Multiple Access Performance of Ultra-wideband Transmitted Reference Systems in Multipath Environments

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***Abstract*— The multiple access (MA) performance of an ultra-wideband differential transmitted reference (UWB DTR) system and an ultra-wideband transmitted reference (UWB TR) system in multipath environments is investigated through analysis. The Gaussian assumption of the multiple access interference is verified by simulation and used in the analysis. Numerical examples are also given in this paper according to the analytical results and the channel models proposed by IEEE P802.15 working group. Results show that the MA performance of these two systems depends on the multipath situation, and the MA capacity of an UWB DTR system is twice that of an UWB TR system. A transmission strategy for these two systems is also proposed to improve the MA performance.**

***Index Terms*—Ultra-wideband, transmitted reference system, multiple access performance, multipath environment.**

I. INTRODUCTION

Ultra-wideband (UWB) impulse radio systems transmit data by modulation of subnanosecond pulses. These narrow pulses are distorted by the channel, but often can resolve many distinct propagation paths (multipath) because of their fine time-resolution capability [1]. However, a Rake receiver that implements tens or even hundreds of correlation operations may be required to take full advantage of the available signal energy [2]. On the other hand, a receiver using a single correlator matched to one transmission path may be operating at a 10 - 15dB signal energy disadvantage relative to a full Rake receiver.

Recently, Hoctor and Tomlinson proposed an UWB delay-hopped transmitted-reference (DHTR) system with a simple receiver structure to capture all of the energy available in an UWB multipath channel [3]. In this transmitted reference (TR) system, a reference pulse is transmitted before each data-modulated pulse for the purpose of determining the current multipath channel response. Since the reference and data pulses are transmitted within the coherence time of the channel, it is assumed that the channel responses to these two pulses are the same. The proposed receiver correlates the data signal with the reference to use all the energy of the data signal without requiring additional channel estimation and Rake reception. It is worth noting that the TR approach is not new, but dates back to the early days of communication theory [5],[6],[7].

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Generally TR systems waste communication resources, i.e., power and time, to transmit reference signals. In [4], an ultra-wideband differential transmitted reference (UWB DTR) system, which uses the concept of a TR system without really transmitting references, was investigated. This UWB DTR system differentially encodes the information bits in the transmitter, so the receiver can use a prior data-bearing waveform as a reference. The receiver structure is still simple and implementable, and it saves resources for communication. The performance of UWB TR and UWB DTR systems using different receiver structures in a single user environment was analyzed in [4], but the multiple access performance has not been investigated yet. Section II reviews the UWB DTR system and receiver structure, analyzes the multiple access interference (MAI), verifies the Gaussian assumption of MAI, and evaluates the MA performance of the UWB DTR system. Section III introduces an UWB TR system with multiple access capability, reviews a simple transmitted reference (STR) receiver, and evaluates the MA performance of the TR system using an STR receiver. In section IV, numerical examples are given according to the analytical results in section II and III. The channel realizations in this section are generated using models proposed by IEEE P802.15 working group for wireless personal area network [8]. Fixing the data rate, a strategy to arrange communication resources in UWB DTR and UWB TR systems is proposed to improve the MA performance. Section V is the conclusion.

II. UWB DTR SYSTEM AND DTR RECEIVER

An UWB DTR system uses a prior data-bearing waveform as a reference. In order to do so, the transmitter includes an encoder which differentially encodes the information data bits before an antipodal modulation. Therefore the information is buried in the phase difference of two signals in consecutive frames. The UWB DTR transmitted signal of transmitter n is

$$s_{\mathrm{tr}}^{(n)}(t) = \sum_{i=-\infty}^{\infty} d_i^{(n)} g_{\mathrm{tr}}(t - iT_{\mathrm{f}} - c_i^{(n)} T_{\mathrm{c}}), \qquad (1)$$

where $d_i^{(n)} = d_{i-1}^{(n)} b_{\lfloor i/N_{\mathrm{s}} \rfloor}^{(n)}$ is the encoded bit, and $b_{\lfloor i/N_{\mathrm{s}} \rfloor}^{(n)} \in \{+1, -1\}$ is the information bit in the i^{th} frame of transmitter

n. The index $\lfloor i/N_s \rfloor$, i.e., the integer part of i/N_s, represents the index of the information bit in the i^{th} frame. Hence each bit is transmitted in N_s successive frames to achieve an adequate bit energy in the receiver, and the channel is assumed invariant over this bit time. The hopping sequence $\{c_i^{(n)}\}$ is a pseudo-random code with period $N_p >> N_s$. It is the pulse shift pattern of transmitter n which can eliminate catastrophic collisions because the patterns are different for each transmitter. Each element of the hopping sequence in one period is a random variable uniformly distributed in $\{0, 1, \ldots, N_h - 1\}$. Here $g_{tr}(t)$ is a transmitted monocycle pulse that is non-zero only for $t \in (0, T_p)$, T_f is its repetition time (frame time), and T_c is the duration of one hopping time slot. The frame time which is needed to prevent the interframe interference is that $T_f = (N_h - 1)T_c + T_p + T_{mds}$ where T_{mds} is the multipath channel delay spread.

