Generalized Transmitted-Reference UWB Systems

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Abstract: Transmitted-reference (TR) ultra-wideband (UWB) wireless communication systems [1] can relax the difficult UWB timing requirements and can provide a simple receiver that gathers the energy from the many resolvable multipath components. However, TR-UWB's relatively poor bit error rate (BER) performance and low data rate have limited its application. In this paper, the TR-UWB idea is generalized to address both of these issues. In particular, the aspects of the system that provide the desirable multipath gathering ability and timing attributes are retained, while the remainder of the system is optimized, resulting in a significantly different signaling scheme and receiver back-end. Numerical results for two examples indicate a significant BER improvement over standard TR-UWB under the same timing requirement.

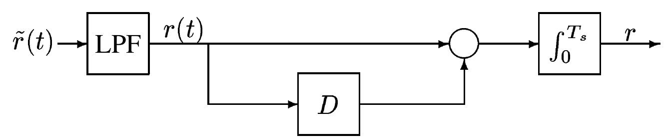

Figure 1: Receiver for a standard TR-UWB communication system, where $\tilde{r}(t)$ is the received signal, and $r(t)$ is a lowpass-filtered version of such. The signal is multiplied by a delayed version of itself and the result integrated over the symbol period T_s, which can consist of many frames, each of duration T_f, in low data rate UWB systems. A threshold decision is made on r to decode the data bit for the current symbol.

1 Introduction

Because their extremely large bandwidth provides a number of potential advantages over other communication strategies, UWB communication systems have emerged as a promising alternative for short-distance, low-power wireless applications. From a regulatory standpoint, the extremely low power density of UWB communications has led the federal communications commission (FCC) of the United States to allow it to operate over the top of other bands, thus helping to solve the frequency allocation problem that often limits high data rate wireless communication systems. From a technical standpoint, the extremely wide bandwidth offers a number of *potential* advantages versus narrowband alternatives, including the ability to carry very high data rates, diversity against multipath, and the mitigation of interference (both multi-user and non-system interference).

However, from a theoretical perspective, recent results have shown that, although the inherent capacity of the wideband multipath fading channel is equal asymptotically to that of the additive white Gaussian noise (AWGN) channel, the achievement of such capacity requires "peaky" signals, and the capacity of systems which spread their energy very finely in the frequency domain, as in the typically envisioned and FCC-approved UWB systems, diminishes to zero in the limit of large bandwidth [2, 3]. The explanation for this result is that channel estimation becomes very difficult as the number of resolvable signal paths that need to be estimated grows to infinity.

Although the studies mentioned in the previous paragraph are largely theoretical, implementation problems due to the large bandwidth, or, equivalently, the extremely short pulse, have slowed the development of practical UWB systems. In particular, the short pulse makes the acquisition of the code and frame timing extremely difficult, and such acquisition, as with many spread-spectrum systems, can limit overall system performance. Practical channel estimation can indeed be difficult, and, even *given* accurate channel estimation, the standard UWB system requires a rake receiver with a large number of fingers and thus of prohibitive complexity [4].

One method of addressing the timing and channel estimation problems is through the use of the TR-UWB system [1]. Each of the signals in the TR-UWB signal set:

$$
\begin{aligned}
s_0(t) &= \sqrt{E_s/2}\, p(t) + \sqrt{E_s/2}\, p(t-D) \\
s_1(t) &= \sqrt{E_s/2}\, p(t) - \sqrt{E_s/2}\, p(t-D),
\end{aligned}
$$

consists of a pair of pulses, where $s_0(t)$ and $s_1(t)$ are the signals transmitted for an information bit 0 and an information bit 1, respectively, during each frame of duration T_f, E_s is the transmitted energy per symbol, D is a fixed delay, and $p(t)$ is the unit-energy UWB pulse shape. The TR-UWB receiver is shown

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in Figure 1.

The TR-UWB architecture has a number of attractive properties:

1. Multipath gathering is achieved, since the first pulse (i.e. "reference pulse") in each of the possible transmitted-reference signals goes through the same channel as the second pulse (i.e. "data pulse"). Thus, the reference arm of the receiver provides a perfect (but, unfortunately, noisy) template to which to match the data pulse without explicit channel estimation or the need for a rake receiver with *many* branches.

