# Joint Estimation and Detection of UWB Signals with Timing Offset Error and Unknown Channel

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**Abstract**: An ultra-wideband (UWB) receiver that jointly estimates and detects the received signal when the channel is unknown and the transmitter and receiver clocks are not synchronized is presented. Instead of using a RAKE receiver, which requires knowledge of the received monocycle, the entire receiver pulse is estimated directly. By formulating the time-varying pulse samples in each frame as the interpolated versions of a fixed template pulse, an Extended Kalman Filter (EKF) can be used to estimate the channel and the timing offset. Because the EKF is computationally demanding, a simplified EKF with negligible loss in performance is also proposed. The EKF operates jointly with a per-survivor processing (PSP) unit to detect the received signal.

## 1 INTRODUCTION

Ultra-wideband (UWB) radio systems are defined as those with a 10dB bandwidth that exceed 20% of their center frequency or with a total bandwidth of more than 500MHz. Because of its large bandwidth, the UWB signal can provide significant multipath diversity. The large bandwidth, however, also implies that a digital UWB receiver is highly sensitive to sampling time mismatches caused by the difference in the transmitter and receiver clock frequencies. A key challenge in UWB systems, therefore, is to accurately estimate the channel in the presence of sampling time mismatches.

The most common UWB receiver structure described in the literature is the RAKE receiver, which assumes that the received signal is a superposition of known received monocycle waveform with different amplitudes and delays. This assumption, however, is not accurate, since the actual received monocycle waveform may differ significantly from the expected one due to distortions caused by the propagation channel, the antennas including their orientations, and the receiver/transmitter implementation limitations [1]. The lack of accurate knowledge of the received monocycle waveform motivates us to abandon the RAKE receiver structure and to instead estimate the entire received signal pulse directly.

The estimation of the received signal pulse is complicated by the fact that in actual UWB systems, the receiver and transmitter clocks are not synchronized. Consequently, the sampled pulse changes from frame to frame even in the absence of additive noise. A common approach for correcting the timing mismatch is to employ a PLL with a timing error detector [2]. It is unclear, however, how the timing error detector (TED) would operate, since most TEDs rely on a known structure of the received signal waveform. As described earlier, such structure may not be available in an UWB system. The PLL also takes a long time to synchronize.

In this paper, we present a receiver that rapidly estimates and detects the received signal in the presence of sample time mismatches and unknown channel. The unknown parameters are estimated using the per-survivor processing (PSP) technique [3]. The PSP operates as in the standard Viterbi algorithm but uses the data sequence associated to each survivor as the data-aiding sequence for parameter estimation. These per-survivor estimates are then employed in the computation of the transition (or branch) metric. In our receiver, the per-survivor estimates of the pulse response in each frame are used to update the branch metric.

Since the transmitter and receiver clocks are not synchronized, the received pulse in each frame is sampled at different initial times. This causes the sampled signal pulse to be time-varying, which complicates the estimation of the pulse response. To minimize the number of parameters to estimate, the received pulses are viewed as interpolated versions of a fixed template pulse. The estimates of the fixed template pulse and the timing offset, which will be defined later, are then used to accurately estimate the pulse response in each frame. The parameter estimation is achieved using the Extended Kalman Filter (EKF) [4]. Since the EKF is computationally demanding, a simplified version of EKF with negligible loss in performance is also devised by modifying the state-space model of the EKF and making appropriate approximations.

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The organization of the paper is as follows. Section 2 provides the signal model. The branch metric for PSP is given in section 3. The parameter estimation process is described in section 4. Section 5 presents simulation results and section 6 draws conclusions.

## 2 SYSTEM MODEL

The UWB signal is a time-hopping pulse train. To simplify the problem, we assume the pulse position in each frame is fixed, although this work can be easily extended to a time hopping sequence. We will mention wherever necessary how the current work could be extended to time-hopping systems. A block of received single user UWB signal using antipodal signaling is

$$r(t) = \sum_{k=0}^{N_f-1} a_k s(t - kT_f) + v(t) \qquad (1)$$

where $T_f$ is the frame time and $N_f$ is the number of frames in each transmission block. The transmitted binary symbols $a_k$ are taken from $\{+1, -1\}$. $v(t)$ is the AWGN with two-sided noise variance $N_0/2$, and $s(t)$ is the received signal pulse, which is the result of the transmitted pulse passing through a multipath channel and the receiver. The channel is assumed constant within the block, and the pulse duration is less than $T_f$ so there is no interference between frames.

