 + X + X_0)E_2(t) + XE_1(t) \tag{11}$$

equations (11) and (12) can be written in vector form as

$$\dot{\mathbf{E}}(t) = \mathbf{AE}(t) \tag{12}$$

where

$$\mathbf{E}(t) = \begin{bmatrix} E_1(t) \\ E_2(t) \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} -(a_1 + X + X_0) & X \\ X & -(a_2 + X + X_0) \end{bmatrix} \tag{13}$$

$$\dot{\mathbf{E}}(t) = \frac{d}{dt}\mathbf{E}(t)$$

The solution to this system of differential equation is unique knowing its initial conditions. We assume that the transmitter is in room 1 and transmits an impulse at time zero so that the normalized energy at room 1 at time zero is equal to 1, i.e.,

$$\mathbf{E}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{14}$$

so the solution to (12) is of the form

$$E_1(t) = K_{11}\exp(m_{11}t) + K_{12}\exp(m_{12}t) \tag{15}$$

$$E_1(0) = 1 \Rightarrow K_{11} + K_{12} = 1$$

$$E_2(t) = K_{21}\exp(m_{21}t) + K_{22}\exp(m_{22}t) \tag{16}$$

$$E_2(0) = 0 \Rightarrow K_{21} + K_{22} = 0$$

2.2. MORE COMPLEX STRUCTURES

Consider a structure of 12 rooms which is shown in Figure 3. The transmitter is in room 7. We assume that the transmission coefficient between room i and j is equal to $\rho_{ij} = \rho_{ji}$, and of the outside walls is equal to ρ_{j0} for the outer room j. With the same method as we did in section 2.1, we determine the following equation:

$$\dot{\mathbf{E}}(t) = \mathbf{AE}(t) \tag{17}$$

where $\mathbf{E}(t) = [E_1(t) \quad E_2(t) \quad \cdots \quad E_{12}(t)]^t$

In constructing the matrix **A,** neighboring rooms are important for each room. Appendix I shows the matrix **A** elements for the construction of Figure 3.

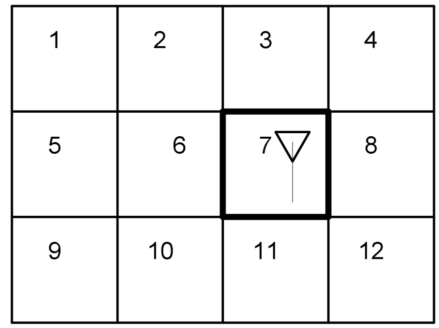

Figure 3. A structure with 12 rooms. Transmit antenna is in room 7.

Solving (17) will result

$$\mathbf{E}(t) = \exp(\mathbf{A}t)\mathbf{E}(0) \tag{18}$$

with initial condition

$$E_i(0) = \begin{cases} 0 & ;i \neq 7 \\ 1 & ;i = 7 \end{cases}$$

Figures 4 and 5 show the plot of a sample solution to the system of differential equations (17) for two and 12 room scenarios respectively. To get reasonable responses, we've assumed $a = a_1 = a_2 = 1\times10^7$ and $X = 1\times10^7$. Since later in data regression section we work with the envelope of the received signal, which is proportional to the square root of the energy, this Figure has also been drawn in terms of square root of the energy envelope.

2.3. MODIFIED MODEL

Some modifications are needed to get a more realistic diffusion model for UWB indoor propagation. One problem of this model is that it ignores the propagation delay due to finite wave propagation speed. For example in the model of Figure 3, although the transmitter at room 7 is far from room 5, but the response plots at Figure 5 show an immediate energy at room 5. All adjacent rooms to room 7 may have immediate energy since we're averaging energy in the room based on all possible locations for transmitter and receiver antenna. Therefore in the above model, rooms 1, 5 and 9 are the only rooms that don't take instant average diffused energy. In average sense, the energy should travel the length of room 6 before it gets in to rooms 1, 5 or 6. Therefore a delay of $t_0 = L/D$ where L

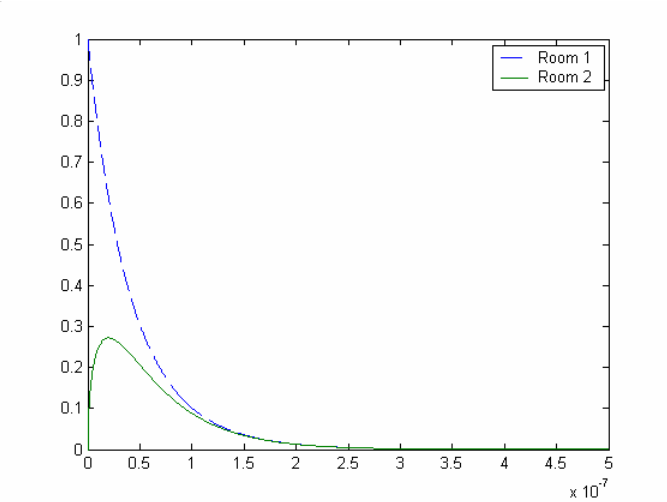

Figure 4. Average received signal envelope for the model with two rooms.