2. Simple timing acquisition can be achieved by repeating $s_i(t)$ as many times per symbol as desired and having the receiver integrate across these many frames; thus, timing is only required at the symbol (rather than frame) level, which can be an important gain in low data rate applications, where often $T_s \gg T_f$.

3. Since the reference pulse and the data pulse are transmitted within one frame, the channel need only be constant over the frame time. This can be significant for systems operating in a highly mobile environment.

However, despite the simplicity and robust performance of the TR-UWB system, it has not found wide acceptance, because the bit-error rate performance and data rate do not approach that promised by antipodal UWB communication systems [1].

Because of TR-UWB's promise and the prominence of the UWB concept, there has been significant recent work on trying to improve the performance of the TR-UWB system [4, 5, 6] (see also [7], which proposes a differential scheme that can be viewed as a variant of TR-UWB [4]). These proposed schemes have generally focused on the key idea of providing pilots (i.e. a "transmitted reference"), and have optimized the placement of the pilots and channel estimation based on the received signal from such. However, schemes which attempt to use multiple references for channel estimation require timing at the frame level [4, 5, 6], and differential schemes require either timing at the frame level or channel stability over a symbol (rather than a frame) interval [4, 7]. In this paper, the parts of the TR-UWB system that provide for its simplicity of timing and gathering of multipath energy are maintained, while the rest of the system is optimized.

The organization of this paper is as follows. In Section 2, the framework for the proposed system is presented. In Section 3, the receiver output statistics are derived. The receiver output statistics are used in Section 4 to choose signal sets, and numerical results and comparisons to other systems are presented. Finally, Section 5 presents the conclusions and ideas for future work.

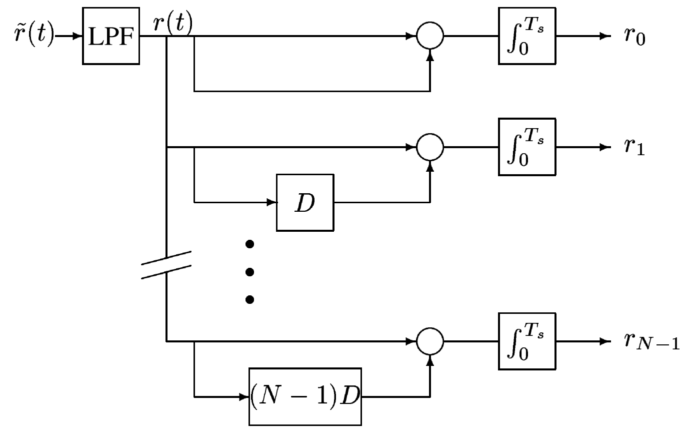

Figure 2: Receiver front-end for the proposed generalized transmitted-reference system. Note that, instead of a threshold rule being applied to each of the outputs r_i, a joint decision is made on $\underline{r}$ to decode each symbol, as described in the text.

2 System Description

Throughout this paper, a baseband UWB system will be assumed. The transmitted signal $s(t)$, which will include the effects of both the transmit and receive antennas, will be chosen from the M-ary signal set $\{s_i(t) : i = 0, 1, \ldots, M-1\}$. It will be assumed that there is one frame per symbol period (i.e. $T_s = T_f$) to simplify the already cumbersome equations; however, the results apply *directly* to low data rate applications ($T_s \gg T_f$), which are of the main interest here. Operation over a multipath fading channel will generally be considered, where the channel is defined by:

$$\tilde{r}(t) = \int h(\tau)s(t-\tau)d\tau + \tilde{n}(t)$$

where $\tilde{r}(t)$ is the received signal, $\tilde{n}(t)$ is a zero-mean white Gaussian noise process with (two-sided) power spectral density $S_n(f) = \frac{N_0}{2}$, and $h(\tau)$ is the channel impulse response, which will be assumed to follow the Gaussian wide-sense stationary uncorrelated scattering (GWSSUS) model and will be assumed to have support for $\tau \in [0, \tau_{max}]$. Since it will be assumed that the channel is constant over one frame, the variation of the channel with time has been suppressed for notational convenience.