The received signal is sampled at the sampling frequency $f_s$. Since the transmitter and the receiver clocks are not synchronized, the frame period $T_f$ and the sampling period $T_s$ ($T_s = 1/f_s$) have an unknown relation:

$$T_f = (N_l - \mu)T_s \qquad (2)$$

where $N_l$ represents the nominal integer number of samples between frames, which is known at the receiver, and $\mu$ is the timing offset, which is unknown at the receiver and modeled as a random variable. In the ideal case, the transmitter and the receiver clocks are synchronized and the timing offset $\mu = 0$.

The receiver initiates sampling of the received signal in the $k$th frame at time $t = kN_lT_s$ and collects $N_p$ samples. Since $\mu$ is generally nonzero, the sampling times of the signal pulse in the $k$th frame shifts by $k\mu T_s$ with respect to the sampling times of the signal pulse in the zeroth frame. This shift in sampling times with respect to the zeroth frame is subsequently referred to as the timing shift. The maximum timing shift in a block is $(N_f - 1)\mu T_s$.

The received signal samples collected in the $k$th frame is

$$r_k(n'T_s) = a_k s(n'T_s + k\mu T_s) + v_k(n'T_s) \qquad (3)$$

where $n' = 0, 1, \ldots, N_p - 1$. Using vectors,

$$\mathbf{r}_k = a_k \mathbf{s}_k + \mathbf{v}_k \qquad (4)$$

where

$$\mathbf{s}_k = \begin{bmatrix} s(k\mu T_s) \\ s((1 + k\mu)T_s) \\ \vdots \\ s((N_p - 1 + k\mu)T_s) \end{bmatrix} \qquad (5)$$

$$\mathbf{r}_k = \begin{bmatrix} r_k(0) \\ r_k(T_s) \\ \cdots \\ r_k((N_p - 1)T_s) \end{bmatrix}, \mathbf{v}_k = \begin{bmatrix} v_k(0) \\ v_k(T_s) \\ \cdots \\ v_k((N_p - 1)T_s) \end{bmatrix} \qquad (6)$$

## 3 SIGNAL DETECTION

In this section, the branch metric based on maximum-likelihood (ML) detection for use in PSP is described. Denote the data and received sample sequence up to time $k$ by $\mathbf{a}_k$ and $\mathbf{z}_k$, respectively, where $\mathbf{a}_k = [a_0, \ldots, a_k]^T$ and $\mathbf{z}_k = [\mathbf{r}_0^T, \ldots, \mathbf{r}_k^T]^T$. The optimal decision rule at the $k$th frame is

$$\hat{\mathbf{a}}_k = \arg\max P(\mathbf{a}_k | \mathbf{z}_k) \qquad (7)$$

Using a similar procedure described in [5], the recursion for the decision metric becomes

$$m_k(\mathbf{a}_k) = \Delta m_k(\mathbf{a}_k) + m_{k-1}(\mathbf{a}_{k-1}) \qquad (8)$$

The branch metric $\Delta m_k(\mathbf{a}_k)$ can be approximated as

$$\Delta m_k(\mathbf{a}_k) \approx \| \mathbf{r}_k - a_k \hat{\mathbf{s}}_{k,k|k-1} \|^2 \qquad (9)$$

where $\hat{\mathbf{s}}_{k,k|k-1}$ is the one-step prediction of $\mathbf{s}_k$. The following section describes how $\hat{\mathbf{s}}_{k,k|k-1}$ is obtained.

## 4 PARAMETER ESTIMATION

### *4.1 Model of Sampled Pulses*

The estimation of $\mathbf{s}_k$, which is needed for the decision metric given in (9), varies from frame to frame because the transmitter and the receiver clocks are not synchronized, i.e., $\mu \neq 0$. To minimize the number of parameters to estimate, $\mathbf{s}_k$ is viewed as an interpolated version of a fixed template pulse. Defining the signal pulse in the zeroth frame $\mathbf{s}_0$ as the template pulse, $\mathbf{s}_k$ can be interpolated by time shifting $\mathbf{s}_0$ by $k\mu T_s$.

