COMPARISON OF TRANSMITTED- AND STORED-REFERENCE SYSTEMS FOR ULTRA-WIDEBAND COMMUNICATIONS

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Abstract— *Performance of a transmitted-reference system (TR) with a cross-correlation receiver is compared with that of a stored-reference system (SR) with a Rake receiver for ultra-wideband (UWB) communications. The effect of channel estimation error that appears in SR systems is taken into account, eliminating some unfair advantage in comparisons with TR systems. Some insight on system design for multipath channels is drawn from the comparison.*

I. INTRODUCTION

One form of UWB radio is to transmit short pulses with duration on the order of a nanosecond [1], which is usually dubbed an impulse radio. When a UWB signal travels through the channel, it usually gets distorted, which leads to the reception of replicas of the signal with different delays, amplitudes, phases or even different shapes. Usually, the distortion is partially compensated for or equalized at the receiver by way of channel estimation and proper receiver design. To perform the channel estimation, some knowledge of the transmitted waveform is usually assumed at the receiver, and channel estimation can then proceed. A receiver design for multipath channels based on this philosophy is called a *stored-reference* system (SR) since knowledge about the waveform or the signal transmitted is available at the receiver. With channel estimation available in a SR receiver, signals with different delays due to multipath effects can be combined by a Rake receiver to provide decision statistics for data detection.

However, for highly dispersive channels such as the ones encountered by UWB radios, effective channel estimation and the associated receiver design to equalize the channel and to collect enough energy for data detection seems difficult. Therefore, a receiver design based on another philosophy is called a *transmitted-reference* system (TR) [2]. One of the primitive forms of TR system transmits a pair of pulses in each frame. The first pulse is unmodulated and used to provide the multipath channel's pulse response function to the demodulator, while the second pulse is data-modulated. At the receiver side, a simple delay line and a correlator is used to demodulate the data symbols. The advantage of the TR system compared to the SR system is its ease of implementation since complex channel estimation is not required. However, the major drawback for TR systems is the noise-cross-noise term that appears at the correlator output, which severely degrades the performance at low pulse signal-to-noise ratios (SNR).

When SR and TR systems are compared, the channel estimation error that appears in SR systems is usually neglected, which gives the SR system some unfair advantage in comparisons with the TR systems. In this paper, we will take this effect into account and compare their performances. In addition, we will investigate the effects of various system parameters on the performance and draw some insight on system design for multipath channels.

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II. PERFORMANCE OF A TRANSMITTED-REFERENCE SYSTEM

The TR system considered here is the one introduced in [2]. The advantage of this delay-hopped TR system is that only a delay line and a correlator is required at the receiver to perform data detection, which excludes the need for estimation of the complex multipath channels usually encountered by UWB systems. The transmit signal is given by

$$s_{\mathrm{TR}}(t) = \sum_m p(t - mT_f) + b_{\lfloor m/N_s \rfloor} p(t - T_d - mT_f) \quad (1)$$

where $p(t)$ is the monocycle waveform or pulse with support in $t \in \{0, T_w\}$, m the frame index and T_f the frame time. The first pulse in each frame provides the pulse response function for data detection, while the second one transmitted T_d seconds later is modulated by the binary data $b_i \in \{\pm 1\}$. Each bit is transmitted over N_s consecutive frames to provide adequate energy for data detection. For a TR system operating in a multipath channel with multipath delay spread T_{mds}, it is usually designed that $T_f \geq 2T_d \geq 2T_{mds}$ [3].