Per Section 1, the goal is to maintain the aspects of the transmitted-reference system that provide simple timing and multipath gathering ability in the application area of low data rate systems in highly mobile environments. This will be done by employing the receiver front-end shown in Figure 2, where N is the maximum number of pulses in a transmitted signal, and choosing the M possible transmitted signals from the set

$$s_i(t) = \sum_{j=0}^{N-1} s_{i,j} p(t - jD), \quad i = 0, 1, \ldots M-1 \tag{1}$$

where $s_{i,j}$ is the representation of the i^{th} signal on the j^{th} delayed pulse, and $D > \tau_{max}$. It should be noted that the restriction $D > \tau_{max}$ simply eliminates "self-interference" at the expense of data rate, and the tradeoffs associated with the relaxation of this requirement are an object of future study. Per the caption of Figure 2, it is important to note that threshold decisions are *not* made on the individual elements of $\underline{r} = (r_0, r_1, \ldots, r_{N-1})^T$; instead, the vector will be processed jointly as described in Section 2.3.

The receiver of Figure 2 and the signal format described in (1) maintain the multipath gathering ability of the TR-UWB system, as shown here. Assuming that s_i is sent, ignoring the noise (just for this portion of the paper), and, for notational simplicity, assuming that the lowpass filter at the front-end of the receiver does not distort the received signal (an assumption that is easily relaxed):

$$\begin{aligned} r_l &= \int_0^{T_s} \left(\int_0^{\tau_{max}} h(\tau_1) s_i(t-\tau_1) d\tau_1 \right. \\ &\qquad \left. \int_0^{\tau_{max}} h(\tau_2) s_i(t - lD - \tau_2) d\tau_2 \right) dt \\ &= \sum_{j=0}^{N-1} \sum_{n=0}^{N-1} s_{i,j} s_{i,n} \int_0^{T_s} \left(\int_0^{\tau_{max}} h(\tau_1) p(t-\tau_1 - jD) d\tau_1 \right. \\ &\qquad \left. \int_0^{\tau_{max}} h(\tau_2) p(t - lD - nD - \tau_2) d\tau_2 \right) dt \\ &= \sum_{j=0}^{N-1} s_{i,j} s_{i,j-l} \int_0^{T_s} \left(\int_0^{\tau_{max}} h(\tau_1) p(t-\tau_1) d\tau_1 \right)^2 dt \\ &= E_r \sum_{j=0}^{N-1} s_{i,j} s_{i,j-l} \end{aligned} \tag{2}$$

where the assumption that $D > \tau_{max}$ has been exploited in the second to last line, and E_r has been defined as the total received energy from all of the multipath components.

Equation (2) also illustrates a key design issue - that the noiseless vector receiver output for the i^{th} signal is the deterministic discrete-time autocorrelation function of the sequence $(s_{i,0}, s_{i,1}, \ldots, s_{i,N-1})^T$; in other words, maximizing the Euclidean distance in $\underline{r}$ for disparate transmitted signals means maximizing the Euclidean distance between the autocorrelation function of the signaling sequences. Of course, the desire to maximize Euclidean distance presumes an effective AWGN channel; in other words, it presumes that the vector noise affecting $\underline{r}$ is: (1) additive, (2) jointly Gaussian, (3) independent between dimensions, (4) identical between dimensions, (5) identical for different transmitted signals. Whereas properties (1) and (2) are at least approximately true, properties (3)-(5) are decidedly not true as discussed below.

3 Statistics at the Receiver Output

Since the effective channel of the generalized transmitted-reference system from $\underline{s}_i$ to $\underline{r}$ is not an AWGN channel (or other standard channel), the signal design criteria must be established. Thus, in this section, the statistics of $\underline{r}$ given the transmitted vector $\underline{s}_i$ are calculated. This allows the derivation of the optimal receiver, supports the derivation of the performance of such, and, finally, allows for optimal signal set selection.