Using the Cubic Lagrange Interpolator [5],

$$\mathbf{s}_k = \begin{bmatrix} q_0^k & q_1^k & q_2^k & 0 & \dots & \dots & 0 \\ q_{-1}^k & q_0^k & q_1^k & q_2 & \dots & \dots & 0 \\ 0 & q_{-1}^k & q_0^k & q_1^k & q_2^k & \dots & 0 \\ 0 & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & q_{-1}^k & q_0^k & q_1^k & q_2^k \\ 0 & \dots & \dots & 0 & q_{-1}^k & q_0^k & q_1^k \\ 0 & \dots & \dots & \dots & 0 & q_{-1}^k & q_0^k \end{bmatrix} \mathbf{s}_0 \quad (10)$$

where the interpolator coefficients are

$$q_{-1}^k = -\frac{1}{6}(k\mu)^3 + \frac{1}{2}(k\mu)^2 - \frac{1}{3}k\mu \quad (11)$$

$$q_0^k = \frac{1}{2}(k\mu)^3 - (k\mu)^2 - \frac{1}{2}k\mu + 1 \quad (12)$$

$$q_1^k = -\frac{1}{2}(k\mu)^3 + \frac{1}{2}(k\mu)^2 + k\mu \quad (13)$$

$$q_2^k = \frac{1}{6}(k\mu)^3 - \frac{1}{6}k\mu \quad (14)$$

Substituting (11)-(14) into (10) and rearranging, $s_k$ can be written as

$$\mathbf{s}_k \cong \left[(k\mu)^3\mathbf{H}_{C3} + (k\mu)^2\mathbf{H}_{C2} + (k\mu)\mathbf{H}_{C1} + \mathbf{H}_{C0}\right]\mathbf{s}_0 \quad (15)$$
$$\triangleq \mathbf{H}(k\mu)\mathbf{s}_0$$

where $\mathbf{H}_{C3}$, $\mathbf{H}_{C2}$, $\mathbf{H}_{C1}$ and $\mathbf{H}_{C0}$ are the polynomial coefficient matrices corresponding to different orders of $k\mu$.

In the case of a time-hopping sequence, the timing shift in the $k$th frame is $(k + c(k))\mu T_s$, where $c(k)$ is the time-hopping code. Since $c(k)$ is known at the receiver, the analysis in this paper can be extended to time-hopping systems by replacing all occurrences of $k\mu$ by $(k + c(k))\mu$.

### *4.2 Extended Kalman Filter Estimator*

EKF is used to estimate the sampled pulse $\mathbf{s}_k$ in the $k$th frame given all the past received samples $\mathbf{z}_{k-1}$. To remove the data dependency, the observed vector in the $k$th frame is

$$\mathbf{y}[k] = \hat{a}_k\mathbf{r}_k \quad (16)$$

where $\hat{a}_k$ is the $k$th frame symbol of the survivor path. The UWB communication system is modeled by the following nonlinear state-space equations:

$$\mathbf{x}[k+1] = \mathbf{x}[k] \quad (17)$$

$$\mathbf{y}[k] = \mathbf{c}[k, \mathbf{x}[k]] + \tilde{\mathbf{v}}_k \quad (18)$$

where the state vector is defined as

$$\mathbf{x}[k] = \begin{bmatrix} \mu \\ \mathbf{s}_0 \end{bmatrix}, \quad (19)$$

the measurement vector is

$$\mathbf{c}[k, \mathbf{x}[k]] = \mathbf{H}(k\mu)\mathbf{s}_0 \quad (20)$$

and $\tilde{\mathbf{v}}_k$ is measurement noise.