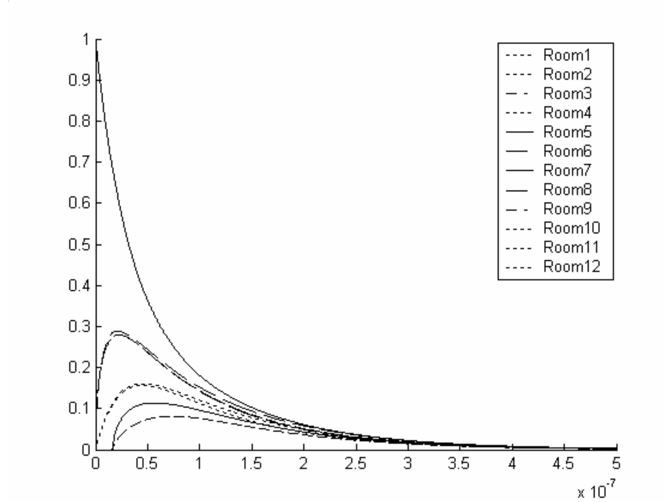

Figure 6. Average received signal envelope for the modified model with 12 rooms.

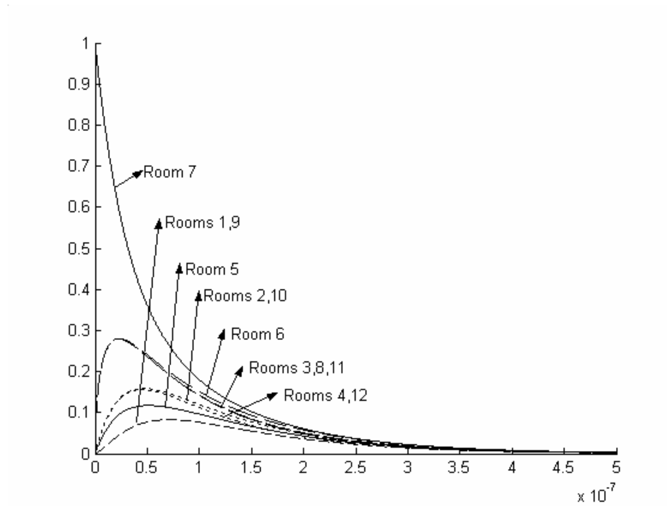

Figure 5. Average received signal envelope for the model with 12 rooms.

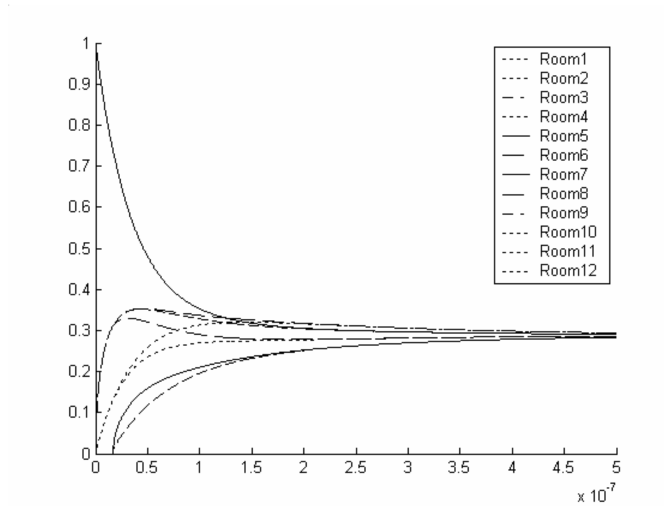

Figure 7. Average received signal envelope for the modified model with 12 rooms, no loss or escape energy.

is the length of room 6 and *D* is the diffusion coefficient (velocity) is necessary before rooms 1, 5 and 6 practically affect the system. We compensate for this delay by changing the system of differential equations (17) in to a time varying system where before t_0 we have

$$X_{1,2} = X_{2,1} = X_{6,5} = X_{5,6} = X_{10,9} = X_{9,10} = 0$$

and after t_0 they change to their final value according to appendix I. In fact we solve the system of diffusion model with rooms 1,3,4,6,7,8,10,11 and 12 before time t_0. Then the solution at time t_0 will act as the initial condition for the system with 12 rooms from time t_0 on. The initial condition for rooms 1,5 and 9 will be zero at time t_0 . Figure 6 shows the solutions to the modified model with delay for L = 5 m.