The received signal of an UWB DTR receiver is

$$r(u,t) = \sum_{n=1}^{N_u} \sum_{i=-\infty}^{\infty} d_i^{(n)} g_i^{(n)}(t - iT_f - c_i^{(n)} T_c - \tau_n) + n_t(u,t) \tag{2}$$

where $n_t(u,t)$ represents a bandpass Gaussian receiver noise with two-sided power spectral density $\frac{N_0}{2}$, and N_u is the number of active transmitters. The received waveform from transmitter n in the i^{th} frame, $g_i^{(n)}(t)$, is the convolution of a single transmitted pulse $g_{tr}(t)$ and the channel impulse response, which includes effects of antennas. The time asynchronism between the clocks of transmitter n and the receiver is τ_n.

Assuming the desired signal is from transmitter 1 without loss of generality, the received signal in (2) can be divided into three portions which are signals from transmitter 1 $s(t)$, signals from undesired transmitters $n_m(u,t)$, and the receiver noise $n_t(u,t)$.

$$r(u,t) = s(t) + n_m(u,t) + n_t(u,t), \tag{3}$$

where

$$s(t) = \sum_{i=-\infty}^{\infty} d_i^{(1)} g_i^{(1)}(t - iT_f - c_i^{(1)} T_c - \tau_1), \tag{4}$$

$$n_m(u,t) = \sum_{n=2}^{N_u} \sum_{i=-\infty}^{\infty} d_i^{(n)} g_i^{(n)}(t - iT_f - c_i^{(n)} T_c - \tau_n). \tag{5}$$

A DTR receiver correlates the signals in two successive frames, and sums the N_s results that are affected by a single information bit to be a decision statistic [4]. Suppose the information bit we want to detect is $b_0^{(1)}$, the detection here is based on the hypothesis testing of $b_0^{(1)}$ and the assumption of perfect synchronization. This implies the receiver knows $\{c_i^{(1)}\}$ and τ_1 completely, and the N_s repetitions of $b_0^{(1)}$ can be added coherently. Let $D_d(u)$ be the decision statistic of $b_0^{(1)}$, then

$$D_d(u) = \sum_{i=0}^{N_s-1} D_d(i,u), \tag{6}$$

$$\begin{aligned} D_d(i,u) &= \int_{iT_f + c_i^{(1)} T_c + \tau_1}^{iT_f + c_i^{(1)} T_c + \tau_1 + T_{corr}} r(u,t) \times r(u, t - T_f + (c_{i-1}^{(1)} - c_i^{(1)}) T_c) dt \\ &= s(i) + \sum_{j=1}^{8} n_i(j), \end{aligned} \tag{7}$$

where T_{corr} is the integration time of the correlator, and the signal $s(i)$ and noises/interferences $n_i(j)$ are explained in the following. Defining the time interval $R_i = [iT_f + c_i^{(1)} T_c + \tau_1, iT_f + c_i^{(1)} T_c + \tau_1 + T_{corr}]$, the receiver noise and undesired transmitters' signals can interfere the statistic $D_d(i,u)$ if their arrival times are in R_i and R_{i-1}. Let $s_i(t)$ be the signal in the i^{th} frame of transmitter 1, $n_{mi}(u,t)$ be undesired transmitters' signals arriving in R_i, and $n_{ti}(u,t)$ be the receiver noise in the time interval R_i, then noises/interferences $n_i(j)$, $j = 1, 2, \ldots, 8$ can be defined in Table I by using these notations. The first column and row in Table I denote the sources which can cause the interference to $D_d(i,u)$. Let $t_{l,k}$ be the element in the l^{th} row and the k^{th} column in Table I, then $t_{l,k}$ is the correlation of $t_{l,1}$ and $t_{1,k}$.

	$s_i(t)$	$n_{m(i)}(u,t)$	$n_{t(i)}(u,t)$
$s_{i-1}(t)$	$s(i)$	$n_i(3)$	$n_i(6)$
$n_{m(i-1)}(u,t)$	$n_i(1)$	$n_i(4)$	$n_i(7)$
$n_{t(i-1)}(u,t)$	$n_i(2)$	$n_i(5)$	$n_i(8)$

TABLE I
SIGNAL AND NOISES IN A DECISION STATISTIC OF A DTR RECEIVER IN A MULTIPLE ACCESS ENVIRONMENT.

In the following derivation of the MA performance of a DTR receiver, some reasonable assumptions are made.

(1) The information bit $b_i^{(n)} \in \{+1, -1\}$ with equal probability, $b_i^{(n)}$ and $b_j^{(n)}$ are independent if $i \neq j$, and $b_i^{(n)}$ and $b_j^{(m)}$ are independent for all i, j if $n \neq m$.

(2) The encoded bits $d_i^{(n)}$ and $d_j^{(m)}$ are independent for all i, j if $n \neq m$. This assumption is directly derived from the previous assumption.

