HYBRID FIXED-DWELL-TIME SEARCH TECHNIQUES FOR RAPID ACQUISITION OF ULTRA-WIDEBAND SIGNALS

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Abstract—**Spread spectrum systems, such as Ultra-Wideband impulse radio or Wideband-CDMA, are often required to work in dense multipath situations due to the frequency selective nature of the channels involved. Recent studies have concluded that the process of acquiring these wideband signals can be expedited by the presence of multipath. An acquisition analysis is presented here using a generalized signal flow graph approach which allows for an arbitrary search pattern and a multiple detection scenario generated by the multipath. A linear, or consecutive, search of the uncertainty region is shown to yield longer average search times when compared with a more efficient nonconsecutive serial search, namely the so called *bit reversal search*. Consideration is also given to a hybrid parallel/serial search scheme, arising from the use of multiple correlators in the receiver. It is shown that the bit reversal search leads to a near optimal hybrid search technique.**

I. Introduction

Ultra-Wideband (UWB) signals are those with large fractional bandwidths and are used in communications systems where the wide bandwidth yields an advantage over a more narrowband signal. For example, the narrow time resolution of the UWB signals involved allows for precise location tracking. In addition, systems required to operate in dense multipath channels, such as indoor or urban environments, are well suited for UWB signals. This is because the individual multipath components are resolvable and do not lead to destructive interference, or nulls, as with narrowband signals. The UWB system and multipath channel considered here are discussed in section II.

The paper investigates the acquisition of time-hopped UWB signals in a dense multipath channel. This is an extension of [1] which investigated an uncoded UWB signal and a different detection scheme than is considered here. The acquisition approach chosen here is based upon a serial search in time, resolving both the time-hopping code and frame boundaries, in a manner analogous to CDMA code acquisition. A common method used to investigate the acquisition process is through the use of signal flow graph techniques. Fixed-dwell-time, serial search techniques can be modeled nicely using this approach. While much of the initial studies in CDMA code acquisition neglected multipath [2], much effort has been put forth fairly recently into investigating acquisition in multipath ([3], [4], [5] and the references contained therein). A generalized signal flow graph and the accompanying moment generating function are presented in section III which encompasses much of the aforementioned CDMA code acquisition studies, as well as the UWB acquisition problem studied in section IV.

Other non-serial or non-fixed-dwell-time techniques exist and tend to offer slightly better performance when compared to fixed-dwell-time serial search techniques. The cost of increased performance, however, is increased complexity in the receiver architecture [5] [6]. Obviously, a fully parallel search would be preferable but the associated complexity is often prohibitive. One trade-off is a hybrid search approach where multiple locations, but not all, are searched simultaneously. This will be accomplished through the use of multiple correlators, already present as taps of a RAKE receiver. A near optimal hybrid search scheme is discussed in section V.

II. System Overview

The received pulse shape considered here is a 2nd derivative Gaussian pulse:

$$p(t) = \sqrt{\frac{4}{3\sigma\sqrt{\pi}}}\left(1 - \left(\frac{t}{\sigma}\right)^2\right)\exp\left(-\frac{1}{2}\left(\frac{t}{\sigma}\right)^2\right) \quad (1)$$

This pulse has unit energy and the scale factor, σ, determines the width of the pulse in time. Here σ will be set equal to $(2\sqrt{\pi})^{-1} \cdot 0.95$ nsec.

The specular multipath channel assumed here has the impulse response:

$$h(t) = \sum_{l=0}^{L_p-1} a_l \delta\left(t - \tau_l\right) \quad (2)$$

The amplitude coefficients and time delays (which are ordered so that $\tau_0 < \tau_1 < \cdots$) considered here will be fixed based upon a single realization of a UWB channel, namely a measured UWB signal in an office environment as was done in [1]. This channel is comparable to a single realization using the worst case multipath channel (Channel Model 4) in

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the IEEE 802.15.SG3a channel model final report [7]. The received signal, with no data modulation, at the output of the multipath channel is

$$r(t)=\sqrt{E_p}\sum_{n}\sum_{l=0}^{L_p-1} a_l\cdot p(t-nT_f-c_nT_c-\tau_l)+n(t) \quad (3)$$

The additive noise, $n(t)$, is a mean zero Gaussian random process with autocorrelation function $N_0\delta(t_1-t_2)$. The time hopping code, c_n, is a length N_c sequence of nonnegative integers and T_c is the code chip time. The frame time, T_f, is assumed to be an integer multiple of the code chip time, i.e., $T_f = N_f T_c$. The receiver and transmitter frame times are assumed to be equal. The distance between the transmitter and receiver is not known, however, giving rise to a uniformly random direct path arrival time over the period of the received signal. This implies that the direct path delay, τ_0, is uniform on $[0, N_cT_f)$.