An assumption which practically simplifies our calculations is to consider the whole building as two rooms, the room of interest, and all the other parts of the building. As a result of this assumption we can always use the two room model, and the solution consists of two exponential terms. In this case, other walls act like objects that contribute in constructing the diffusion phenomena.

It is informative to look at the limiting rather theoretical case where there is no dissipation or escaped energy from the system. The result is shown in Figure 7.

3. MEASUREMENTS, REGRESSION AND DATA ANALYSIS

For data analysis and model validity investigation, a set of propagation data taken by Win [4] in an office building was used. At each room, data was gathered in a spatial grid of 7 by 7. Data at each point also has been averaged on 32 different runs. Although this 7 by 7 grid does not cover the whole area of the room and therefore averaging the envelope of these signals is not the true spatial average of the room, but with good approximation it shows the behavior of the averaged signal. Figure 8 shows the plan of the building where measurements have been done.

To do the regression, we use the simplified model that

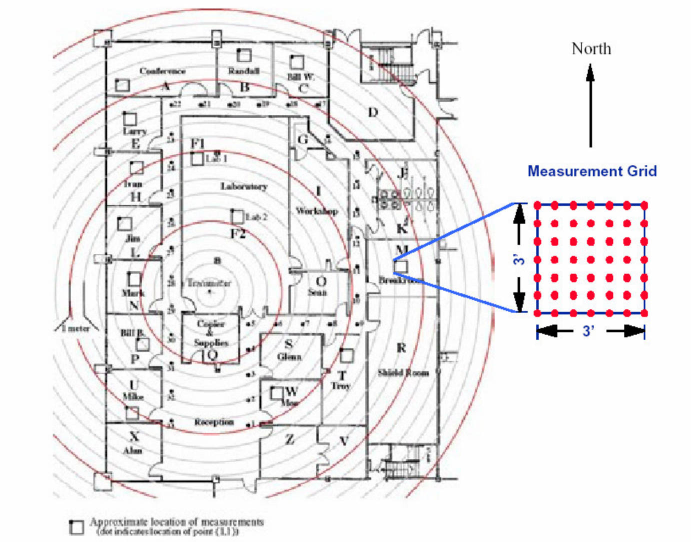

Figure 8. Building plan where measurements have been done.

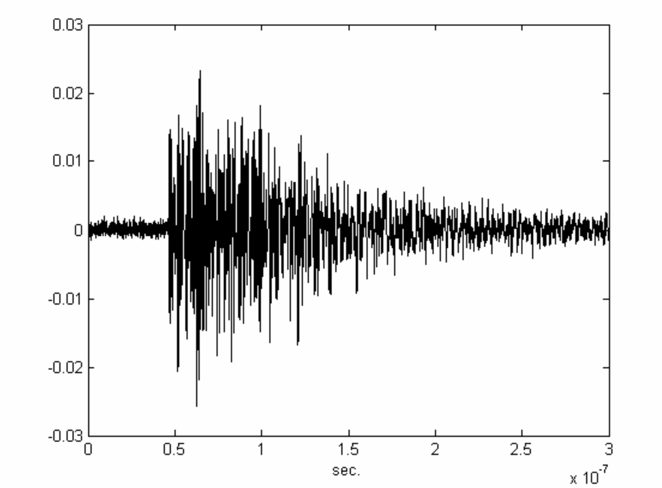

Figure 9. Measured data in a sample point in room M.

