On the Power Spectral Density of Wireless Multiple-Access UWB Impulse Radio under Realistic Propagation Conditions *

Ali Taha †, Keith M. Chugg ‡

†Pulse Link Inc., 9155 Brown Deer Road, Suite 8, San Diego, CA 92121

‡Dept. of Electrical Engineering-Systems,University of Southern California,Los Angeles, CA 90089

ataha@pulselink.net , chugg@usc.edu

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***Abstract*-The multiple-access Power Spectral Density (PSD) under realistic multipath channels extracted from experimental measurements, for both time-hopping pulse position and direct-sequence bit flipping data modulation, is analytically derived and closed form solutions are given. Using a proper pulse shape compatible with FCC spectral mask, examples of PSD's are given. The magnitude of power spectral density fluctuations in realistic environments is investigated and compared with line of sight measurements in an anechoic chamber, which may be used for FCC compliance tests.**

I. INTRODUCTION

In this paper, the Power Spectral Density (PSD) of multiple access digital impulse radio utilizing direct-sequence or time-hopping spreading technique with bit-flipping or pulse position data modulation is investigated. According to Federal Communication Commission's Report and Order on Ultra Wide Bandwidth (UWB) systems, the transmitted power radiated by such devices should be restricted within spectral masks associated to different UWB applications. For an indoor wireless multiuser scenario the aggregate interference should comply with these regulations in order to limit the spurious emission level caused by such systems. This motivates investigating the power spectral density of multiple access UWB impulse radio under realistic propagation conditions for system design and power control issues based on estimated power level fluctuations in typical UWB applications.

This paper is organized as follows. In Section II, power spectral density of a finite power random process is defined. This definition is then applied to direct-sequence and time-hopping multiple-access UWB systems in Sections III and IV, respectively. A pulse shape designed to meet FCC spectral mask is given in Section V, which will be used to investigate the effects of multipath on UWB PSD in Section VI. Concluding remarks are followed in Section VII.

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II. POWER SPECTRAL DENSITY OF A FINITE POWER RANDOM SIGNAL

The following approach is taken in computing the power spectral density, $S(f)$, of a finite power signal, $s(t)$, throughout this paper.

$$S(f) = \lim_{T\to\infty} \frac{\mathrm{E}\{|\mathrm{F}\{s_{2T}(t)\}|^2\}}{2T} \tag{1}$$

where $\mathrm{F}\{s(t)\}$ represents the Fourier Transform of $s_{2T}(t)$ and

$$s_{2T}(t) = s(t), t \in [-T, T] \tag{2}$$

and $s_{2T}(t) = 0$ outside the above interval. This approach can be used regardless of a stochastic process being wide sense stationary or not. Albeit, the power spectral density of a wide sense stationary process computed using (1) is equivalent to the Fourier transform of its autocorrelation function. The method used in [3] computes the power spectral density of a time-hopping system based on computing the autocorrelation function of a single user scenario under ideal propagation conditions. In this paper, we derive closed form formulas for both time-hopping and direct-sequence multiple-access power spectral densities. The effects of UWB multipath on the power spectral density is also investigated.

III. DIRECT-SEQUENCE WITH BIT-FLIPPING MODULATION

The wireless multiple access digital impulse radio signal under multipath propagation conditions utilizing direct-sequence spreading with bit-flipping data modulation can be represented as

$$s(t)=\sum_{k=1}^{N_u}\sum_{m=0}^{L^k-1} g_m^k f^k(t-\tau_m^k) \tag{3}$$

where

$$f^k(t)=\sum_l a_l^k d^k_{\lfloor \frac{l}{N_s}\rfloor} w(t-lT_f) \tag{4}$$

is the transmitted signal of User k with no asynchronous delay. The mth path's delay of the kth user is represented by $\tau_m^k = \tau_0^k + \lambda_m^k$, where λ_m^k is the excess delay of the kth user's mth path and τ_0^k is his/her corresponding asynchronous delay. The amplitude of the mth path of the kth user is denoted by g_m^k. We assume there are N_u active users simultaneously present and that User k has a multipath channel with L^k taps. Each transmitted data is repeated N_s times, therefore, the kth user's data at the lth frame is shown as $d_{\lfloor \frac{l}{N_s}\rfloor}$. The transmitted data $d_i^k \in \{+1,-1\}$ and there is an additional direct-sequence spreading modulation, a_l^k, which represents the polarity of user k's spreading sequence at the lth frame. The spreading sequences are assumed random. At the beginning of each frame duration, T_f, one pulse, $w(t)$, is transmitted.