(3) Without network synchronization, the time difference $\tau_n - \tau_1$, $n = 2, \ldots, N_u$ are i.i.d. random variables, and $\tau_n - \tau_1$ mod T_f is uniformly distributed on $[0, T_f)$. Without loss of generality, we can assume $\tau_1 = 0$.

(4) Information bits and differentially encoded bits are independent of hopping sequences and time asynchronisms.

A. Gaussian Assumption

If no signals have the dominating power, the MAI can be approximated by a Gaussian random variable as the numbers of transmitters and arrival paths go large. In order to know how many transmitters and arrival paths can validate this Gaussian assumption, IEEE P802.15 model CM1 and CM3 are used to generate channel realizations to simulate the distribution of $n_i(1) + n_i(3) + n_i(4)$ for different numbers of transmitters. In this simulation, the received pulse is a second order derivative Gaussian pulse with $T_p = 0.7$ nsec, $T_f = 999.6$ nsec, and the signal energies of all transmitters are normalized to 100. The correlator integration times are equal to 50 nsec and 160 nsec for model CM1 and CM3 respectively, and the magnitude of $n_i(1)+n_i(3)+n_i(4)$ is simulated 1000 times using each model to produce the distribution figures. Results show when the numbers of transmitters are greater than 40 and 20 for model CM1 and CM3, the value of $n_i(1)+n_i(3)+n_i(4)$ can be modelled by a Gaussian random variable. Part of the results is shown in Figure 1. Besides, all the other noise/inteference terms and a large bit repetition N_s make this Gaussian assumption of $\sum_{i=0}^{N_s-1}\sum_{j=1}^{8} n_i(j)$ more valid.

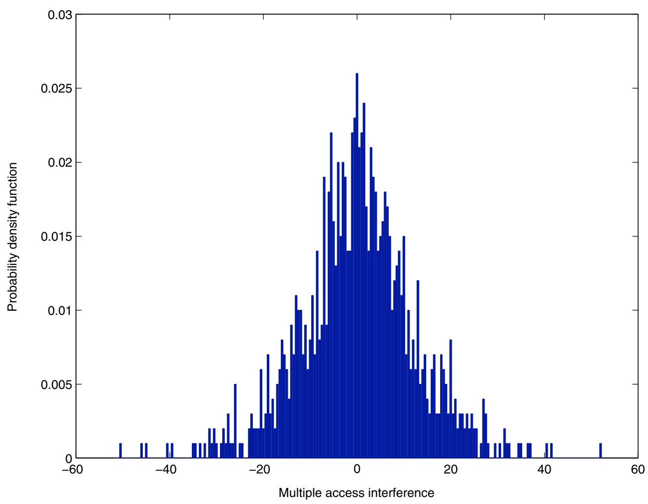

Fig. 1. The simulated probability density function of $n_i(1)+n_i(3)+n_i(4)$ with 20 transmitters. Channel realizations are generated by IEEE P802.15 model CM3.

B. Performance Analysis

Using the Gaussian assumption of the MAI, the bit error probability (BEP) is a Q-function of the signal energy to noise/interference power ratio. Because channels are assumed invariant over one bit time, the subscript i of $g_i^{(n)}(t)$ will be dropped in the following to keep the notation simple. The superscript n is remained to remark differences between the waveforms from different transmitters. Let $n(j)$ denote $\sum_{i=0}^{N_s-1} n_i(j)$. Using the above assumptions (1)-(5) and the mean value of the receiver noise is zero, it can be verified that $\mathbb{E}\{n(j)\}=0$, $j\in\{1,\ldots,8\}$, and $\mathbb{E}\{n(k)n(j)\}=0$ except that $(k,j)=(1,3)$, $(2,6)$, and $(5,7)$. Therefore, given the channel realizations of all users,

$$\mathbb{E}\{D_d(u)|b_0^{(1)}\} = \sum_{i=0}^{N_s-1} s(i) = b_0^{(1)} N_s \int_0^{T_{corr}} [g^{(1)}(t)]^2 dt, \quad (8)$$

$$\mathbb{V}\mathrm{ar}\{D_d(u)\} = \sum_{j=1}^{8}\mathbb{E}\{n^2(j)\} + 2\mathbb{E}\{n(1)n(3)\} + 2\mathbb{E}\{n(2)n(6)\} + 2\mathbb{E}\{n(5)n(7)\}. \quad (9)$$

The bit error probability is

$$P_{bit}^{DTR} = Q\left(\frac{N_s\int_0^{T_{corr}}[g^{(1)}(t)]^2dt}{\sqrt{\mathbb{V}\mathrm{ar}\{D_d(u)\}}}\right). \quad (10)$$

Table I shows that two noises/interferences which are symmetric of the diagonal are caused by the same reason. Both $n_i(2)$ and $n_i(6)$ are the correlation of signals from transmitter 1 and the receiver noise. Signals from transmitter 1 and undesired transmitters produce $n_i(1)$ and $n_i(3)$. Undesired transmitters' signals and the receiver noise generate $n_i(5)$ and $n_i(7)$. Hence two noises/interferences which are symmetric have the same variance. The computation of $\mathbb{V}\mathrm{ar}\{D_d(u)\}$ is messy but doable. The details are eliminated due to the space limitation, and the result is listed below.