The received signal is modeled as

$$r(t) = h(t) * s_{\mathrm{TR}}(t) + n(t) \quad (2)$$

where $h(t)$ is the channel impulse response function and $n(t)$ is the additive white Gaussian noise process (AWGN) with two-sided power spectral density (PSD) $N_0/2$. In this paper, we assume a quasi-static specular multipath channel whose impulse response stays fixed for the duration of a few bits. The channel impulse response is modelled as

$$h(t) = \sum_i \alpha_i \delta(t - \tau_i) \quad (3)$$

where i is the path index, α_i and τ_i are the path amplitude and delay, respectively. For convenience of analysis, we discretize the channel response according to the timing resolution limitation of either the receiver sampling rate or the bandwidth of the pulse, i.e., we let $\tau_i = i\Delta$ where Δ is the minimum timing resolution. Therefore, the range of the path index is $0 \le i \le \lfloor T_{mds}/\Delta \rfloor$. In the following analysis, the summation limits of the path index will be dropped with the understanding that they lie in this range. Without loss of generality, we also assume an energy-normalized channel impulse response $\sum_i \alpha_i^2 = 1$ and define the pulse energy as $E_p \triangleq \int_{-\infty}^{\infty} p^2(t)dt$.

Some variations of the primitive form of TR system were considered in [3, 4] and the associated optimal and sub-optimal receivers were proposed. In this paper, however, we will only consider the TR system introduced in [2] with the simple cross-correlation receiver. The decision statistic z for b_0, after integrating over N_s frames conditioned on $b_0 = 1$, is given by

$$\begin{aligned} z &= \sum_{m=0}^{N_s-1} \int_{mT_f+T_d}^{mT_f+T_d+T_{mds}} r(t-T_d)r(t)dt \\ &= m_Z + N_1 + N_2 + N_3 \end{aligned} \tag{4}$$

where

$$m_Z = N_s E_p \left[1 + \sum_i \sum_{\substack{j \\ i \neq j}} \alpha_i \alpha_j R_p[(i-j)\Delta] \right] \tag{5}$$

and

$$\begin{aligned} N_1 &\triangleq \sum_{m=0}^{N_s-1} \int_{mT_f+T_d}^{mT_f+T_d+T_{mds}} n(t-T_d) \\ &\quad \cdot \left[\sum_j \alpha_j p(t - j\Delta - mT_f) \right] dt \\ N_2 &\triangleq \sum_{m=0}^{N_s-1} \int_{mT_f+T_d}^{mT_f+T_d+T_{mds}} \left[\sum_i \alpha_i p(t - i\Delta - mT_f) \right] \\ &\quad \cdot n(t)dt \\ N_3 &\triangleq \sum_{m=0}^{N_s-1} \int_{mT_f}^{mT_f+T_d+T_{mds}} n(t-T_d) \cdot n(t)dt. \end{aligned} \tag{6}$$

The function $R_p(\tau)$ is the normalized auto-correlation function of $p(t)$ given by $R_p(\tau) \triangleq \int_{-\infty}^{\infty} p(t)p(t-\tau)dt/E_p$. Note m_Z is the desired correlator output and is the mean of the random variable z. The N_i's are three uncorrelated and zero-mean random variables because $n(t)$ is assumed white, and they represent noise components in the detection process. Note that N_1 comes from the product of the noise in the reference part of the frame and the desired signal in the data part, while N_2 from the product of the desired signal in the reference and the noise in the data part. N_3 is the noise-cross-noise term that dominates and seriously degrades the TR system performance in the low pulse SNR region.

After some straightforward calculations, we have the following expressions the variances of N_i's.

$$\begin{aligned} \mathbb{V}\text{AR}\{N_1\} &= N_s E_p \frac{N_0}{2} \left[1 + \sum_i \sum_{\substack{j \\ i \neq j}} \alpha_i \alpha_j R_p[(i-j)\Delta] \right] \\ \mathbb{V}\text{AR}\{N_2\} &= \mathbb{V}\text{AR}\{N_1\} \\ \mathbb{V}\text{AR}\{N_3\} &= N_s \frac{N_0^2}{2} T_{mds} W \end{aligned} \tag{7}$$

where W is the one-sided noise bandwidth of the receiver. Let's define the detection SNR at the correlator output as

$$SNR_{\text{TR}} \triangleq \frac{m_Z^2}{\mathbb{V}\text{AR}\{N_1\} + \mathbb{V}\text{AR}\{N_2\} + \mathbb{V}\text{AR}\{N_3\}}. \tag{8}$$

