MMSE Matching for Low Noise Amplifier

Jongrit Lerdworatawee, Won Namgoong
Department of Electrical Engineering
University of Southern California
lerdwora@usc.edu, namgoong@usc.edu

ABSTRACT

This paper presents a new theoretical framework for designing the minimum mean squared error (MMSE) matching network for LNA. The MMSE matching network is determined by reformulating the matching problem as a continuous-time estimation problem and solving for the Wiener-Hopf equation. Since the MMSE matching network is in general difficult to implement, a constrained MMSE matching network is also presented. The resulting noise factor of LNA is computed and compared against different matching strategies.

1. INTRODUCTION

In a communication receiver, the received signal from the antenna is interfaced to the low noise amplifier (LNA), whose purpose is to amplify the received signal with as little distortion as possible. This is generally achieved by designing the LNA to present a specific impedance to the antenna so that the power transfer is maximized and the amount of noise added minimized. Since these two matching objectives are generally contradictory, the LNA design requires a careful balance between them.

Traditionally, the LNA is designed to present an input impedance of 50 ohms to minimize the signal distortion resulting from the cable line. Such termination is useful only if the antenna presents a source impedance of 50 ohms across the entire frequency band of interest. This condition is generally not satisfied in many broadband systems, such as in the ultra-wideband radio. Furthermore, as the receiver becomes more integrated, the antenna is placed closer to the LNA, and the use of a cable line along with the 50-ohm requirement becomes unnecessary. Hence, this paper assumes an arbitrary source impedance.

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The design goal in the LNA matching network is to minimize the noise factor (NF) (or noise figure in dB), which is defined as the ratio of the signal-to-noise ratio (SNR) at the input of the LNA to the SNR at the output of the LNA. Compared to the matching problem for a narrowband signal, which is assumed to be a single tone [1], the more general matching problem for a broadband signal is much more difficult. There are two main broadband matching techniques. One is to maximize the total signal power transferred to the LNA output [3]. The other is to minimize the total noise added at the LNA output in the frequency band of interest [4]. Since these broadband matching techniques fail to account for the combined effects of both the signal transferred and the noise added, they generally do not yield the minimum NF.

In this paper, we propose to design the matching network that balances their combined effects to further reduce the NF. This is achieved by reformulating the matching problem as a continuous time estimation problem and solving for the Wiener-Hopf equation. Since the resulting MMSE matching network is in general difficult to realize in practice, we also present an approach for designing a MMSE matching network with a constrained structure.

The paper is organized as follows. In Section 2, the circuit and system model for LNA-Antenna is developed. In Section 3, the general solutions to the unconstrained and constrained LNA matching networks are derived. The performance results are presented in Section 4, and conclusions are drawn in Section 5.

2. CIRCUIT AND SYSTEM MODEL

Throughout this paper, capital letters are used to denote the Fourier transforms (e.g. $X(f)$) of signal voltages (or system responses) in the time domain, which are written in the corresponding lower case letters (e.g. $x(t)$). Sometimes the terms f and t are omitted for notational brevity unless needed for clarity.

2.1 Circuit model of LNA-Antenna

With no loss in generality, the antenna and the LNA are modeled as shown in Fig. 1. The antenna is represented

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as a voltage generator $v(t)$ with a source impedance Z_S. The LNA is assumed to be a common source MOS amplifier with a noiseless matching network interfaced between the antenna and the MOS transistor. There exists three noise sources: the thermal voltage noise from the antenna resistance $v_s(t)$, the MOS gate voltage noise $v_g(t)$, and the MOS drain current noise $i_d(t)$.

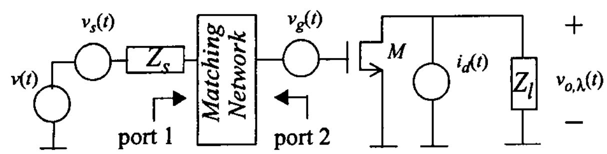

Figure 1 : General front-end receiver architecture.

The power spectral density (PSD) of the drain and gate noise are given by

$$S_{i_d}(f) = 4kT\gamma g_{do}, \tag{1}$$

$$S_{v_g}(f) = 4kT\delta r_g \tag{2}$$

where $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant, T is the absolute temperature, g_{do} is the zero-bias drain conductance, γ and δ are bias-dependent factors, and r_g is the gate resistance [5][6], which is given by

$$r_g = \frac{1}{5g_{do}} \tag{3}$$

In general, $v(t)$ and $v_s(t)$ are assumed independent to all other processes, but $v_g(t)$ is correlated to $i_d(t)$ with a correlation coefficient

$$|c| = \frac{S_{v_g i_d}(f)}{\sqrt{S_{v_g}(f)} \cdot \sqrt{S_{i_d}(f)}}, \tag{4}$$

where $c = 0.395j$ for long channel [5].