$$\begin{aligned}
\mathbb{V}\mathrm{ar}\{D_d(u)\} \cong\ & \frac{4N_s-2}{T_f}\sum_{n=2}^{N_u}\int_{-\infty}^{\infty} R_{1n}^2(\tau_n)d\tau_n \\
& + \frac{2N_s(N_s-2)I_{\{N_s>1\}}}{T_f N_h}\sum_{n=2}^{N_u}\int_{-\infty}^{\infty} R_{1n}^2(\tau_n)d\tau_n \\
& + (2N_s-1)N_0\int_0^{T_{corr}}[g^{(1)}(t)]^2dt + \frac{N_s T_{corr} N_0^2 W}{2} \\
& + \frac{N_0 T_{corr}(N_s N_h + N_s - 1)}{T_f N_h}\sum_{n=2}^{N_u}\int_{-\infty}^{\infty}[g^{(n)}(\tau_n)]^2 d\tau_n \\
& + \frac{1}{T_f N_h^6}[N_s N_h^2 + 2N_h(N_s-1) + (N_s-1)(N_s-2)] \\
& \times \sum_{n=2}^{N_u}\sum_{x=-(N_h-1)}^{N_h-1}\sum_{y=-(N_h-1)}^{N_h-1}(N_h-|x|)(N_h-|y|) \\
& \times\{\int_{-T_{corr}}^{T_{mds}}\left[\int_{\tau}^{T_{corr}+\tau} g^{(n)}(t)g^{(n)}(t+(y-x)T_c)dt\right]^2 d\tau \\
& + \frac{1}{2}\int_{-T_{corr}}^{T_{mds}}\left[\int_{\tau}^{T_{corr}+\tau} g^{(n)}(t)g^{(n)}(t+T_f+(y-x)T_c)dt\right]^2 d\tau\} \\
& + \frac{1}{T_f^2 N_h^2}[N_s N_h^2 + 2N_h(N_s-1) + (N_s-1)(N_s-2)] \\
& \times \sum_{n=2}^{N_u}\sum_{\substack{m=2\\ m\neq n}}^{N_u}\int_0^{T_{corr}}\int_{-v}^{T_{corr}-v} C_n(x)C_m(x)dxdv. \quad (11)
\end{aligned}$$

where $R_{1n}(v)=\int_0^{T_{corr}} g^{(1)}(t)g^{(n)}(t-v)dt$ is similar to a cross-correlation function of $g^{(1)}(t)$ and $g^{(n)}(t)$. But instead of integrating from $-\infty$ to ∞, the integration interval is $[0,T_{corr}]$. Because the channel has delay spread T_{mds}, $R_{1n}(v)\neq 0$ if $v\in[-T_{mds},T_{corr}]\in[-T_f,T_f]$. The autocorrelation function of the signal from transmitter n is $C_n(\tau)=\int_0^{T_{mds}} g^{(n)}(t)g^{(n)}(t-\tau)dt$. The indicator function, $I_{\{N_s>1\}}$, is equal to 1 if the condition in the braces is valid, otherwise, it equals 0.

III. UWB TR SYSTEM AND STR RECEIVER

An UWB TR system transmits one reference pulse before every data-modulated pulse, and the modulation scheme is binary antipodal modulation. The transmitted signal of transmitter n is

$$s_{\rm tr}^{(n)}(t) = \sum_{i=-\infty}^{\infty} d_i^{(n)}[g_{\rm tr}(t-iT_{\rm f}-c_i^{(n)}T_{\rm c}) + b_{\lfloor i/N_{\rm s}\rfloor}^{(n)} g_{\rm tr}(t-iT_{\rm f}-c_i^{(n)}T_{\rm c}-T_{\rm d}^{(n)})]. \quad (12)$$

Each frame contains two monocycle pulses. The first is a reference and the second, $T_{\rm d}^{(n)}$ seconds later, is a data-modulated pulse. Here $\{d_i^{(n)}\}$ is a pseudo-random sequence with period $N_{\rm d} >> N_{\rm s}$ which randomizes polarities of the reference and data-modulated pulses in one frame together. So the information is still buried in the phase difference between the reference and data pulses. Each element of the hopping sequence $\{c_i^{(n)}\}$ in one period is uniformly distributed on $\{0, 1, ..., N_{\rm h}^{(n)} - 1\}$. In order to prevent the interpulse interference, $T_{\rm d}^{(n)}$ should be at least equal to the channel delay spread. The frame time $T_{\rm f} = (N_{\rm h}^{(n)} - 1)T_{\rm c} + T_{\rm p} + T_{\rm d}^{(n)} + T_{\rm mds}$ so no interframe interference exists. Other parameters have been defined in section II.