Substituting the expressions gives

$$\begin{aligned} SNR_{\text{TR}}^{-1} &= \left(\frac{N_0}{E_p}\right) \frac{1}{N_s} \frac{1}{1 + \sum_i \sum_{j, i\neq j} \alpha_i \alpha_j R_p[(i-j)\Delta]} \\ &\quad + \left(\frac{N_0}{E_p}\right)^2 \frac{T_{mds} W}{2N_s} \frac{1}{1 + \sum_i \sum_{j, i \neq j} \alpha_i \alpha_j R_p[(i-j)\Delta]}. \end{aligned} \tag{9}$$

In the absence of inter-pulse interference, i.e., $R_p(i\Delta) = 0$ for $i \neq 0$ and hence all the i and j cross-terms in the expression disappear, the result reduces to

$$SNR_{\text{TR}} = \left\{ \left(\frac{N_0}{E_p}\right) \frac{1}{N_s} + \left(\frac{N_0}{E_p}\right)^2 \frac{T_{mds} W}{2N_s} \right\}^{-1}. \tag{10}$$

Since this result is valid for both $b_0 = \pm 1$, using appeals to the central limit theorem to approximate N_3 as Gaussian [3], the probability of bit error P_e is given by $P_e = Q\left(\sqrt{SNR_{\text{TR}}}\right)$ where $Q(x) \triangleq \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \exp\{-\frac{y^2}{2}\}dy$. Note this result is identical to Eq. (27) in [3] after replacing E_p with $E_f/2$ where E_f is the frame energy. There is a factor of two because there are two pulses in each frame.

There are two terms in the SNR expression (10), one proportional to (E_p/N_0), the other proportional to $(E_p/N_0)^2$. For low pulse SNR (E_p/N_0) region, the second term dominates the first one. This happens when

$$\begin{aligned} & \left(\frac{N_0}{E_p}\right)^2 \frac{T_{mds} W}{2N_s} \gg \left(\frac{N_0}{E_p}\right) \frac{1}{N_s} \\ \Leftrightarrow \quad & \left(\frac{E_p}{N_0}\right) \ll \frac{T_{mds} W}{2}. \end{aligned} \tag{11}$$

As we will see in the numerical examples below, this condition is usually met in the P_e range of interest $(P_e \approx 10^{-4})$ with only a moderate value of N_s. Therefore, SNR_{TR} can be approximated by

$$SNR_{\text{TR}} \approx \left(\frac{E_f}{N_0}\right)^2 \frac{N_s}{2T_{mds} W} \tag{12}$$

where we have replaced E_p by $E_f/2$.

III. PERFORMANCE OF A STORED-REFERENCE SYSTEM WITH A RAKE RECEIVER

For SR UWB systems, the transmitted signal is simply a sequence of pulses modulated by binary data.

$$s_{\mathrm{SR\_DATA}}(t) = \sum_{m=0}^{N_b N_s} b_{\lfloor m/N_s \rfloor} p(t - mT_f) \tag{13}$$

where N_s is the number of frames used to transmit a single data bit and N_b is the number of bits in each block. Before the data-modulated pulses, β unmodulated pulses are transmitted to facilitate the channel estimation.

$$s_{\mathrm{SR\_CE}}(t) = \sum_{m=-\beta}^{-1} p(t - mT_f) \tag{14}$$

The overall SR signal is the sum of these two, i.e., $s_{\mathrm{SR}}(t) = s_{\mathrm{SR\_CE}}(t) + s_{\mathrm{SR\_DATA}}(t)$. In the following analysis, we assume that the channel stays static for the duration of a block of $(\beta + N_b N_s)$ frames. The received signal $r(t)$ is given by (2) with $s_{\mathrm{TR}}(t)$ replaced by $s_{\mathrm{SR}}(t)$.