2.2 System model of LNA-Antenna

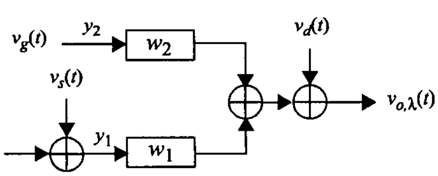

Figure 2 : System block diagram of LNA.

The system model of the LNA is shown in Fig. 2. It consists of two paths: the top represents the reflection path at port 2 (see Fig. 1) while the bottom is the transfer path from port 1 to port 2. We assume the matching network is a lossless passive two-port network, which is completely characterized using an open-circuit impedance matrix. Since there are two degrees of freedom in the choice of z-parameters, filters w_1 and w_2 can be designed to have arbitrary impulse responses. Without loss of generality, Z_l is assumed to be a resistor with resistance r_l. The output voltage corresponding to the drain current noise is $v_d(t)$, i.e., $v_d(t) = r_l i_d(t)$.

Signal $v_{o,\lambda}(t)$, which represents the LNA output voltage, should ideally be $v(t)$ scaled by the signal gain λ, i.e., $\lambda v(t)$. However, the presence of additive noise $v_g(t)$, $v_s(t)$, and $v_d(t)$ distorts the output signal. Hence, filters $W_1(f)$ and $W_2(f)$ are designed so that $v_{o,\lambda}(t)$ best approximates $\lambda v(t)$ in the mean squared sense. Filters $W_1(f)$ and $W_2(f)$ must exploit the correlation between $v_g(t)$ and $v_d(t)$ as well as the statistics of $v_s(t)$ and $v(t)$ to maximize the output SNR.

To compute the NF, the SNR at the input and output of the LNA needs to be defined. In a narrowband system, defining the SNR is straightforward since the signal is assumed to be a single tone. However, for a broadband system with a non-flat signal and noise spectrum, the definition of SNR is less obvious. In this paper, the SNR is defined as the matched filter bound (MFB) [7], which represents the maximum achievable performance in a data transmission system. The MFB is readily computed based on the signal and noise spectrum. The input SNR, SNR_i, is defined as

$$SNR_i = \int_{-\infty}^{\infty} \frac{S_v(f)}{S_{v_s}(f)} df \tag{5}$$

and the output SNR, SNR_o, as

$$SNR_o = \tag{6}$$

$$\int_{-\infty}^{\infty} \frac{S_v|W_1|^2}{S_{v_s}|W_1|^2 + S_{v_g}|W_2|^2 + S_{v_d} + 2Re\{W_2\}|c|\sqrt{S_{v_g}S_{v_d}}} df$$

3. MMSE SIGNAL ESTIMATION

3.1 Optimal matching

The output voltage $v_{o,\lambda}(t)$ is given by

$$v_{o,\lambda}(t) = v_d(t) + w_1(t) \otimes y_1(t) + w_2(t) \otimes y_2(t) \tag{7}$$

where $y_1(t) = v(t) + v_g(t)$ and $y_2(t) = v_d(t)$, as shown in Fig. 2. To achieve the MMSE estimation, we invoke the orthogonality principle, which states that the error must be uncorrelated with the observable space spanned by $y_1(t)$ and $y_2(t)$, i.e.,

$$E\left\{(\lambda v(t) - v_{o,\lambda}(t))\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}^H\right\} = 0 \tag{8}$$

Substituting (7) into (8), the causal system transfer functions can be solved by the Wiener-Hopf technique [8]:

$$\begin{aligned} W_1^*(f) &= \frac{1}{S_{y_1}^+(f)}\left\{\frac{\lambda S_{vy_1}(f)}{S_{y_1}^-(f)}\right\}_+ \\ W_2^*(f) &= -\frac{1}{S_{y_2}^+(f)}\left\{\frac{S_{v_d y_2}(f)}{S_{y_2}^-(f)}\right\}_+ \end{aligned} \tag{9}$$

where

$$\begin{aligned} S_{y_1}(f) &= S_v(f) + S_{v_s}(f) \\ S_{y_2}(f) &= S_{v_g}(f) \\ S_{vy_1}(f) &= S_v(f) \\ S_{v_d y_2}(f) &= r_l S_{i_d v_g}(f) \end{aligned} \tag{10}$$

In (9), $\{\ \}^+$ is the canonical spectral factor of $\{\ \}$ and $\{\ \}_+$ is the causal component of $\{\ \}$.