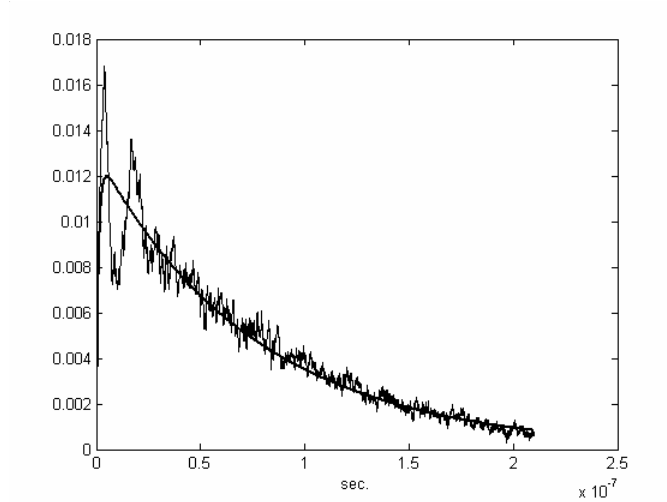

Figure 10. Averaged envelope of the 49 sample spatial points and fitted curve for room M.

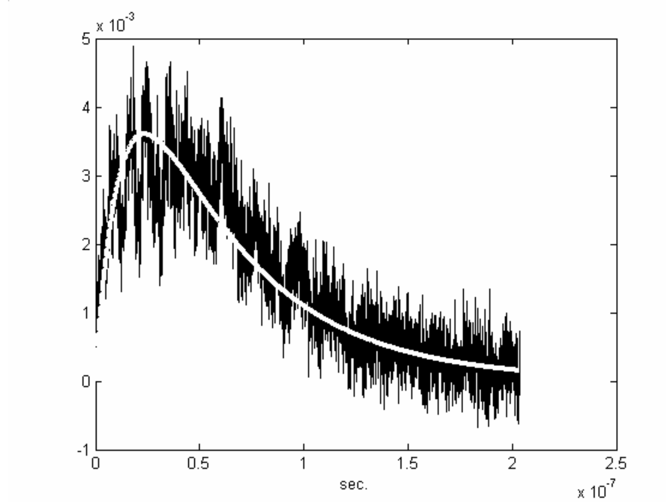

Figure 11. Averaged envelope of the 49 sample spatial points and fitted curve for room E.

partitions the building in to two parts, the room of interest and the rest of the building. Hence the model that we apply regression on has two exponential terms. We have used expectation maximization (EM) and least square estimation methods to minimize the L_2 distance between the averaged envelopes of measured data and the estimated curve. We also use Hilbert transform and analytical signal magnitude to calculate the envelope of the received signal as

$$P(t) = \left| r(t) + j\hat{r}(t) \right| \qquad (19)$$

where *r(t)* is the received UWB multipath signal and $\hat{r}(t)$ is its Hilbert transform of *r(t)*.

Figure 9 shows a sample measured data in room M. The averaged envelope of the 49 spatial sample points, together with the fitted curve is shown in Figure 10. It can be seen that this model is closely matched with the measured data at the tails. In the first part of the signal, due to lack of enough spatial measurement points, and non-uniform distribution of points in the room, the averaged envelope is affected by the line-of-sight signal front wave and the first few strong reflections. This is the interval that based on insufficient reflections, the signal behavior is not completely matched with diffusion phenomena. However after that, the signal behavior matches completely with diffusion model.

The estimated parameters of (16) for room M are:

$$K_{21} = -K_{22} = 0.0130$$

$$m_{21} = -0.1308 \times 10^8$$

$$m_{22} = -8.3058 \times 10^8$$

and the relative residual sum of squares defined as $\left\| \mathbf{Y} - \hat{\mathbf{Y}} \right\|^2 / \left\| \mathbf{Y} \right\|^2$ where $\mathbf{Y}$ is the vector of data and $\hat{\mathbf{Y}}$ is the vector of estimated points according to regressed curve is 2.8%. This value is mainly because of the first part of the signal. The results of similar analysis on data gathered in room E is shown in Figure 11. Numerical estimated parameters are

$K_{21} = -K_{22} = 0.0075$

$m_{21} = -0.19048 \times 10^8$

$m_{22} = -.78836 \times 10^8$

and the relative residual sum of squares is 6.8%. In both cases the error vector is orthogonal to the regressed vector. For room E, the diffusion model seems to be valid even for the first part of the signal which can be due to weak line of sight and specular reflection signals, and more diffused reflections before signal reaches to this room.

Room E has big windows toward outside which causes more escaped energy. It seems that this has caused smaller decay time constant, i.e., faster energy decay at room E. On the other hand the shield room R which is next to room M causes strong reflections at room M. Also regarding the rising part of the signal envelopes, we note that the position of rooms M and E relative to transmitter is similar to the position of rooms 5 and 1 in the 12 room model respectively (Except the direction of room M), i.e., the average energy first hits to the edge for room E and to the side for room M. The different time constants at the rising part of the average envelopes which can be seen for data in Figures 10 and 11 is predicted by model at Figure 6.