Using the following notation for state prediction at the $k$th frame given $\mathbf{z}_{k-1}$

$$\hat{\mathbf{x}}_{k|k-1} = \begin{bmatrix} \hat{\mu}_{k|k-1} \\ \hat{\mathbf{s}}_{0,k|k-1} \end{bmatrix} \quad (21)$$

and linearizing (20) around the most recent state estimate, the following matrix is constructed

$$\mathbf{C}[k, \hat{\mathbf{x}}_{k|k-1}] = \frac{\partial}{\partial \mathbf{x}}\mathbf{c}[k, \mathbf{x}]\Big|_{\mathbf{x} = \hat{\mathbf{x}}_{k|k-1}} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{H}(k\hat{\mu}_{k|k-1}) \end{bmatrix} \quad (22)$$

where

$$\mathbf{c}_1 = \mathbf{T}_1\hat{\mathbf{s}}_{0,k|k-1} \quad (23)$$

$$\mathbf{T}_1 = 3k(k\hat{\mu}_{k|k-1})^2\mathbf{H}_{C3} + 2k(k\hat{\mu}_{k|k-1})\mathbf{H}_{C2} + k\mathbf{H}_{C1} \quad (24)$$

Based on the above equations, the standard EKF equations can be applied to estimate $\hat{\mathbf{s}}_{0,k|k-1}$. This estimate can then be used to interpolate $\hat{\mathbf{s}}_{k,k|k-1}$, which is needed in the decision metric given in (9):

$$\hat{\mathbf{s}}_{k,k|k-1} \cong \mathbf{H}(k\hat{\mu}_{k|k-1})\hat{\mathbf{s}}_{0,k|k-1} \quad (25)$$

Most interpolators, including the Cubic Lagrange Interpolator, are most effective when $0 \le k\mu < 1$. Thus, if $k\mu \ge 1$ or $k\mu < 0$, we need to replace $\mathbf{r}_k$ with a new signal vector $\mathbf{r}_k^s$ so that $0 \le k\mu < 1$. This can be achieved by initiating the sampling in the $k$th frame a few samples early or late based on the estimate of $\hat{\mu}_{k|k-1}$. More specifically, the sampling in the $k$th frame occurs $N_I - \lfloor k\hat{\mu}_{k|k-1} \rfloor$ samples after the ($k$-1)th frame instead of $N_I$ samples, where $N_I$ is the nominal integer number of samples between frames and $\lfloor X \rfloor$ is the largest integer not exceeding $X$. The new timing shift in the $k$th frame then becomes $(k\hat{\mu}_{k|k-1} - \lfloor k\hat{\mu}_{k|k-1} \rfloor)T_s$. This shifting of the received samples can be accounted for in the EKF equations described above by replacing all occurrences of $\mathbf{r}_k$ and $k\hat{\mu}_{k|k-1}$ with $\mathbf{r}_k^s$ and $(k\hat{\mu}_{k|k-1} - \lfloor k\hat{\mu}_{k|k-1} \rfloor)$, respectively.

### *4.3 Simplified EKF Estimator*

Solving the EKF directly is computationally demanding as it requires a matrix inversion. This operations can be avoided by reformulating the EKF state-space model and making appropriate approximations. For notational brevity, we subsequently use $\mathbf{r}_k$ and $k\hat{\mu}_{k|k-1}$ instead of $\mathbf{r}_k^s$ and $(k\hat{\mu}_{k|k-1} - \lfloor k\hat{\mu}_{k|k-1} \rfloor)$, respectively.

To reduce the computational requirements, the nonlinear state-space equations are modified as follows:

$$\mathbf{x}'[k+1] = \mathbf{x}'[k] \quad (26)$$

$$\hat{a}_k = \mathbf{r}_k^T \mathbf{c}'[k, \mathbf{x}'[k]] + v_{ck} \quad (27)$$

where $v_{ck}$ is the measurement noise at the $k$th frame,

$$\mathbf{x}'[k] = \begin{bmatrix} \mu \\ \mathbf{s}'_0 \end{bmatrix}, \ \mathbf{s}'_0 = \mathbf{s}_0 / \|\mathbf{s}_0\|^2 \quad (28)$$

and

$$\mathbf{c}'[k, \mathbf{x}'[k]] = \mathbf{H}(k\mu)\mathbf{s}'_0 \quad (29)$$

By making the observation vector in (27) into a scalar, the matrix inversion in the Kalman filter equations becomes a simple scalar division. To further simplify the EKF, the state-error correlation matrix **K**, which is used to update the Kalman filter, is approximated by keeping only the first row and column (i.e., the correlation terms between the prediction errors of $\mu$ and $\mathbf{s}'_0$) and the diagonal terms. This simplification removes a matrix multiplication in the EKF update equations.