In the MMSE matching network, filter $W_1^*(f)$ optimally shapes $y_1(t)$ to minimize the mean squared error, while filter $W_2^*(f)$ optimally filters the gate noise to cancel the effect of the drain noise. The resultant output voltage is denoted as $v_{o,\lambda}^*(t)$. Simplifying the output SNR by substituting (4) and (9) into (6), the corresponding NF is given by

$$NF = \frac{\int_{-\infty}^{\infty}\frac{S_v(f)}{S_{v_s}(f)}df}{\int_{-\infty}^{\infty}\frac{S_v|W_1^*|^2}{S_{v_s}|W_1^*|^2 + (1-|c|^2)S_{v_d}}df}. \tag{11}$$

3.2 Constrained matching

Since $W_1^*(f)$ and $W_2^*(f)$ in (9) is in general too complex to realize in practice, the MMSE matching network can be designed with a constrained structure. For example, the matching network in Fig. 1 can be constrained to be a second order linear ladder filter. The constrained MMSE estimate is obtained conceptually as illustrated in Fig. 3. The plane represents the causal linear subspace spanned by observation y_1 and y_2, and the darkened area represents the constrained subspace. The constrained MMSE estimate, which is denoted as $\hat{v}_{o,\lambda}(t)$, is obtained by choosing $\hat{W}_1(f)$ and $\hat{W}_2(f)$ so that the L_2 distance $v_{o,\lambda}^*(t) - \hat{v}_{o,\lambda}(t)$ is minimized.

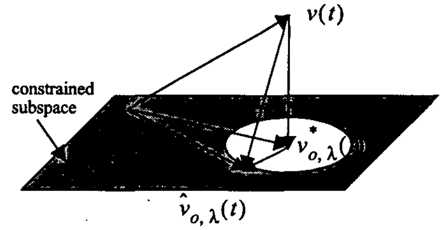

Figure 3 : Geometric diagram for MMSE estimation.

4. PERFORMANCE RESULTS

Throughout this section, the PSD of the signal is assumed to have the frequency response of a second order Butterworth bandpass filter with center frequency f_o and bandwidth f_B. The antenna source impedence Z_s is assumed to be second order and given by $Z_s = r_s + j\omega L_s + 1/(j\omega C_s)$. The constrained matching network is a second order latter network with capacitance C and inductance L. The frequency responses of the corresponding filters $\hat{W}_1(f)$ and $\hat{W}_2(f)$ in the s-domain are given by

$$\begin{aligned} \hat{W}_1(s) &= \frac{1}{Xs^2 + Ys + Z} \\ \hat{W}_2(s) &= \hat{W}_1(s) + \frac{C}{C + C_{gs}}[1 - \hat{W}_1(s)] \end{aligned}, \tag{12}$$

where

$$\begin{aligned} X &= (C + C_{gs})(L + L_s) \\ Y &= r_s(C + C_{gs}) \\ Z &= 2 + C_{gs}/C \end{aligned}. \tag{13}$$

Without loss of generality, the signal gain λ is set to be 10dB and the signal-to-antenna noise ratio, the signal-to-gate noise ratio, and the signal-to-drain noise ratio are each 0dB.

The frequency responses of $W_1^*(f)$ and $\hat{W}_1(f)$ are shown in Fig. 4. The frequency and the magnitude are normalized to the center frequency f_o and the signal gain λ, respectively. Filters $W_1^*(f)$ and $\hat{W}_1(f)$ are derived numerically from (9) and (12).

Increasing f_o/f_B from 1 to 10, as indicated by the arrow in Fig. 4, filter $W^*{}_1(f)$ becomes a bandpass filter with the passband gradually narrowing. This occurs because filter

$W^*_1(f)$ emphasizes the frequency bands with high SNR in favor of those with low SNR to increase the overall SNR.