The first step toward power spectral density computation is to take the Fourier Transform of $s_{2nT_f}(t)$ using (3).

$$\mathrm{F}\{s_{2nT_f}(t)\}=\sum_{k=1}^{N_u}\sum_{m=0}^{L^k-1} g_m^k F^k_{2nT_f}(f) e^{-j2\pi f\tau_m^k} \tag{5}$$

where $F^k_{2nT_f}(f)$ is the Fourier Transform of $f^k_{2nT_f}(t)$. But using (4),

$$F^k_{2nT_f}(f)=\sum_{l=-n}^{n-1} a_l^k d^k_{\lfloor \frac{l}{N_s}\rfloor} W(f) e^{-j2\pi f lT_f} \tag{6}$$

where $W(f)$ represents the Fourier Transform of $w(t)$. Note that the support of $w(t)$ is much less than one frame duration. To compute $|\mathrm{F}\{s_{2nT_f}(t)\}|^2$, we have

$$|\mathrm{F}\{s_{2nT_f}(t)\}|^2=\sum_{k=1}^{N_u}\sum_{k'=1}^{N_u}\sum_{m=0}^{L^k-1}\sum_{m'=0}^{L^{k'}-1} g_m^k g_{m'}^{k'} F^k_{2nT_f}(f) F^{k'}_{2nT_f}(f)^* e^{-j2\pi f\tau_m^k} e^{+j2\pi f\tau_{m'}^{k'}} \tag{7}$$

Using (6)

$$F^k_{2nT_f}(f)F^{k'}_{2nT_f}(f)^* = \sum_{l=-n}^{n-1}\sum_{l'=-n}^{n-1} a_l^k a_{l'}^{k'} d^k_{\lfloor \frac{l}{N_s}\rfloor} d^{k'}_{\lfloor \frac{l'}{N_s}\rfloor} |W(f)|^2 e^{-j2\pi f(l-l')T_f} \tag{8}$$

Therefore, $\mathrm{E}\{|\mathrm{F}\{s_{2nT_f}(t)\}|^2\}$ can be computed as

$$\mathrm{E}\{|\mathrm{F}\{s_{2nT_f}(t)\}|^2\}=\sum_{k=1}^{N_u}\sum_{k'=1}^{N_u}\sum_{m=0}^{L^k-1}\sum_{m'=0}^{L^{k'}-1} g_m^k g_{m'}^{k'} \sum_{l=-n}^{n-1}\sum_{l'=-n}^{n-1} \mathrm{E}\{X(k,k',m,m',l,l')\}|W(f)|^2 \tag{9}$$

where

$$X(k,k',m,m',l,l')=a_l^k a_{l'}^{k'} d^k_{\lfloor \frac{l}{N_s}\rfloor} d^{k'}_{\lfloor \frac{l'}{N_s}\rfloor} e^{-j2\pi f(l-l')T_f} e^{-j2\pi f(\tau_m^k-\tau_{m'}^{k'})} \tag{10}$$

Since different users have independent random direct sequences, a_l^k is independent from $a_{l'}^{k'}$ for $k \neq k'$, and even for the same user, a_l^k is independent from $a_{l'}^k$ when $l \neq l'$. From this observation and from the fact that $\Pr.\{a_l^k=+1\}=\Pr.\{a_l^k=-1\}=\frac{1}{2}$, we conclude that $\mathrm{E}\{X(k,k',m,m',l,l')\}=0$ when $k \neq k'$ and/or $l \neq l'$. Hence,

$$\mathrm{E}\{|\mathrm{F}\{s_{2nT_f}(t)\}|^2\}=\sum_{k=1}^{N_u}\sum_{m=0}^{L^k-1}\sum_{m'=0}^{L^k-1} g_m^k g_{m'}^k \sum_{l=-n}^{n-1} \mathrm{E}\{X(k,k,m,m',l,l)\}|W(f)|^2 \tag{11}$$