An STR receiver correlates each received data waveform with the reference received $T_{\rm d}^{(n)}$ seconds earlier if the desired signal is from transmitter n, and sums the $N_{\rm s}$ results over the $N_{\rm s}$ frames that are affected by a single data bit. In this TR system, $T_{\rm d}^{(n)}$ and $\{c_i^{(n)}\}$ are different for each transmitter in order to provide the multiple access capability. Since all the transmitters have different time separation $T_{\rm d}^{(n)}$, only the desired one can have the reference and data-modulated signal alignment because the value of the delay mechanism in the STR receiver is equal to the time separation of the desired transmitter. A Good choice of $T_{\rm d}^{(n)}$ for all users is to make $\int_{-\infty}^{\infty} g_{\rm rx}(t)g_{\rm rx}(t + T_{\rm d}^{(n)} - T_{\rm d}^{(m)})dt$ as close to zero as possible for $n \neq m$, where $g_{\rm rx}(t)$ is a received pulse. Besides, we also want $N_{\rm h}^{(n)}, n = 1, \ldots, N_{\rm u}$, as large as possible to provide a better capability to avoid MA collisions. Due to this reason, $T_{\rm d}^{(n)}$, which is greater than or equal to $T_{\rm mds}$, should be as small as possible for $n = 1, \ldots, N_{\rm u}$ if the frame duration is fixed. And the number of hopping time slots $N_{\rm h}^{(n)}$ is therefore different for each user if all the users have the same frame duration. By using a second order derivative Gaussian received pulse with $T_{\rm p} = 0.7$ nsec as an example, a rule to assign $T_{\rm d}^{(n)}$ and $N_{\rm h}^{(n)}$ is provided in (13) and (14),

$$T_{\rm d}^{(n)} = T_{\rm mds} + \frac{(n-1)T_{\rm p}}{2}, \quad (13)$$

$$N_{\rm h}^{(n)} = N_{\rm h}^{(N_{\rm u})} + \left\lfloor \frac{N_{\rm u} - n}{2} \right\rfloor, \quad (14)$$

where $N_{\rm u}$ is the number of users. As long as $|m - n| \geq 2$, $\int_{-\infty}^{\infty} g_{\rm rx}(t)g_{\rm rx}(t + T_{\rm d}^{(n)} - T_{\rm d}^{(m)})dt = 0$. For $|m-n|$=1, $\int_{-\infty}^{\infty} g_{\rm rx}(t)g_{\rm rx}(t+T_{\rm d}^{(n)} - T_{\rm d}^{(m)})dt$ is 0.07 which is close to zero.

Assuming that desired signals are from transmitter 1, the received signal is composed of three parts which are signals from transmitter 1, signals from other transmitters, and the receiver noise.

$$r(u,t) = s(t) + n_{\rm m}(u,t) + n_{\rm t}(u,t), \quad (15)$$

	$s_{\rm id}(t)$	$n_{\rm m(id)}$	$n_{\rm t(id)}(u,t)$
$s_{\rm ir}(t)$	$s(i)$	$n_i(3)$	$n_i(6)$
$n_{\rm m(ir)}(u,t)$	$n_i(1)$	$n_i(4)$	$n_i(7)$
$n_{\rm t(ir)}(u,t)$	$n_i(2)$	$n_i(5)$	$n_i(8)$

TABLE II

SIGNAL AND NOISES OF A DECISION STATISTIC OF A STR RECEIVER IN A MULTIPLE ACCESS ENVIRONMENT.

where $n_{\rm t}(u,t)$ represents a bandpass Gaussian receiver noise with two-sided power spectral density $\frac{N_0}{2}$, and

$$s(t) = \sum_{i=-\infty}^{\infty} d_i^{(1)}[g_i^{(1)}(t-iT_{\rm f}-c_i^{(1)}T_{\rm c}-\tau_1) + b_{\lfloor i/N_{\rm s}\rfloor}^{(1)} g_i^{(1)}(t-iT_{\rm f}-c_i^{(1)}T_{\rm c}-T_{\rm d}^{(1)}-\tau_1)], \quad (16)$$

$$n_{\rm m}(u,t) = \sum_{n=2}^{N_{\rm u}} \sum_{i=-\infty}^{\infty} d_i^{(n)}[g_i^{(n)}(t-iT_{\rm f}-c_i^{(n)}T_{\rm c}-\tau_n) + b_{\lfloor i/N_{\rm s}\rfloor}^{(n)} g_i^{(n)}(t-iT_{\rm f}-c_i^{(n)}T_{\rm c}-T_{\rm d}^{(n)}-\tau_n)]. \quad (17)$$