Conditioned on the transmitted signal s_i, the vector of system observables $\underline{r}$ will be assumed to be jointly Gaussian, where the justification follows that of previous work for standard transmitted-reference [1, 5]. It will be observed in Section 3 that the receivers derived under such an assumption perform very well. Thus, it remains only to find the first order (i.e. $\{E[r_l|s_i], l = 0, 1, \ldots, N-1\}$) and second order (i.e. $\{E[r_l r_k|s_i], l = 0, 1, \ldots, N-1, k = 0, 1, \ldots, N-1\}$) statistics of $\underline{r}$. Straightforward derivation yields the first-order statistics as:

$$E[r_l|s_i] = \begin{cases} E_r \sum_{j=0}^{N-1} s_{i,j}^2 + N_0 W T_s, & l = 0 \\ E_r \sum_{j=0}^{N-1} s_{i,j} s_{i,j-l}, & l > 0 \end{cases}$$

where W is the bandwidth of the lowpass filter at the front of the receiver. Tedious calculation leads to the covariance of r_l and r_k given s_i:

$$\begin{aligned} \mathrm{Cov}[r_k, r_l|s_i] &= \sum_{j=0}^{N-1} \sum_{n=0}^{N-1} s_{i,j} s_{i,n} \\ &\quad \left\{ \int_0^{T_s} \int_0^{T_s} u(s - jD) u(t - nD) R_n(t - s + kD - lD) ds dt \right. \\ &+ \int_0^{T_s} \int_0^{T_s} u(s - jD - kD) u(t - nD) R_n(t - s - lD) ds dt \\ &+ \int_0^{T_s} \int_0^{T_s} u(s - jD) u(t - nD - lD) R_n(t - s + kD) ds dt \\ &+ \left. \int_0^{T_s} \int_0^{T_s} u(s - jD - kD) u(t - nD - lD) R_n(t - s) ds dt \right\} \\ &+ [\delta(k-l) + \delta(k)\delta(l)] \int_0^{T_s} \int_0^{T_s} R_n^2(t - s) ds dt \end{aligned} \tag{3}$$

where $u(t) = h(t) * p(t)$ is the convolution of the multipath fading channel with the transmitter pulse shape and $R_n(\tau) = E[n(t)n(t+\tau)]$ is the autocorrelation function of the filtered noise. The fact that $R_n(\tau) \approx 0$ if $\tau \geq D$ has been used in the last term.

Although it might not be obvious from the above complicated expression, the variance of the noise not only varies across components of $\underline{r}$ but also actually depends on the transmitted signal itself! Thus, it is clear that the first idea of "choosing the signal whose deterministic autocorrelation function is closest in Euclidean distance to the receiver vector $\underline{r}$", which may be the first instinct based on Section 2.2, can be far from optimal. This was indeed observed in the consideration of four-point signal sets to find a good candidate in Section 3 below. Thus, instead of simply employing the minimum Euclidean distance, the maximum-likelihood receiver chooses the signal s_i that maximizes:

$$p(\underline{r}|s_i) = \frac{1}{(2\pi)^{\frac{N}{2}} [\det(K_{\underline{r}}(s_i))]^{\frac{1}{2}}} \exp\left[-\frac{1}{2} (\underline{r} - E[\underline{r}|s_i])^T K_{\underline{r}}(s_i)^{-1} (\underline{r} - E[\underline{r}|s_i]) \right]$$

where $K_{\underline{r}}(s_i)$ is the covariance matrix of $\underline{r}$ given that s_i was transmitted. Note that the statistics required by this optimal receiver depend on the channel through $u(t)$, and, hence, must be estimated. However, this small number of parameters can be estimated adaptively at the receiver and can be contrasted to the large number of parameters that must be estimated for the standard rake receiver. Most pertinently, the number of parameters that must be estimated is *fixed* regardless of the number of resolvable paths, and thus there is no increase in complexity of the parameter estimation as the system bandwidth scales, which, per Section 1, is one of the greatest challenges of UWB communications.

4 Signal Design: An Example and Numerical Results

In this section, two signal sets are designed and their performance simulated to demonstrate some key points of the generalized TR-UWB architecture.