where

$$\mathrm{E}\{X(k,k,m,m',l,l)\}=e^{-j2\pi f(\lambda_m^k-\lambda_{m'}^k)} \tag{12}$$

since ${a_l^k}^2=1$ and ${d^k_{\lfloor \frac{l}{N_s}\rfloor}}^2=1$ and $,\tau_m^k-\tau_{m'}^k=\lambda_m^k-\lambda_{m'}^k$. Hence,

$$\mathrm{E}\{|\mathrm{F}\{s_{2nT_f}(t)\}|^2\}=\sum_{k=1}^{N_u}\sum_{m=0}^{L^k-1}\sum_{m'=0}^{L^k-1} g_m^k g_{m'}^k\, 2n|W(f)|^2 e^{-j2\pi f(\lambda_m^k-\lambda_{m'}^k)} \tag{13}$$

Using (1) with $T=nT_f$, we have

$$S(f)=\lim_{n\to\infty}\frac{\mathrm{E}\{|\mathrm{F}\{s_{2nT_f}(t)\}|^2\}}{2nT_f} \tag{14}$$

or the power spectral density of multiple access digital impulse radio utilizing direct-sequence spreading with bit-flipping data modulation under multipath propagation conditions is given by

$$S(f)=\frac{|W(f)|^2}{T_f}\sum_{k=1}^{N_u}\left|\sum_{m=0}^{L^k-1} g_m^k e^{-j2\pi f\lambda_m^k}\right|^2 \tag{15}$$

As can be seen, there are no discrete parts present in the power spectral density of a direct-sequence signal utilizing antipodal data modulation. As expected, the multiuser power spectral density is simply the sum of individual power spectral densities corresponding to different users and the effect of kth user's multipath channel is demonstrated through $\left|\sum_{m=0}^{L^k-1} g_m^k e^{-j2\pi f\lambda_m^k}\right|^2$. There is also a scale factor of $\frac{1}{T_f}$ in the power spectral density formula.

IV. TIME-HOPPING WITH PULSE POSITION MODULATION

The wireless multiple access digital impulse radio signal under multipath propagation conditions utilizing time-hopping spreading with pulse position data modulation can be represented as

$$r_{2nN_sT_f}(t) = \sum_{k=1}^{N_u}\sum_{m=0}^{L^k-1} g_m^k S_{2nN_sT_f}^k(t-\tau_m^k) \quad (16)$$

where

$$S_{2nN_sT_f}^k(t) = \sum_{l=-nN_s}^{nN_s-1} w(t-lT_f-c_l^kT_c-\delta d^k_{\lfloor \frac{l}{N_s}\rfloor}) \quad (17)$$

and $d_i^k \in \{0,1\}$ and δ accounts for the pulse position modulation parameter. Taking the Fourier Transform of $r_{2nN_sT_f}(t)$,

$$R_{2nN_sT_f}(f) = \sum_{k=1}^{N_u}\sum_{m=0}^{L^k-1} g_m^k \sum_{l=-nN_s}^{nN_s-1} W(f) e^{-j2\pi f(lT_f+c_l^kT_c+\delta d^k_{\lfloor \frac{l}{N_s}\rfloor}+\tau_m^k)} \quad (18)$$

Therefore, considering $\tau_m^k = \lambda_m^k + \tau_0^k$,

$$|R_{2nN_sT_f}(f)|^2 = \sum_{k=1}^{N_u}\sum_{k'=1}^{N_u}\sum_{m=0}^{L^k-1}\sum_{m'=0}^{L^{k'}-1} g_m^k g_{m'}^{k'} e^{-j2\pi f(\lambda_m^k-\lambda_{m'}^{k'})} \sum_{l=-nN_s}^{nN_s-1}\sum_{l'=-nN_s}^{nN_s-1}|W(f)|^2 e^{-j2\pi f((l-l')T_f)} e^{-j2\pi f(c_l^k-c_{l'}^{k'})T_c} e^{-j2\pi f\delta(d^k_{\lfloor \frac{l}{N_s}\rfloor}-d^{k'}_{\lfloor \frac{l'}{N_s}\rfloor})} e^{-j2\pi f(\tau_0^k-\tau_0^{k'})} \quad (19)$$