A. CHANNEL ESTIMATION

The task of the channel estimator is to estimate the gain of each path, α_i, in the discrete-time channel model using the β unmodulated pulses. The log-likelihood function of the received signal $r(t)$ conditioned on path gains α_i's is given by

$$\begin{aligned}\ln \Lambda(r(t)|\{\alpha_i\}) =& \beta E_p \sum_i \sum_j \alpha_i \alpha_j R_p[(i-j)\Delta] \\ & -2 \sum_i \alpha_i \int_{-\beta T_f}^{0} r(t) \left[\sum_{m=-\beta}^{-1} p(t - i\Delta - mT_f) \right] dt.\end{aligned} \tag{15}$$

In this paper, we consider an idealized situation where there is no inter-pulse interference for channel estimation. Taking the derivative of $\ln \Lambda(r(t)|\{\alpha_i\})$ with respect to α_i and equating it to zero gives the maximum-likelihood estimate of α_i as

$$\hat{\alpha}_i = \frac{1}{\beta E_p} \int_{-\beta T_f}^{0} r(t) \left[\sum_{m=-\beta}^{-1} p(t - i\Delta - mT_f) \right] dt. \tag{16}$$

The estimate $\hat{\alpha}_i$ can be shown to be an unbiased estimate of α_i and a Gaussian random variable with mean and variance given by

$$\hat{\alpha}_i \sim \mathcal{N}\left(\alpha_i, \frac{1}{2\beta} \frac{N_0}{E_p} \right). \tag{17}$$

Note the channel estimation technique described here is an *unstructured* one [6], in that we discard the structure of the channel impulse response and assign uniform spacing Δ between paths. One drawback of this technique is that it may lead to the estimation of many paths that do not even exist. There are techniques to alleviate this problem. However, we will not discuss them here.

B. SNR PERFORMANCE ANALYSIS

Suppose the SR system employs a maximal-ratio combining (MRC) Rake receiver with L fingers and is able to ideally locate the L strongest paths of the channel impulse response. The Rake receiver then equivalently generates a template $\hat{h}_L(t)$ to correlate the received signal $r(t)$, where $\hat{h}_L(t)$ is given by

$$\hat{h}_L(t) = \sum_{i \in I_L} \hat{\alpha}_i p(t - i\Delta). \tag{18}$$

The set I_L is an index set of size L containing indices of the L strongest paths. For purposes of comparisons with the TR systems, we rewrite the channel estimate $\hat{\alpha}_i$ as $\hat{\alpha}_i = \alpha_i + \nu_i$ where ν_i is a zero-mean Gaussian random variable with variance $\left(\frac{1}{2\beta}\frac{N_0}{E_p}\right)$. The decision statistic z for b_0 conditioned on $b_0 = 1$ is then given by

$$\begin{aligned} z &= \sum_{m=0}^{N_s-1} \int_{mT_f}^{(m+1)T_f} r(t) \hat{h}_L(t - mT_f) dt \\ &= m_Z + N_1 + N_2 + N_3 \end{aligned} \tag{19}$$

where

$$m_Z = N_s E_p \left[\sum_{i \in I_L} \alpha_i^2 + \sum_{i \in I_L} \sum_{j \notin I_L} \alpha_i \alpha_j R_p[(i-j)\Delta] \right] \tag{20}$$

and

$$\begin{aligned} N_1 &\triangleq \sum_{m=0}^{N_s-1} \int_{mT_f}^{(m+1)T_f} \left[\sum_{j \in I_L} \nu_j p(t - j\Delta - mT_f) \right] \cdot \left[\sum_i \alpha_i p(t - i\Delta - mT_f) \right] dt \\ N_2 &\triangleq \sum_{m=0}^{N_s-1} \int_{mT_f}^{(m+1)T_f} \left[\sum_{j \in I_L} \alpha_j p(t - j\Delta - mT_f) \right] \cdot n(t) dt \\ N_3 &\triangleq \sum_{m=0}^{N_s-1} \int_{mT_f}^{(m+1)T_f} \left[\sum_{j \in I_L} \nu_j p(t - j\Delta - mT_f) \right] \cdot n(t) dt. \end{aligned} \tag{21}$$