4. CONCLUSION

We presented a new model for UWB indoor propagation based on diffusion phenomena for different structures. It seems that this model can describe and predict some of the important propagation characteristics for indoor ultra-wide band communications systems. The model is fitted with data and shows reasonable behavior.

APPENDIX I

Elements of matrix **A** which is a 5 diagonal matrix for structure of Figure 3 are:

$(\mathbf{A})_{1,2} = X_{12}$ $(\mathbf{A})_{2,1} = X_{21}$

$(\mathbf{A})_{2,3} = X_{23}$ $(\mathbf{A})_{3,2} = X_{32}$

$(\mathbf{A})_{3,4} = X_{34}$ $(\mathbf{A})_{4,3} = X_{43}$

$(\mathbf{A})_{4,5} = 0$ $(\mathbf{A})_{5,4} = 0$

$(\mathbf{A})_{5,6} = X_{56}$ $(\mathbf{A})_{6,5} = X_{65}$

$(\mathbf{A})_{6,7} = X_{67}$ $(\mathbf{A})_{7,6} = X_{76}$

$(\mathbf{A})_{7,8} = X_{78}$ $(\mathbf{A})_{8,7} = X_{87}$

$(\mathbf{A})_{8,9} = 0$ $(\mathbf{A})_{9,8} = 0$

$(\mathbf{A})_{9,10} = X_{910}$ $(\mathbf{A})_{10,9} = X_{109}$

$(\mathbf{A})_{10,11} = X_{1011}$ $(\mathbf{A})_{11,10} = X_{1110}$

$(\mathbf{A})_{11,12} = X_{1112}$ $(\mathbf{A})_{12,11} = X_{1112}$

$(\mathbf{A})_{1,5} = X_{15}$ $(\mathbf{A})_{5,1} = X_{51}$

$(\mathbf{A})_{2,6} = X_{26}$ $(\mathbf{A})_{6,2} = X_{62}$

$(\mathbf{A})_{3,7} = X_{37}$ $(\mathbf{A})_{7,3} = X_{73}$

$(\mathbf{A})_{4,8} = X_{48}$ $(\mathbf{A})_{8,4} = X_{84}$

$(\mathbf{A})_{5,9} = X_{59}$ $(\mathbf{A})_{9,5} = X_{95}$

$(\mathbf{A})_{6,10} = X_{610}$ $(\mathbf{A})_{10,6} = X_{106}$

$(\mathbf{A})_{7,11} = X_{711}$ $(\mathbf{A})_{11,7} = X_{117}$

$(\mathbf{A})_{8,12} = X_{812}$ $(\mathbf{A})_{12,8} = X_{128}$

$(\mathbf{A})_{1,1} = -a_1 - X_{12} - X_{15} - X_{10}$

$(\mathbf{A})_{2,2} = -a_2 - X_{21} - X_{26} - X_{23} - X_{20}$

$(\mathbf{A})_{3,3} = -a_3 - X_{32} - X_{37} - X_{34} - X_{30}$

$(\mathbf{A})_{4,4} = -a_4 - X_{43} - X_{48} - X_{40}$

$(\mathbf{A})_{5,5} = -a_5 - X_{51} - X_{56} - X_{59} - X_{50}$

$(\mathbf{A})_{6,6} = -a_6 - X_{65} - X_{62} - X_{67} - X_{610}$

$(\mathbf{A})_{7,7} = -a_7 - X_{73} - X_{76} - X_{711} - X_{78}$

$(\mathbf{A})_{8,8} = -a_8 - X_{84} - X_{87} - X_{812} - X_{80}$

$(\mathbf{A})_{9,9} = -a_9 - X_{95} - X_{910} - X_{90}$

$(\mathbf{A})_{10,10} = -a_{10} - X_{106} - X_{109} - X_{1011} - X_{100}$

$(\mathbf{A})_{11,11} = -a_{11} - X_{117} - X_{1112} - X_{1110} - X_{110}$

$(\mathbf{A})_{12,12} = -a_{12} - X_{128} - X_{1211} - X_{120}$

where

$X_{i,j} = \ell_{i,j} D\rho / S_i$

$X_{i0} = \ell_{i0} D\rho_0 / S_i$

$a_i = \bar{a}_i / (S_i h)$

$\ell_{i,j}$ is the length of wall between room *i* and room *j*, and S_i is the area of room *i*.

Other elements of matrix **A** are equal to zero.

5. REFERENCES

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