When using the simplified EKF, the branch metric is slightly modified. Assuming the predicted signal power $\|\hat{\mathbf{s}}_{k,\,k|k-1}\|^2$ is approximately constant, minimizing the branch metric (9) is equivalent to minimizing the following branch metric:

$$\Delta m'_k(\mathbf{a}_k) \approx \left|a_k - \mathbf{r}_k^T \hat{\mathbf{s}}'_{k,\,k|k-1}\right|^2 \quad (30)$$

where $\hat{\mathbf{s}}'_{k,\,k|k-1}$ is the prediction of $\mathbf{s}_k / \|\mathbf{s}_k\|^2$. $\hat{\mathbf{s}}'_{k,\,k|k-1}$ is obtained by interpolating using the EKF estimate of $\hat{\mathbf{s}}'_{0,\,k|k-1}$ as in (25).

Table 1 shows the required computational complexity in a survivor path of a simplified EKF receiver with simplified **K** and full **K**. All terms not related to $N_p$ are ignored. The interpolators are all assumed to be implemented with a Farrow structure [6]. Also for comparison purposes, the computational complexity of a cubic interpolator is included. Except for the interpolator, the number of each computational operation is obtained directly based on the equation without assuming a structure. The computational requirements of the EKF with simplified **K** increases linearly with $N_p$ and is comparable with the computational complexity of a cubic interpolator.

TABLE 1. Computational Complexity

| | Cubic Interpolator | EKF with simplified **K** | EKF with full **K** |
|---|---|---|---|
| Scale by constant | $3N_p$ | $5N_p$ | $5N_p$ |
| Add/Subtract | $11N_p$ | $21N_p$ | $1.5N_p^2+15.5N_p$ |
| Multiply/ Divide | $3N_p$ | $16N_p$ | $1.5N_p^2+11.5N_p$ |

## 5 SIMULATION

The received monocycle, which we assume is unknown at the receiver, is the second derivative of a gaussian pulse. The model used here is

$$w_{rec}(t+1) = (1 - 4\pi t^2)\exp(-2\pi t^2) \quad (31)$$

The received noise free signal pulse is a superposition of monocycles with different delays and amplitudes. The multipath model used is the CM1 channel model recommended by the IEEE P802.15-02/368r5-SG3a. The estimation and detection are based on the samples for the first 20 nanoseconds(ns) of each frame, which includes about 50 paths. The energy in the 20ns occupies about 90% of the total energy of one frame. The sampling period is $T_s = 0.167ns$. The number of samples in each frame is $N_p = 120$.

The receiver works in PSP mode with four states. The states in each time index represent two adjacent non-overlapping bits. For example, for transmitted bits $\{a_0, a_1, a_2, a_3, \ldots\}$, $\{a_0, a_1\}$ represent the states in the first time index, $\{a_2, a_3\}$ the states in the second time index, and so on.

The signal energy is defined as $E_b = E\{\|\mathbf{s}_k\|^2\}$. Unless stated otherwise, the following operating conditions are assumed: $\mu = 0.1$, $N_f = 100$, and the first eight symbols are known.