A bank of M correlators, with $M \leq L_p$, is present in the receiver and the received signal in (3) acts as the input to each of these correlators. The template waveform of the m^{th} correlator is:

$$l^{(m)}(t)=\sum_{j}\sum_{k=0}^{N_c-1} p(t-\alpha_{j,k}^{(m)}-jN_cT_f) \quad (4)$$

The time offset for the m^{th} correlator template, which can vary over a code period, is set by the term $\alpha_{j,k}^{(m)}$ which is:

$$\alpha_{j,k}^{(m)}=\Big(\big(kN_f+c_k+k_{\beta,j}^{(m)}\big) \bmod N_fN_c\Big)T_c+\beta_{r,j}^{(m)} \quad (5)$$

This term accounts for the proper code phase as well as the proper timing offset within each frame for an arbitrary time shift of $\beta_j^{(m)} = k_{\beta,j}^{(m)}T_c + \beta_{r,j}^{(m)}$ which varies over $[0, N_cT_f)$. The integer term $k_{\beta,j}^{(m)}$ is a nonnegative integer and the remainder term $\beta_{r,j}^{(m)}$ varies over $[0, T_c)$. The frame time will be divided into N bins so that $\beta_j^{(m)}$ can be selected from a set of $N \cdot N_c$ time offsets.

Since the codes considered here are short codes, the correlator dwell time is assumed to be one code period in length. The resulting output for the m^{th} correlator is $z_j^{(m)} = s_j^{(m)} + n_j^{(m)}$ where the correlator noise sequence, $n_j^{(m)}$, is an i.i.d. sequence of mean zero, variance N_cN_0 Gaussian random variables. The correlator output mean is

$$s_j^{(m)}=\sqrt{E_p}\sum_{n=(j-1)N_c}^{(j+1)N_c-1}\sum_{k=0}^{N_c-1}\sum_{l=0}^{L_p-1} a_l\cdot\gamma\big(\tau_l-\alpha_{j,k}^{(m)} +c_{(n \bmod N_c)}T_c-(jN_c-n)T_f\big) \quad (6)$$

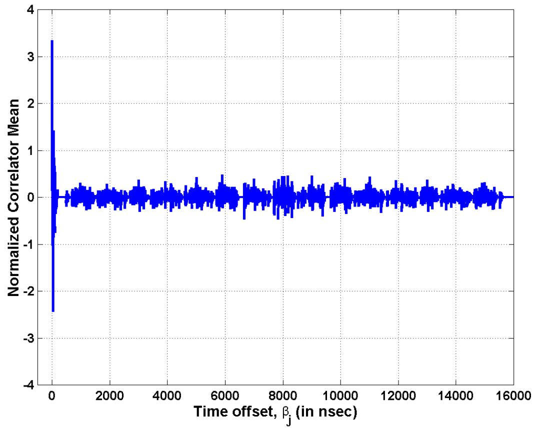

Fig. 1. Normalized correlator mean for $\tau_0 = 0$ nsec

The pulse autocorrelation function of (1) is given as:

$$\gamma(\tau)=\left(1-\left(\tfrac{\tau}{\sigma}\right)^2+\tfrac{1}{12}\cdot\left(\tfrac{\tau}{\sigma}\right)^4\right)\exp\left(-\tfrac{1}{4}\cdot\left(\tfrac{\tau}{\sigma}\right)^2\right) \quad (7)$$

The correlation mean, normalized by $\sqrt{E_p}$ and computed at each of the bin centers for $\beta_j = j \cdot T_f/N$ with $j = 0, 1, \cdots, N \cdot N_c - 1$, is shown in Figure 1 for $\tau_0 = 0$ nsec, $T_c = 10$ nsec, $T_f = 1000$ nsec, $N_f = 100$, $N_c = 16$, $N = 256$, and a code sequence, $\{c_n\}_{n=0}^{N_c-1} = \{0, 13, 52, 43, 61, 30, 26, 48, 21, 21, 48, 26, 30, 61, 43, 52\}$. This code sequence, which is based upon techniques found in [8], is a sequence of integers between 0 and 70 where the upper limit of 70 is set to provide some guard time in each frame. Although a different modulation scheme is used, a method of code design for rapid acquisition is studied in [9].