All the parameters in above equations have been defined before. Suppose the information bit we want to detect is $b_0^{(1)}$, and let $D_{\rm s}(u)$ be the decision statistic of this bit, then

$$D_{\rm s}(u) = \sum_{i=0}^{N_{\rm s}-1} D_{\rm s}(i,u), \quad (18)$$

$$D_{\rm s}(i,u) = \int_{iT_{\rm f}+c_i^{(1)}T_{\rm c}+T_{\rm d}^{(1)}+\tau_1}^{iT_{\rm f}+c_i^{(1)}T_{\rm c}+T_{\rm d}^{(1)}+\tau_1+T_{\rm corr}} r(u,t) \times r(u,t-T_{\rm d}^{(1)})dt = s(i) + \sum_{j=1}^{8} n_i(j), \quad (19)$$

where $T_{\rm corr}$ is the integration time of the STR receiver, and the signal $s(i)$ and noises/interferences $n_i(j)$ are explained in the following. Defining the time interval $R_{i{\rm r}} = [iT_{\rm f} + c_i^{(1)}T_{\rm c} + \tau_1, iT_{\rm f} + c_i^{(1)}T_{\rm c} + \tau_1 + T_{\rm corr}]$ and $R_{i{\rm d}} = [iT_{\rm f} + c_i^{(1)}T_{\rm c} + T_{\rm d}^{(1)} + \tau_1, iT_{\rm f} + c_i^{(1)}T_{\rm c} + T_{\rm d}^{(1)} + \tau_1 + T_{\rm corr}]$, the receiver noise and undesired transmitters' signals can interfere the statistic $D_{\rm s}(i,u)$ if their arrival times are in $R_{i{\rm r}}$ and $R_{i{\rm d}}$. Let $s_{i{\rm r}}(t)$ and $s_{i{\rm d}}(t)$ be the reference and data-modulated waveforms of transmitter 1 in the $i^{\rm th}$ frame, $n_{{\rm m}(i{\rm a})}(u,t)$ be signals from undesired transmitters arriving in $R_{i{\rm a}}$ for a=r, m, and $n_{{\rm t}(i{\rm a})}(u,t)$ be the receiver noise in $R_{i{\rm a}}$ for a=r, m, then noises/interferences $n_i(j)$,$j = 1, \ldots, 8$, can be defined in Table II by using these notations. The first column and row in Table II denote the sources which can cause the interference to $D_{\rm d}(i,u)$. Let $t_{l,k}$ be the element in the $l^{\rm th}$ row and the $k^{\rm th}$ column in Table II, then $t_{l,k}$ is the correlation of $t_{l,1}$ and $t_{1,k}$.

Like in the discussion of an UWB DTR receiver, we model $\sum_{i=0}^{N_{\rm s}-1}\sum_{j=1}^{8} n_i(j)$ by a Gaussian random variable, so the

BEP of the STR receiver is a Q-function. Let $n(j)$ denote $\sum_{i=0}^{N_s-1} n_i(j)$, $j = 1, \ldots, 8$. Assuming that each element of the pseudo-random sequence in one period, $\{d_i^{(n)}\}$, is in $\{+1, -1\}$ with equal probability, and the pseudo-random sequences of different users are independent, also using assumptions (1), (3), and (4) in section II, it is easy to verify that $\mathbb{E}\{n(j)\} = 0$ and $\mathbb{E}\{n(j)n(k)\} = 0$, $j, k = 1, \ldots, 8$. Given channel realizations, the mean of $D_s(u)$ is equal to $\sum_{i=0}^{N_s-1} s(i)$, and the variance of $D_s(u)$ can be obtained after some manipulation. The result is listed in (21) without computation details. The BEP of an STR receiver can be got immediately by substituting (21) into (20).

$$P_{\text{bit}}^{\text{STR}} = Q\left(\frac{N_s \int_0^{T_{\text{corr}}}[g^{(1)}(t)]^2 dt}{\sqrt{\mathbb{V}\text{ar}\{D_s(u)\}}}\right), \tag{20}$$

$$\begin{aligned} \mathbb{V}\text{ar}\{D_s(u)\} &\cong \frac{4N_s}{T_f}\sum_{n=2}^{N_u}\int_{-\infty}^{\infty} R_{1n}^2(\tau_n)d\tau_n \\ &+N_s N_0 \int_0^{T_{\text{corr}}}[g^{(1)}(t)]^2 dt + \frac{N_s N_0^2 W T_{\text{corr}}}{2} \\ &+\frac{2N_s N_0 T_{\text{corr}}}{T_f}\sum_{n=2}^{N_u}\int_{-\infty}^{\infty}[g^{(n)}(\tau_n)]^2 d\tau_n \\ &+\sum_{n=2}^{N_u}\frac{N_h^{(n)}N_s + N_s(N_s-1)}{T_f N_h^{(n)}} \\ &\times\int_{-T_{\text{corr}}}^{T_{\text{mds}}}\left[\int_{\tau_n}^{T_{\text{corr}}+\tau_n} g^{(n)}(t)g^{(n)}(t+T_d^{(1)}-T_d^{(n)})dt\right]^2 d\tau_n \\ &+\frac{4N_s}{T_f^2}\sum_{n=2}^{N_u}\sum_{m=2}^{N_u}\int_0^{T_{\text{corr}}}\int_{-y}^{T_{\text{corr}}-y} C_n(x)C_m(x)dxdy. \end{aligned} \tag{21}$$