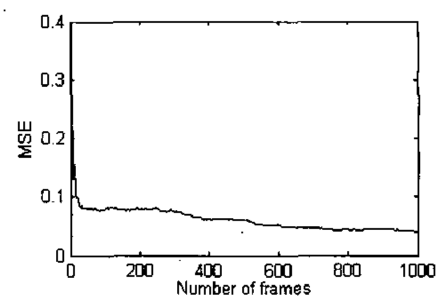


Figure 1 Estimation MSE of the pulse

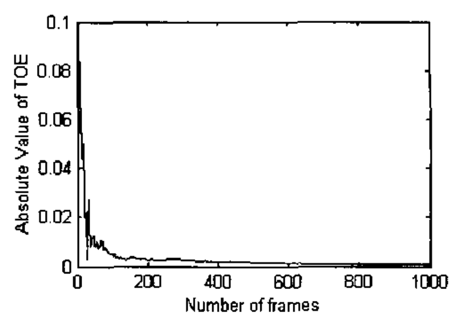


Figure 2 Timing Offset Error

Figure 1 and Figure 2 plot the estimation mean-squared error (MSE) of $\mathbf{s}'_0$ and the absolute value of timing offset error (TOE) as a function of the number of frames received for the simplified EKF receiver (described in section 4.3) assuming $E_b/N_0 = 10dB$ and $N_f$= 1000. Most of the estimation error of the template pulse and the timing offset reduce rapidly in approximately 20 frames.

The performance of the full EKF (FEKF) and simplified EKF (SEKF) receivers described in sections 4.2 and 4.3, respectively, are plotted in Figure 3 as a function of $E_b/N_0$. For comparison purposes, the Comparison PSP (CPSP) receiver is also plotted. The CPSP receiver is defined as a receiver with no timing offset (i.e., $\mu = 0$) and hence, only needs to average along each survivor to estimate the signal pulse. The CPSP receiver, therefore, represents approximately a lower bound on the performance of receivers with unknown channel and $\mu \neq 0$. The CPSP, FEKF, and SEKF receivers all have the same number of PSP states. The performance of the ideal antipodal receiver is also shown as a reference for comparison in Figure 3. The ideal antipodal receiver assumes perfect knowledge of the signal pulse and no timing offset (i.e., $\mu = 0$).

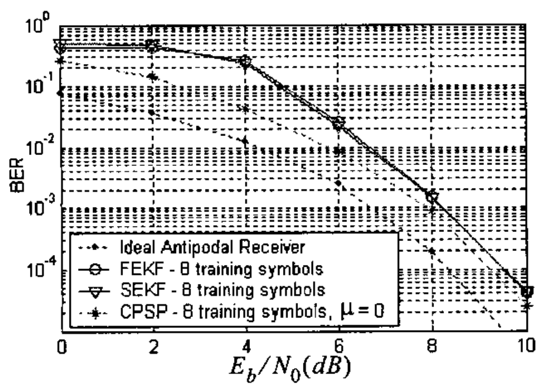


Figure 3 Bit Error Rate

The difference in performance between the FEKF SEKF receivers is small. At some of low SNR values, the SEKF receiver actually outperforms the FEKF receiver. This occurs because when approximating the state-error correlation matrix **K** as a sparse matrix in the SEKF receiver, the effect of the noise in the off-diagonal elements is eliminated. At high SNR values, both the FEKF and SEKF receivers approach the performance of the CPSP.

In our simulation so far, we assumed that the first eight symbols are known. Figure 4 plots the effects of the number of training frames for the SEKF receiver. As the number of training symbols increases, the performance improves. However, the improvement is small beyond eight training symbols.

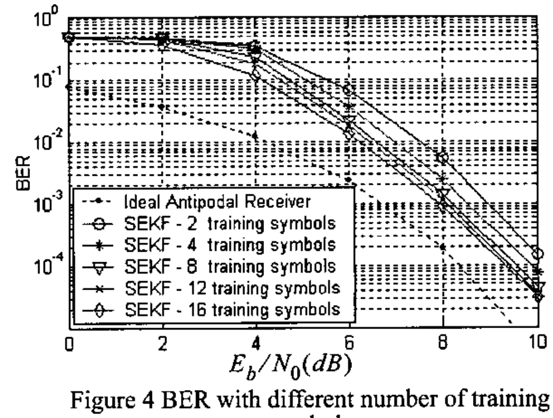


Figure 4 BER with different number of training symbols

## 6 CONCLUSIONS

Using the EKF and PSP, a UWB receiver that jointly estimates and detects the received signal with unknown channel and sampling time mismatches is presented. The use of the EKF to estimate both the channel and the timing offset is made possible by formulating the received pulse samples in each frame as an interpolated version of the zeroth pulse samples. A simplified EKF receiver with complexity that increases linearly with the number of samples in each frame is also presented.

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