It can be shown [2] that when $k \neq k'$,

$$\lim_{n\to\infty}\frac{\mathrm{E}\{|R_{2nN_sT_f}(f;k\neq k')|^2\}}{2nN_sT_f} = 0 \quad (20)$$

where $|R_{2nN_sT_f}(f;k\neq k')|^2$ accounts for those terms of $|R_{2nN_sT_f}(f)|^2$ for which $k \neq k'$. Hence,

$$\lim_{n\to\infty}\frac{\mathrm{E}\{|R_{2nN_sT_f}(f)|^2\}}{2nN_sT_f} = \lim_{n\to\infty}\frac{1}{2nN_sT_f}\sum_{k=1}^{N_u}\sum_{m=0}^{L^k-1}\sum_{m'=0}^{L^{k'}-1} g_m^k g_{m'}^k e^{-j2\pi f(\lambda_m^k-\lambda_{m'}^k)}|W(f)|^2[(\sum_{l=l'=-nN_s}^{nN_s-1} 1) + \sum_{l,l'=-nN_s(l'\neq l)}^{nN_s-1} e^{-j2\pi f((l-l')T_f)}\mathrm{E}\{e^{-j2\pi f c_l^kT_c}\} \mathrm{E}\{e^{j2\pi f c_{l'}^kT_c}\}\mathrm{E}\{e^{-j2\pi f\delta(d^k_{\lfloor \frac{l}{N_s}\rfloor}-d^k_{\lfloor \frac{l'}{N_s}\rfloor})}\}] \quad (21)$$

Dividing the case for $l \neq l'$ into two disjoint subsets, namely, $l \neq l'$ and $\lfloor \frac{l}{N_s}\rfloor = \lfloor \frac{l'}{N_s}\rfloor$ and the case where $l \neq l'$ while $\lfloor \frac{l}{N_s}\rfloor \neq \lfloor \frac{l'}{N_s}\rfloor$, and carrying on some manipulations [2], the power spectral density of multiple access time-hopping digital impulse radio with pulse position modulation in the presence of multipath can be given by

$$S(f) = \sum_{k=1}^{N_u}\left|\sum_{m=0}^{L^k-1} g_m^k e^{-j2\pi f\lambda_m^k}W(f)\right|^2 \{\frac{1}{T_f} + \frac{2\sin^2(\pi f\delta)}{N_h^2N_sT_f}\frac{\sin^2(\pi fN_hT_c)}{\sin^2(\pi fT_c)}\sum_{i=1}^{N_s-1}(N_s-i)\cos(2\pi ifT_f) - \frac{\cos^2(\pi f\delta)}{N_h^2T_f}\frac{\sin^2(\pi fN_hT_c)}{\sin^2(\pi fT_c)} + \frac{1}{N_h^2T_f^2}\sum_{i=-\infty}^{\infty}\frac{\sin^2(\pi i\frac{T_c}{T_f}N_h)}{\sin^2(\pi i\frac{T_c}{T_f})}\cos^2(\pi\frac{i}{T_f}\delta)\delta_D(f-\frac{i}{T_f})\} \quad (22)$$

One should note that when $N_hT_c = T_f$, i.e., when the hopping range is equal to one frame duration, the discrete parts of the power spectral density vanish.

V. PULSE SHAPE DESIGN BASED ON F.C.C. SPECTRAL MASK

A Gaussian pulse can be mathematically represented as

$$w(t) = \frac{A}{\sqrt{(2\pi)\sigma^2}}e^{-\frac{t^2}{2}} \quad (23)$$

By taking the derivatives of a Gaussian pulse, the following recursive relationship for the derivatives of a Gaussian pulse can be obtained [1]

$$w^{(n)}(t) = \frac{1-n}{\sigma^2}w^{(n-2)}(t) - \frac{t}{\sigma^2}w^{(n-1)}(t) \quad (24)$$

where $w^{(n)}(t)$ represents the nth order derivative of a Gaussian pulse. Taking the Fourier transform of $w^{(n)}(t)$,

$$W^{(n)}(f) = A(2\pi f)^n e^{-\frac{(2\pi f\sigma)^2}{2}} \quad (25)$$

where $W^{(n)}(f)$ represents the Fourier transform of the nth derivative of Gaussian pulse shape. To find the maximum value of $W^{(n)}(f)$,

$$\frac{\partial W^{(n)}(f)}{\partial f} = 0 \Rightarrow f_{max} = \frac{\sqrt{n}}{2\pi\sigma} \tag{26}$$

which corresponds to a maximum value of

$$W^{n}(f_{max}) = A\frac{n}{\sigma^2}^{\frac{n}{2}} e^{-\frac{n}{2}} \tag{27}$$

Fig. 1 illustrates the normalized spectral density of an eighth order derivative of Gaussian pulse shape with $\sigma =$ 81.1 ps along with the FCC spectral mask. Fig. 2 shows this pulse in time domain centered at 1 ns.