IV. Numerical Examples and Transmission Strategy

The BEP curves averaged over 100 sets of channel realizations are plotted in Figures 2, 3, and 4. For each set of realization, equal energy channels are generated using IEEE P802.15 model CM1 and CM3 for all the users. These two channel models represent two different indoor environments. Model CM1 is fit to channel measurements with 0-4 meters transmitter-receiver separation with line-of-sight, and model CM3 is fit to channel measurements with 4-10 meters transmitter-receiver separation without line-of-sight [8]. The signal parameters used in this section are in the following. The single received pulse is a second order derivative Gaussian pulse with 0.7 nsec duration. The one-sided receiver bandwidth is 2.0GHz which is around the 99% power bandwidth. The channel delay spreads are 50 nsec and 160 nsec for models CM1 and CM3 respectively. The integration times we used here are 25 nsec and 50 nsec for models CM1 and CM3 respectively which are the optimal values obtained by simulations in a single user environment. The hopping time slot duration $T_c = 0.7$ nsec is equal to a pulse width. The x-axis in the figures is $\frac{E_b}{N_0}$ which is the energy per bit to noise power ratio. In Figure 2 and 3, the frame time $T_f = 999.6$ nsec, and the pulse repetition $N_s = 1$, so the transmission rate of each transmitter is around 1Mbps.

Comparing Figure 2 and 3 for both the DTR and STR receivers, the number of users is less if the channel delay spread is larger. The MA performance depends on the multipath environment. In environments that IEEE P802.15 model CM1 and CM3 represent for, over 80 and 60 active transmitters can exist with 1Mbps transmission rate for each one if BEP=1e-5 is required and a DTR receiver is used. With the same condition, over 40 and 30 active transmitters can exist if an STR receiver is used. This MA capability is attractive for an indoor application especially when a simple receiver structure is considered. The Gaussian assumption of the MAI is not a good approximation if the number of users N_u and the bit repetition time N_s are both small. The performance curves under this conditions might have some error. Increasing N_u or N_s can make the Gaussian assumption more valid. The performance floors in Figure 2 and 3 also show that the MAI dominates the performance as the number of users increases. Among all the interferences, the crosscorrelation of signals from two undesired transmitters degrades the performance most when the number of users is large.

Figure 2 and 3 also show that, compared to an STR receiver, a DTR receiver can double the number of active transmitters and decrease the required $\frac{E_b}{N_0}$ by 3dB when BEP=1e-5. The reasons are an STR receiver spends half of the power on the reference, and the probability of collision is doubled because two pulses are transmitted in a frame. We should keep in mind that an STR receiver needs a correlator with a fixed delay mechanism, but a DTR receiver needs a correlator with a variable delay mechanism [4]. Hence the complexity of a DTR receiver is higher than an STR receiver.

Figure 4 compares the results of different combinations of N_s and T_f for a DTR system with a fixed data rate. Solid lines represent a case in which $T_f = 9996$ nsec and $N_s = 1$, and dot lines represent a case in which $T_f = 499.8$ nsec and $N_s = 20$. In both cases, the data rate is around 100Kbps. Figure 4 shows that concentrating the bit energy in a pulse has better MA performance than distributing the bit energy to more than one pulse. The main reason of this nonlinear behavior in an UWB DTR system can be explained by using $\mathbb{V}\text{ar}\{n(5)\} = N_s N_0 T_{\text{corr}} \sum_{n=2}^{N_u} \int_0^{T_{\text{corr}}}[g^{(n)}(\tau_n)]^2 d\tau_n / 2T_f$ as an example. The noise power in $n(5)$ to the signal energy ratio is

$$\begin{aligned} \frac{\mathbb{V}\text{ar}\{n(5)\}}{[\mathbb{E}\{D_d(u)\}]^2} &= \frac{N_s N_0 T_{\text{corr}}\sum_{n=2}^{N_u}\int_0^{T_{\text{corr}}}[g^{(n)}(\tau_n)]^2 d\tau_n}{2T_f\left[N_s\int_0^{T_{\text{corr}}}[g^{(1)}(\tau_n)]^2 d\tau_n\right]^2} \\ &= \frac{N_0 T_{\text{corr}}\sum_{n=2}^{N_u}\int_0^{T_{\text{corr}}}[g^{(n)}(\tau_n)]^2 d\tau_n}{2N_s T_f\left[\int_0^{T_{\text{corr}}}[g^{(1)}(\tau_n)]^2 d\tau_n\right]^2}. \end{aligned}$$

In scenario 1, let $N_s = 20$, then

$$\frac{\mathbb{V}\text{ar}\{n(5)\}}{[\mathbb{E}\{D_d(u)\}]^2} = \frac{N_0 T_{\text{corr}}\sum_{n=2}^{N_u}\int_0^{T_{\text{corr}}}[g^{(n)}(\tau_n)]^2 d\tau_n}{40T_f\left[\int_0^{T_{\text{corr}}}[g^{(1)}(\tau_n)]^2 d\tau_n\right]^2}, \tag{22}$$

and the data rate is $\frac{1}{20T_f}$. In scenario 2, we transmit each bit only one time, but maintain the same data rate and average power as in scenario 1. So now $N_s = 1$, and the energy in a pulse and T_f increase 20 times. The noise power in $n(5)$ to the signal energy ratio is now

$$\frac{\mathbb{V}\mathrm{ar}\{n(5)\}}{[\mathbb{E}\{D_{\mathrm{d}}(u)\}]^2} = \frac{N_0 T_{\mathrm{corr}} \sum_{n=2}^{N_{\mathrm{u}}} \int_{-\infty}^{\infty} [g^{(n)}(\tau_n)]^2 d\tau_n}{800 T_{\mathrm{f}} \left[\int_{-\infty}^{\infty} [g^{(1)}(\tau_n)]^2 d\tau_n\right]^2}. \quad (23)$$