After some straightforward calculations, as in TR systems, we can show that the N_i's in SR systems are also zero-mean and uncorrelated random variables with variances given by

$$\begin{aligned} \mathrm{VAR}\{N_1\} &= N_s^2 E_p^2 \sum_{i \in I_L} \mathrm{VAR}\{\nu_i\} \left[\sum_j \alpha_j R_p[(i-j)\Delta] \right]^2 \\ \mathrm{VAR}\{N_2\} &= N_s E_p \frac{N_0}{2} \left[\sum_{i \in I_L} \alpha_i^2 + \sum_{i \in I_L} \sum_{\substack{j \in I_L \\ i \neq j}} \alpha_i \alpha_j R_p[(i-j)\Delta] \right] \\ \mathrm{VAR}\{N_3\} &= N_s E_p \frac{N_0}{2} \left[\sum_{i \in I_L} \mathrm{VAR}\{\nu_i\} \right]. \end{aligned} \tag{22}$$

The random variables N_1 and N_2 are exactly Gaussian while N_3 is generally not. For L large enough, we can use the central-limit theorem concept to approximate N_3 as Gaussian. On the other hand, for large N_s, N_1 dominates over N_2 and N_3 (since the variance of N_1 increases with N_s^2 while that of N_2 or N_3 increases with N_s only), so we can approximate $(N_1 + N_2 + N_3)$ as Gaussian as well.

After substituting, the detection SNR at the L-finger MRC-Rake receiver output is given by

$$SNR_{\text{Rake}}^{-1} = \left(\frac{N_0}{E_p}\right) \cdot \left\{ \frac{\sum_{i\in I_L}\left[\sum_j \alpha_j R_p[(i-j)\Delta]\right]^2}{\left[\sum_{i\in I_L}\alpha_i^2 + \sum_{i\in I_L}\sum_{j\notin I_L}\alpha_i\alpha_j R_p[(i-j)\Delta]\right]^2} + \frac{1}{2N_s}\frac{\sum_{i\in I_L}\alpha_i^2 + \sum_{i\in I_L}\sum_{\substack{j\in I_L\\ i\neq j}}\alpha_i\alpha_j R_p[(i-j)\Delta]}{\left[\sum_{i\in I_L}\alpha_i^2 + \sum_{i\in I_L}\sum_{j\notin I_L}\alpha_i\alpha_j R_p[(i-j)\Delta]\right]^2}\right\} + \left(\frac{N_0}{E_p}\right)^2 \frac{L}{4\beta N_s}\frac{1}{\left[\sum_{i\in I_L}\alpha_i^2 + \sum_{i\in I_L}\sum_{j\notin I_L}\alpha_i\alpha_j R_p[(i-j)\Delta]\right]^2}. \tag{23}$$

Define

$$\gamma(L) \triangleq \sum_{i\in I_L}\alpha_i^2. \tag{24}$$

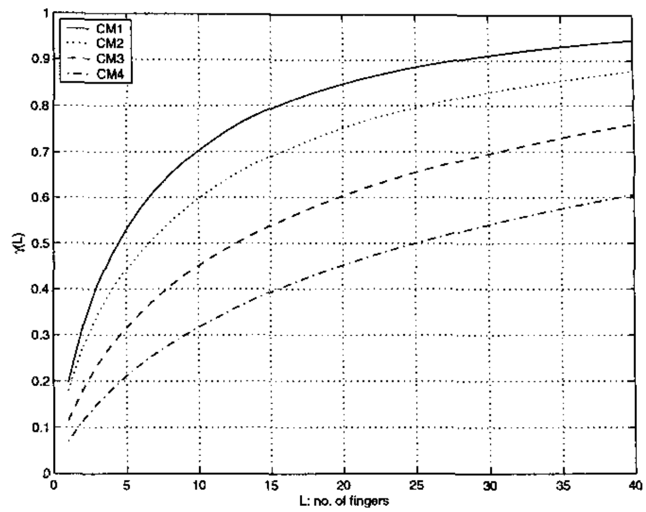
Note $\gamma(L)$ sums to one and hence $\gamma(L)$ represents the fraction of the multipath energy captured by the Rake receiver. In the absence of inter-pulse interference, i.e., all the i and j cross-terms disappear, we have