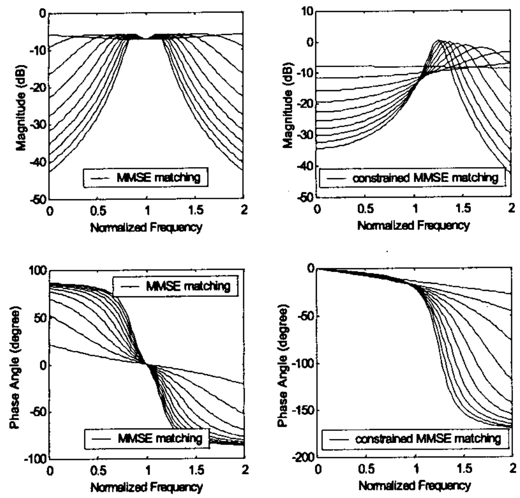

Figure 4 : Normalized frequency response of MMSE and constrained MMSE filters as f_o/f_B is varied from 1 to 10.

The constrained MMSE filter $\hat{W}_1(f)$ is a lowpass filter with peaking at the resonant frequency as shown in Fig. 4. The resonant frequency is greater than f_o, because a second order lowpass filter best approximates a wideband bandpass filter when the resonant frequency is set to be higher than f_o. As f_o/f_B increases, the amount of peaking increases and the resonant frequency shifts closer to f_o.

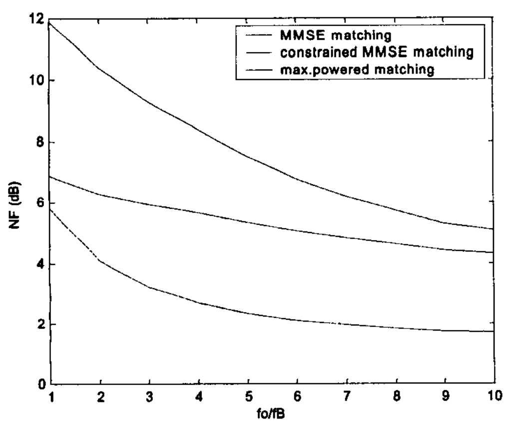

Figure 5 : The NF for MMSE, constrained MMSE, and maximum power matching filters as f_o/f_B is varied from 1 to 10.

Fig. 5 compares the NF for MMSE, constrained MMSE, and maximum power matched networks as a function f_o/f_B. The maximum power matched network is a second order latter network that is designed to maximize the signal power delivered to the output load. The second order latter network is chosen so that a fair comparison can be made with the constrained second order MMSE matched network. The NF of the MMSE matched network serves as a bound and basis for comparison. The MMSE NF decreases with increasing f_o/f_B, and it can be shown using (11) to converge to 1.9dB. The NF of power matched network is high for broadband signals, but it approaches the NF of the constrained MMSE network as the signal bandwidth decreases. Hence, the maximum power matching network is effective for narrowband systems but not for broadband systems.

5. CONCLUSIONS

A generalized approach for designing the matching network of the LNA is presented by reformulating the matching problem in the estimation framework. The SNR was computed using the matched filter bound. The MMSE and the constrained MMSE matching networks were derived. Their performance were compared with a second order latter network that was designed to maximize the power transferred to the output of the LNA. The power matched network performs poorly for broadband systems, but it approaches the constrained MMSE performance as signal bandwidth is reduced.

REFERENCES

[1] D. K. Shaeffer, T. H. Lee, "A 1.5-V, 1.5-Hz CMOS low noise amplifier," IEEE J. Solid-State Circuits, vol. 32, pp. 745-759, May. 1997.

[2] R. S. Tucker, "Low-Noise Design of Microwave Transistor Amplifiers," IEEE Trans. Microwave Theory Tech., vol. MTT-23, pp. 697-700, Aug. 1975.

[3] J. Carlin, B. S. Tarman, "The Double Matching Problem: Analytic and Real Frequency Solutions," IEEE Trans. Circuits Syst., vol. cas-30, pp. 15-28, January 1983.

[4] H. J. Carlin, J. J. Komiak, "A New Method of Broad-Band Equalization Appiled to Microwave Amplifiers," IEEE Trans. Microwave Theory Tech., vol. MTT-27, pp. 93-99, February 1979.

[5] A. van der Ziel, Noise in Solid State Devices and Circuits, New York: Wiley, 1986.

[6] A. Abidi, "High Frequency Noise Measurements on FET's with small dimensions," IEEE Trans. Electron. Devices, vol. ED-33, pp.1801-1805, November, 1986.

[7] J. Cioffi, "EE379A Course Notes," Stanford University.

[8] T. Kailath, A. H. Sayed, B. Hassibi, Linear Estimation, New Jersey: Prentice-Hall, 1999.