The bit energy, data rate, and average power are the same in these two scenarios. Comparing (22) and (23), the bit energy to noise power ratio in scenario 2 is 20 times higher than the one in scenario 1. Without violating the FCC regulation, this nonlinear behavior tells us to concentrate the bit energy in as few pulses as possible and extend the frame time to maintain the data rate and average power. Of course the duration between two pulses should be less than the channel coherent time in order for the correlation receiver to work. This nonlinear behavior appears in both the UWB DTR and UWB TR systems, so the same transmission strategy applies to both of them.

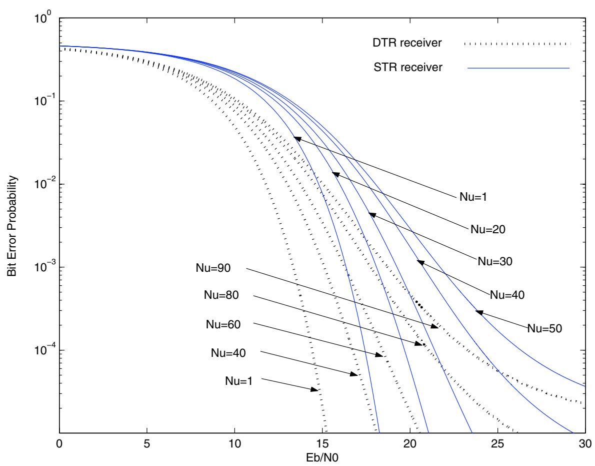

Fig. 2. BEP versus $\frac{E_b}{N_0}$ for DTR and STR receivers with channel model CM1, and $T_{\mathrm{corr}} = 25$ns.

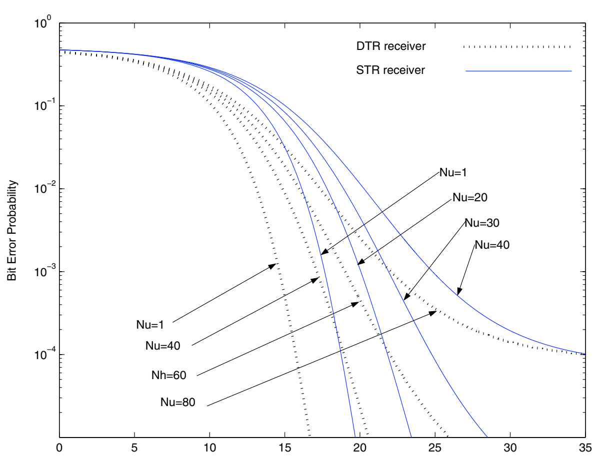

Fig. 3. BEP versus $\frac{E_b}{N_0}$ for DTR and STR receivers with channel model CM3, and $T_{\mathrm{corr}} = 50$ns.

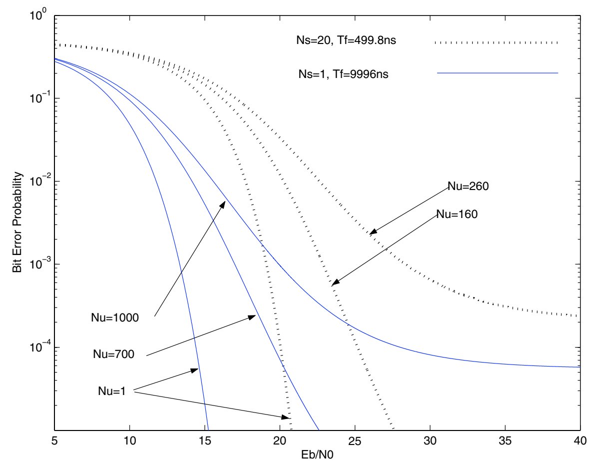

Fig. 4. Comparison of two scenarios with fixed data rate for a DTR receiver. Channel realizations generated using model CM1, and $T_{\mathrm{corr}} = 25$ns.

V. Conclusion

The multiple access performance of the UWB DTR and UWB TR systems is evaluated. Compared to an STR receiver, a DTR receiver doubles the number of users with higher receiver complexity. With 1 Mbps transmission rate, BEP=1e-5, and multipath realizations generated using IEEE P802.15 models CM1 and CM3, over 80 and 60 simultaneous transmitters are predicted for a DTR system, and over 40 and 30 simultaneous transmitters are predicted for a TR system with perfect power control. These capacities are attractive in an indoor application especially if simple receiver structures are considered. For a fixed data rate, in order to achieve a best MA performance, we should keep the bit repetition time as small as possible and lengthen the frame time to maintain the same average power level.

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