Because optimal signal design based on the full expressions from Section 3 has proven to be difficult, the first design presented here is based on maximizing the minimum Euclidean distance between the deterministic autocorrelation vectors corresponding to the transmitted signals, as motivated by Section 2. It will be shown that such signal sets, *when* decoded using the optimal rule derived in Section 3, will perform very well and yield insight into system operation. Based on such a design criteria, an excellent candidate for a raw signal set with two points is given by:

$$\begin{aligned} s_0^{(2)}(t) &= 0 \\ s_1^{(2)}(t) &= p(t) + p(t-D), \end{aligned}$$

which is then normalized to have average energy E_s. Note that this is a generalization of on-off keying to the transmitted-reference case. The raw signal set with four points, which is *not* based on the Euclidean distance criterion, is given by:

$$\begin{aligned} s_0^{(4)}(t) &= 0 \\ s_1^{(4)}(t) &= \sqrt{2.5}\, p(t) + \sqrt{2.5}\, p(t-D), \\ s_2^{(4)}(t) &= p(t) + p(t-D), \\ s_3^{(4)}(t) &= p(t) - p(t-D), \end{aligned}$$

which is then normalized to have average energy E_s.

The simulation parameters are as follows. The pulse shape is the second derivative of a Gaussian with parameter 0.4472 (yielding a width of the pulse of 1.2 ns), $D = 15$ ns, the frame time is 40 ns, the bandwidth of the lowpass filter at the front-end of the receiver is 2.5 GHz, and at least $20/P_b$ bit transmissions were simulated to arrive at a given data point, where P_b is the bit error probability displayed at that point.

Figure 3 displays the bit error probability characteristics when various systems operate over a fixed (but unknown to the system) multipath channel. Most notable is that the proposed scheme outperforms standard TR-UWB by over 2 dB for error rates below 10^{-2}. In addition, the utility of the receiver derived in Section 3 is apparent, since obvious decoders such as one that calculates minimum Euclidean distance (not shown) or a simple energy detector do not perform well. Figure 4 demonstrates the performance of the same schemes when *averaged* over a large number of randomly-generated three-path Rayleigh fading channels, where the second and third paths are 3 dB weaker than the main path. The gains of the proposed scheme increase over the fixed multipath case, particularly for higher signal-to-noise ratio (SNR). Finally, Figure 5 demonstrates the performance of the four-point signal set over a fixed multipath channel. Not only is the observed performance better, but the data rate is twice that of the standard TR-UWB system, while, like standard transmitted reference [1], channel coherence is only required over the delay D (i.e. at the frame level) and timing is only required at T_s (i.e. the symbol level). This makes such a system appropriate for low data rate applications in harsh (i.e. highly mobile) environments.

5 Conclusions and Future Work

In this paper, a generalized version of the transmitted-reference UWB system of [1] has been proposed. In particular, the aspects of the transmitted-reference system that allow for its excellent multipath gathering ability and simple timing acquisition have been retained, but signal set selection and receiver processing have been dramatically modified. The proposed system can be used to increase the bandwidth efficiency of TR-UWB systems or to dramatically improve the BER of the TR-UWB system. Unlike other proposed modifications to TR-UWB [4, 5, 6, 7], these gains are obtained without affecting the desirable properties of the TR-UWB system. Thus, we believe such systems have the potential to greatly impact UWB systems that desire very low-complexity receiver operation.

There is substantial future optimization work that can be considered on the proposed framework. Prominent among this is the mitigation of narrowband interferers, which will be prevalent in both commercial and military UWB systems. Although transmitted-reference systems are susceptible to narrowband interference because of the interference multiplication that can occur in the front end, the potentially large dimensionality signal space here at the output of the receiver should provide the opportunity to mitigate much of their effects.

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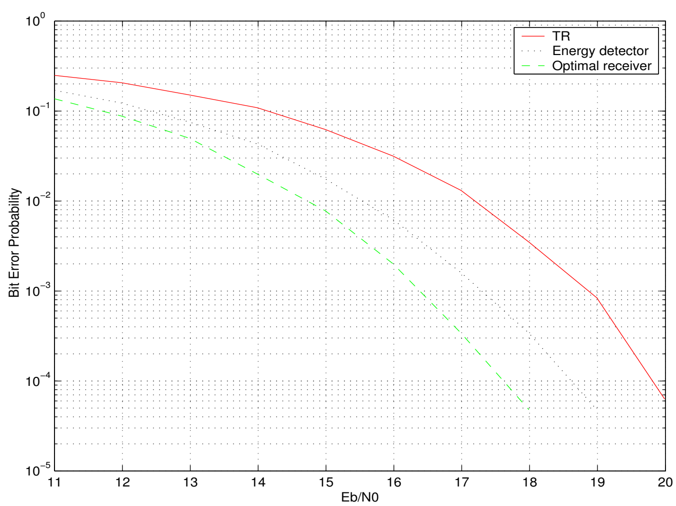