III. Generalized Acquisition Analysis

The generalized signal flow graph used to analyze the UWB acquisition problem is shown in Figure 2. This is an extension of the basic signal flow graph of [2] where only a single bin led to the acquisition state and only consecutive searches were considered. The signal flow graph of Figure 2 overcomes these two important issues and allows for an arbitrary search permutation, $\epsilon(n)$, as well as an arbitraty detection scenario. Here, the N_s states of the flow graph are labeled $\epsilon(n)$ which represents a permutation of the integers $0, 1, \ldots, N_s - 1$. The initial distribution of the states is given by $\pi_{\epsilon(n)}$. The generating function into the acquisition state can be found by various flow graph loop reduction techniques, Mason's gain formula, etc., and is given below, where $\oplus$ represents modulo N_s addition and $\prod_{j=0}^{-1}(\cdot)$ is defined to be unity:
To demonstrate the applicability of (8), consider the example scenario with a linear search pattern, i.e.,

$$P_{ACQ}(z) = \frac{\sum_{k=0}^{N_s-1} \pi_{\epsilon(k)} \sum_{i=0}^{N_s-1} H_{\epsilon(i\oplus k)}(z) \prod_{j=0}^{i-1} G_{\epsilon(j\oplus k)}(z)}{1 - \prod_{i=0}^{N_s-1} G_{\epsilon(i)}(z)} \quad (8)$$

$\epsilon(j) = j$, and path gains leading to the acquisition state which are equal to zero except for $H_{N_s-1}(z) = H_D(z)$ while the path gains between states are set equal to $G_{N_s-1}(z) = H_M(z)$ and $G_n(z) = H_F(z)$ for $n = 0, 1, \ldots, N_s - 2$. This leads to the generating function in equation (4) of [2], Part I:

$$P_{ACQ}^{(LINEAR,1)}(z) = \frac{H_D(z)}{1 - H_M(z) H_F^{N_s-1}(z)} \sum_{i=0}^{N_s-1} \pi_i H_F^{N_s-i-1}(z) \quad (9)$$

The superscript on $P_{ACQ}(z)$ represents the search type and the number of detection states. As a second example, consider a linear search pattern, $\epsilon(j) = j$, and a multiple detection scenario where L consecutive states, $0, 1, \cdots, L-1$, in the flow graph terminate the search. This implies that the path gains are $H_n(z) = H_D(z)$ for $n = 0, 1, \cdots, L-1$ and zero for other n while $G_n(z) = H_M(z)$ for $n = 0, 1, \cdots, L-1$ and $G_n(z) = H_F(z)$ for all other n. If the prior initial distribution of the states is uniform, $\pi_n = 1/N_s$ for all n, then this leads to the generating function in equation (3) of [4]:

$$P_{ACQ}^{(LINEAR,L)}(z) = \frac{1}{N_s} \cdot \frac{H_D(z)}{1 - H_M^L(z) H_F^{N_s-L}(z)} \Big[\sum_{j=0}^{L-1} H_M^j(z) \sum_{i=0}^{N-L} H_F^i(z) + \sum_{i=1}^{L} \Big[L - i + (i-1) H_F^{N_s-L}(z) \Big] \cdot H_M^{i-1}(z) \Big] \quad (10)$$

The generating function found in [3] can also be found using the generalized flow graph of Figure 2. The search permutation found in that particular reference, here termed the *Look and Jump* search, is discussed in more detail below.