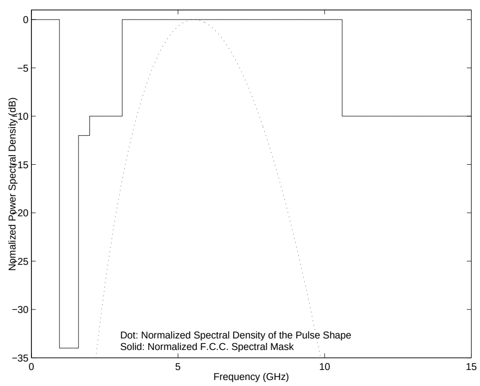

Figure 1: Normalized spectral density of an eighth order derivative of Gaussian pulse shape along with normalized FCC spectral mask

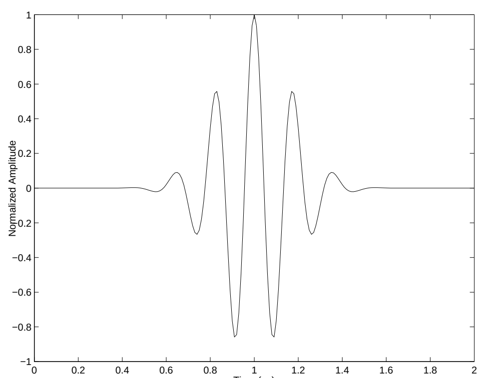

Figure 2: An eighth order derivative of Gaussian pulse shape centered at 1 ns.

Fig. 3 shows a typical UWB channel measurement. This measurement has been normalized to have a unit energy. This multipath channel measurement is one of many taken by Intel Corporation in the frequency domain between 2 GHz to 8 GHz with 3.75 MHz resolution bandwidth. Inverse fast Fourier transform has been applied to this frequency domain measurement to get the time domain representation of Fig. 2.

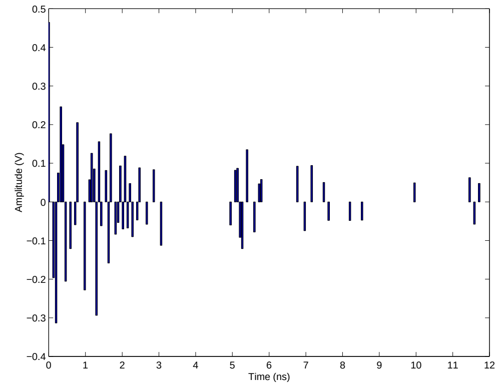

Figure 3: A typical indoor UWB Channel

Throughout this paper we use multipath channel responses as in Fig. 3 with an eighth order derivative of Gaussian pulse shape whose spectral density is shown Fig. 1.

VI. INVESTIGATING THE EFFECTS OF MULTIPATH ON UWB POWER SPECTRAL DENSITY

In this section, the power spectral density fluctuations of single user and multiuser UWB radio in the presence of multipath channels is studied.

Fig. 4 demonstrates the power spectral density of single user direct-sequence UWB radio considering only the fist few arriving paths. This PSD has been normalized by the $\max_f \frac{|W(f)|^2}{T_f}$. As can be seen, the power spectral density