Figure 3: BER of the standard TR-UWB ("TR"), the two-point constellation proposed here with an energy detector ("Energy Detector"), and the two-point constellation proposed here with an optimal receiver based on Section 3 ("Optimal Receiver") for a fixed multipath channel, versus the signal-to-noise ratio $\frac{E_b}{N_0}$, where E_b is the total received energy per information bit.

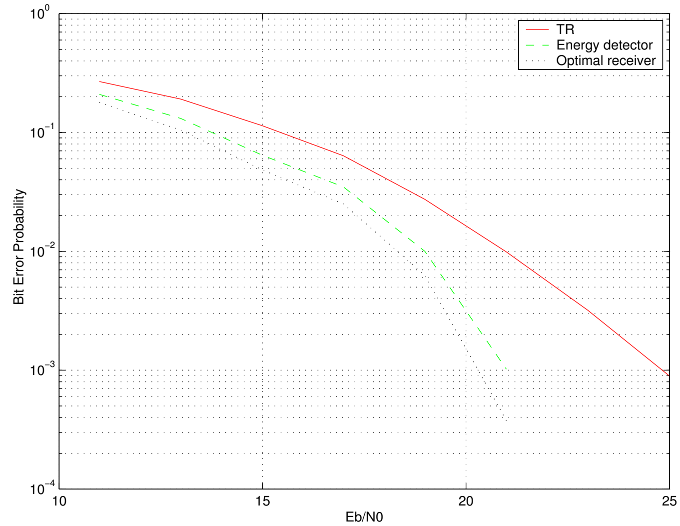

Figure 4: Bit error probability of the standard transmitted-reference ("TR"), the two-point constellation proposed here with an energy detector ("Energy Detector"), and the two-point constellation proposed here with an optimal receiver based on Section 3 ("Optimal Receiver"), averaged over the multipath fading, versus the signal-to-noise ratio $\frac{E_b}{N_0}$, where E_b is the average total received energy per information bit.

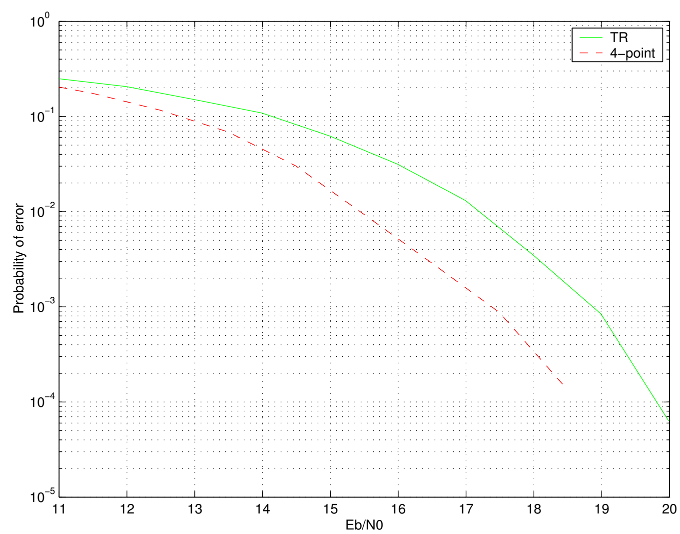

Figure 5: Bit error probability of the four-point constellation described in the text with an optimal receiver based on Section 3 ("4-point") and of standard transmitted-reference ("TR") for a fixed multipath channel, versus the signal-to-noise ratio $\frac{E_b}{N_0}$. Note that the proposed four-point system is transmitting data at twice the rate of the transmitted-reference scheme, while preserving the same timing requirement and with performance as shown.