For the generating function of (8) the mean acquisition time can be found as

$$E(T_{ACQ}) = \frac{\mathrm{d}}{\mathrm{d}z} P_{ACQ}(z) \Big|_{z=1} \quad (11)$$
$$= \frac{Num' \cdot Den - Num \cdot Den'}{Den^2}$$

where Num and Den are the numerator and denominator of (8), respectively, evaluated at $z = 1$. Num' is the derivative of the numerator evaluated at $z = 1$:

$$Num' = \sum_{k=0}^{N_s-1} \pi_{\epsilon(k)} \sum_{i=0}^{N_s-1} \Big(\prod_{j=0}^{i-1} G_{\epsilon(j\oplus k)}(1) \Big) \cdot$$

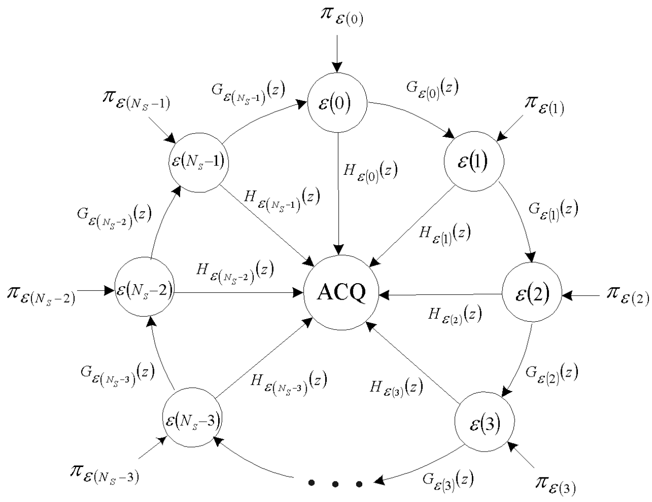

Fig. 2. Generalized acquisition signal flow graph

$$\Big[H'_{\epsilon(i\oplus k)}(1) + H_{\epsilon(i\oplus k)}(1) \sum_{l=0}^{i-1} \frac{G'_{\epsilon(l\oplus k)}(1)}{G_{\epsilon(l\oplus k)}(1)} \Big] \quad (12)$$

Here the summation $\sum_{l=0}^{-1}(\cdot)$ is defined as zero. Den' is the derivative of the denominator evaluated at $z = 1$:

$$Den' = -\sum_{i=0}^{N_s-1} \frac{G'_{\epsilon(i)}(1)}{G_{\epsilon(i)}(1)} \cdot \prod_{j=0}^{N_s-1} G_{\epsilon(j)}(1) \quad (13)$$

Figure 3 gives an example of $E(T_{ACQ})$ where three different search patterns are considered for K consecutive detection states, $N_s = 16$, and the path gains as follows:

$$H_{\epsilon(i)}(z) = \begin{cases} P_D z & \text{if } i \in \mathcal{I} \\ 0 & \text{else} \end{cases} \quad (14)$$

and

$$G_{\epsilon(i)}(z) = \begin{cases} (1-P_D)z & \text{if } i \in \mathcal{I} \\ (1-P_{FA})z + P_{FA} z^{J+1} & \text{else} \end{cases} \quad (15)$$

The index set, $\mathcal{I}$, in the above expressions is obtained based upon the specific search permutation considered. It represents those indices i of $\epsilon(i)$ that lead to the acquisition state. Here these states are assumed to be $0, 1, \cdots, K-1$ so that the size of the set $\mathcal{I}$ is K. Since $\epsilon(j)$ is simply a permutation of the integers, its inverse $\epsilon^{-1}(j)$ exists and can be used to produce the index set. That is $\mathcal{I} = \{\epsilon^{-1}(0), \epsilon^{-1}(1), \cdots, \epsilon^{-1}(K-1)\}$ for the example currently being considered.

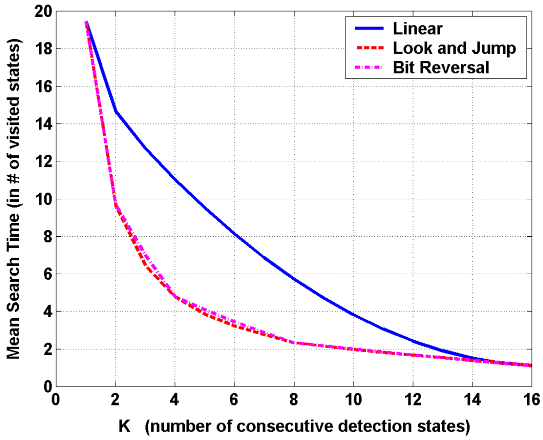

Fig. 3. Mean acquisition time for $P_D = 0.9$, $P_{FA} = 0.1$, $N_s = 16$, and a false alarm penalty time of $J = 10$