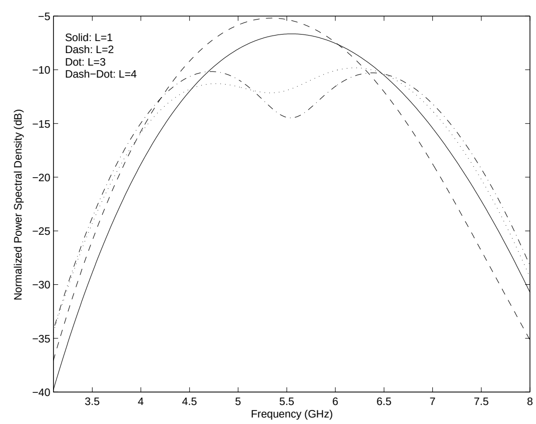

Figure 4: Single user PSD assuming only the first few arriving paths

assuming only the first arriving path is proportional to the energy spectral density of the pulse shape used. Considering the first two arriving paths, the PSD distorts a little bit with respect to the energy spectral density of the pulse shape. Adding the third and fourth paths, we see the frequency selectivity phenomenon and the fluctuations range between more than roughly ±5 dB compared to the no multipath scenario. This suggests the careful design of UWB systems such that under realistic propagation conditions the radiated power level does not exceed the limits imposed by regulatory authorities. Fig. 5 shows these fluctuations using all the paths of the received signal. This

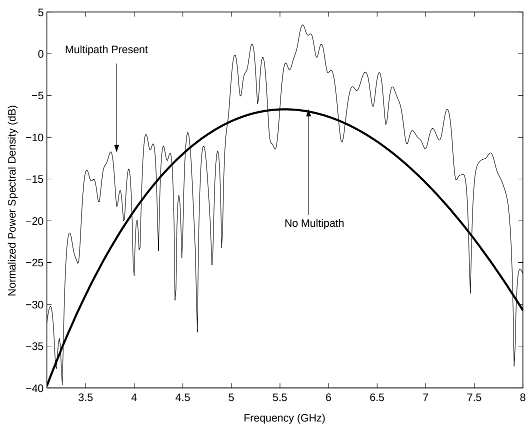

Figure 5: Single user PSD under ideal and realistic propagation conditions

figure illustrates PSD fluctuations more than ±10 dB as compared with the no multipath scenario. Fig. 6 investigates the power spectral density of multiuser UWB radio. Curves for both single user and 10 user scenarios are given for comparison. Here, all the paths have been considered for each user's independent multipath channel. As can

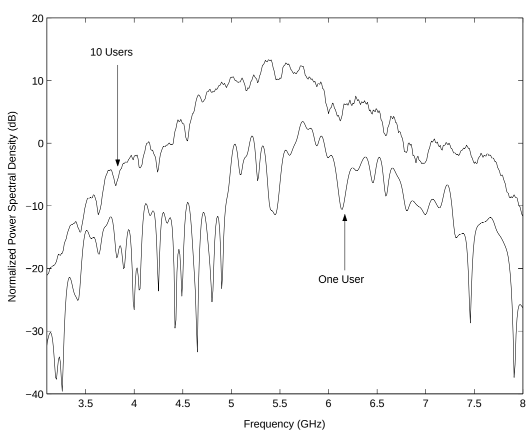

Figure 6: Single user and multiuser PSD comparison

be observed, the multiple access power spectral density is smoother than that of the single user due to the aggregation of different users's PSD's which somehow averages out sharp fluctuations. However, the multiple access PSD is 10 dB stronger than the single user case. This aggregate PSD should be designed such that it meets the spectral mask requirements imposed by regulatory authorities.

VII. CONCLUSION

It is important to know the effects of typical UWB multipath channels on UWB impulse radio power spectral density in order to investigate the spurious emissions from such radios on other narrow band systems working simultaneously at different portions of UWB radio spectrum. This power Spectral density in the presence of multipath demonstrates frequency selectivity on the order of more than ±10 dB fluctuations. These fluctuations should be considered in designing UWB transmitters so as not to cause intolerable spurious emission levels on traditional narrow band systems.

The power spectral density of a multiple-access UWB impulse radio communication system has higher levels proportional to the number of active users communicating at the same time. This suggests reduction in transmitted power levels based on the total number of active users such that the aggregate interference is restricted within specific spectral masks imposed by regulatory authorities. Although the power spectral density levels are higher for multiple-access systems, the power spectral density fluctuations due to multipath phenomenon are smoother because of aggregating different users's power spectral densities which somehow averages such fluctuations.

In order to avoid discrete parts in the power spectral density, one may use direct-sequence UWB with antipodal data modulation. The symmetry of this kind of modulation results in disappearing of discrete components of the power spectral density.

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