The three search permutations shown in Figure 3 are the *linear*, *look and jump*, and *bit reversal* searches. The linear search is simply a consecutive search, $\epsilon(j) = j$ for $j = 0, 1, \cdots, N_s - 1$. The index set for this linear case is $\mathcal{I} = \{0, 1, \cdots, K-1\}$. The look-and-jump search, as the name suggests, is the permutation $0, K, 2K, \cdots, 1, K+1, 2K+1, \cdots$ and the index set is computed as mentioned above. For example, when $K = 3$ the index set is seen to be $\mathcal{I} = \{0, 6, 11\}$ for $N_s = 16$. The look-and-jump search is actually the optimum serial search permutation for K consecutive detection states. The problem with this type of search, however, is that K is related to the number of detectable paths in the multipath channel and this quantity may not be known to the receiver.

In lieu of this fact, the bit reversal search is introduced. This specific permutation, as the name suggests, is obtained from a bit reversal of the binary representation of the integers $0, 1, \cdots, N_s - 1$, assuming N_s is a power of 2. For the example considered here with $N_s = 16$, the bit reversal search pattern is (in binary) $0000, 1000, 0100, 1100, \cdots, 0111, 1111$ or (in decimal) $0, 8, 4, 12, \cdots, 7, 15$. The index set for the bit reversal search is the first K elements of the bit reversal search permutation, namely $\mathcal{I} = \{0, 8, 4, 12, \cdots\}$, since this permutation is its own inverse. As can be seen in Figure 3, the bit reversal search and the look and jump search yield identical mean acquisition times when K is a power of 2 and the two search schemes yield very similar acquisition times for all other values of K. *Thus, when K is not known the optimum search permutation becomes the bit reversal search.*

IV. UWB Acquisition Analysis

The results of the previous section are now applied to study UWB acquisition. As in section II, the code length is set to $N_c = 16$ and each frame is broken into $N = 256$ bins, thus there are $N_s = N \cdot N_c = 4096$ states. The frame time and code chip time are again $T_f = 1000$ nsec and $T_c = 10$ nsec, respectively. The code sequence and multipath channel are also the same as in section II. Thus the normalized correlator mean of (6) and shown in Figure 1 is now used. The acquisition process considered here uses a single correlator and detection occurs when the correlator output crosses a predetermined threshold. This is extended in the next section to use multiple correlators for the hybrid search.

The correlator output, $z_j = s_j + n_j$, is Gaussian with mean s_j given in (6) and variance $N_c N_0$. For a detection threshold of $\Upsilon \cdot \sqrt{E_p}$, where Υ is the normalized detection threshold, the probably of exceeding the threshold for the j^{th} code correlator output is:

$$P_{\epsilon(j)} = \Pr(|z_j| > \Upsilon \cdot \sqrt{E_p}) = Q\Big(\sqrt{\frac{E_p}{N_c N_0}} \cdot (\Upsilon + \frac{s_j}{\sqrt{E_p}})\Big) + Q\Big(\sqrt{\frac{E_p}{N_c N_0}} \cdot (\Upsilon - \frac{s_j}{\sqrt{E_p}})\Big) \tag{16}$$

The quantity s_j is given in (6) and the function $Q(x)$ is the Gaussian integral function. The permutation $\epsilon(j)$ of the integers $0, 1, \cdots, N \cdot N_c - 1$ is related to the search variable β_j as:

$$\beta_j = \epsilon(j) \cdot T_f / N \tag{17}$$

The initial distribution of the states are set by the uniform nature of the first multipath arrival, τ_0, so that $\pi_{\epsilon(j)} = 1/(N \cdot N_c)$. The path gains of the signal flow graph in Figure 2 are

$$H_{\epsilon(j)}(z) = \begin{cases} P_{\epsilon(j)} z & \text{if } j \in \mathcal{I} \\ 0 & \text{else} \end{cases} \tag{18}$$

and

$$G_{\epsilon(j)}(z) = \begin{cases} (1 - P_{\epsilon(j)}) z & \text{if } j \in \mathcal{I} \\ (1-P_{\epsilon(j)})z + P_{\epsilon(j)} z^{J+1} & \text{else} \end{cases} \tag{19}$$

The index set $\mathcal{I}$ is selected based upon the number of detectable paths in the multipath channel and for simplicity was selected as the first $K = 50$ bins. The

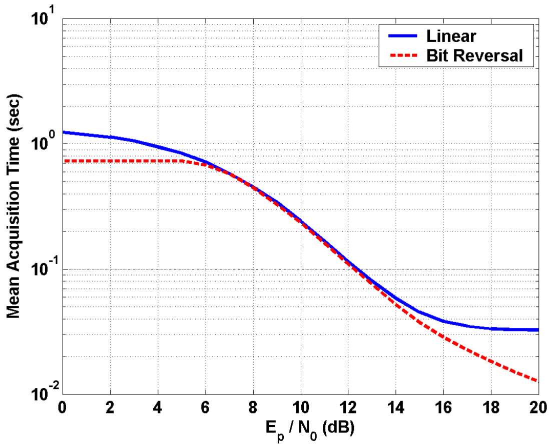

Fig. 4. Mean UWB acquisition time for a single correlator, $T_f = 1000$ nsec, $T_c = 10$ nsec, $N = 256$, $N_c = 16$, $J = 1000$, and optimized threshold, Υ.

mean acquisition time of (12) is shown in Figure 4, where the normalized detection threshold, Υ, was optimized for minimum mean acquisition time at each E_p/N_0. J is the false alarm penalty time.

V. Hybrid Parallel/Serial Search

As was mentioned earlier, a fully parallel search would minimize the mean acquisition time. However, the associated complexity is often too prohibitive. For the example given in the last section $N \cdot N_c = 4096$ correlators would be required, and this is for a very short code length of only $N_c = 16$. A hybrid search offers a reasonable trade-off between receiver complexity and acquisition time. The multiple correlators present in the receiver can each search independent code phases, but in what fashion so as to minimize the mean acquisition time?

A strong candidate for near optimal performance is based upon the bit reversal search. Assuming that there are M correlators and the bit reversal search pattern is listed as $\beta_0, \beta_1, \beta_2, \cdots, \beta_{N \cdot N_c - 1}$, then the first correlator would search as per $\beta_j^{(0)} = \{\beta_0, \beta_M, \beta_{2M}, \cdots\}$, the second correlator would search as per $\beta_j^{(1)} = \{\beta_1, \beta_{M+1}, \beta_{2M+1}, \cdots\}$, etc. Assuming that M and $N \cdot N_c$ are both powers of 2, it is seen that M divides $N \cdot N_c$ into smaller regions of N_h bins, where N_h is also power of 2. Furthermore, each correlator will perform a bit reversal search over this smaller region of N_h bins. Figure 5 shows an example of this phenomenon for $N_s = 16$ search bins and $M = 4$ correlators. Based upon computer simulations it was seen that the hybrid bit reversal search and the hybrid look and jump search had very similar performance. Both of these hybrid searches performed much better than the hybrid linear search.

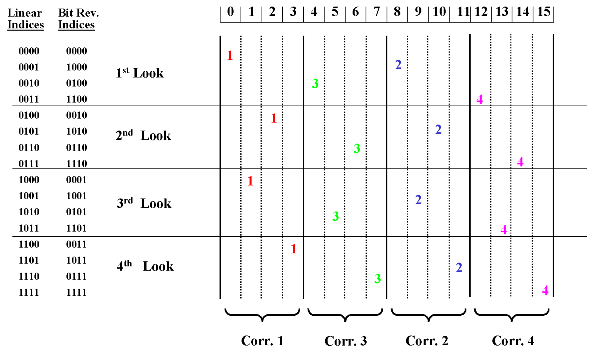

Fig. 5. Hybrid bit reversal search example for $N_s = 16$ bins and $M = 4$ correlators

VI. Concluding Remarks

In this paper a generalized code acquisition signal flow graph and moment generating function were presented. The mean acquisition time derived from this moment generating function for a simple UWB example was examined. Finally a hybrid search scheme was examined which provides near optimum acquisition performance in terms of minimizing the mean acquisition time. This search is the hybrid bit reversal search.

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