$$SNR_{\text{Rake}} = \left\{\left(\frac{N_0}{E_p}\right)\frac{1}{\gamma(L)}\frac{\beta+N_s}{2\beta N_s} + \left(\frac{N_0}{E_p}\right)^2\frac{L}{\gamma^2(L)}\frac{1}{4\beta N_s}\right\}^{-1} . \tag{25}$$

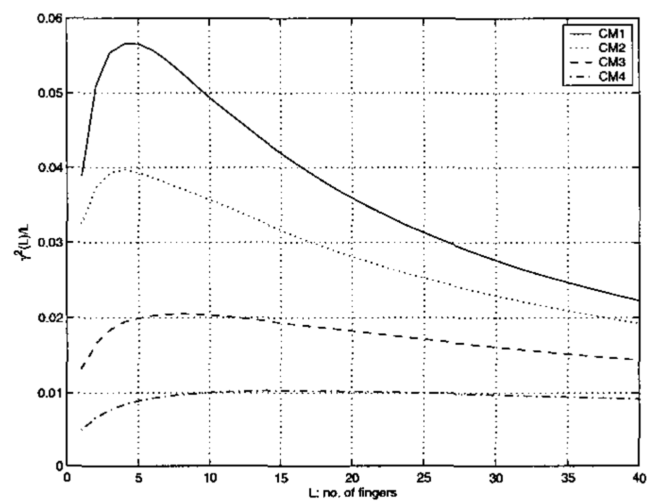
Some observations about (25) are in order. Firstly, as in the TR system, we have the first term proportional to (E_p/N_0), and the second one proportional to $(E_p/N_0)^2$. Also, the first term is also proportional to $\gamma(L)$, while the second term proportional to $\gamma^2(L)/L$. These two functions are plotted in Fig. 1 for all four channel models suggested in [5]. The results are averaged over 100 channel realizations. Apparently, $\gamma(L)$ is an increasing function of L, which means for high pulse SNR, i.e., when the first term dominates, the larger L the better performance as the receiver collects more energy. On the other hand, $\gamma^2(L)/L$ has a maxima, which means for low pulse SNR, i.e., when the second term dominates, there is an optimal L that gives the best performance. Increasing the number of fingers of the Rake receiver beyond that point will only degrade the performance.

Secondly, the first term in the SNR expression dominates when

$$\left(\frac{N_0}{E_p}\right)\frac{1}{\gamma(L)}\frac{\beta+N_s}{2\beta N_s} \gg \left(\frac{N_0}{E_p}\right)^2\frac{L}{\gamma^2(L)}\frac{1}{4\beta N_s}$$
$$\Leftrightarrow \quad \left(\frac{E_p}{N_0}\right) \gg \frac{L}{\gamma(L)}\frac{1}{2(\beta+N_s)}. \tag{26}$$



(a) $\gamma(L)$: fractional of channel energy captured



(b) $\gamma^2(L)/L$

Fig. 1. $\gamma(L)$ and $\gamma^2(L)/L$ of multipath channel models suggested in [5]

The function $L/\gamma(L)$ is plotted in Fig. 2. Note that as L increases, $\gamma(L)$ saturates and approaches one, $L/\gamma(L)$ then increases linearly with L for large L. Therefore, for the condition to be met, the number L cannot be too large. As will be seen in the numerical examples below, in the P_e range of interest and for a practical number of L, this condition is usually satisfied. Therefore, the detection SNR can be approximated by

$$SNR_{\text{Rake}} \approx 2N_s\gamma(L)\left(\frac{E_f}{N_0}\right)\left(\frac{\beta}{\beta+N_s}\right) \tag{27}$$

and it increases with L.

Thirdly, β represents the ratio of the energy used on channel estimation to the energy in one data pulse. As β ap-

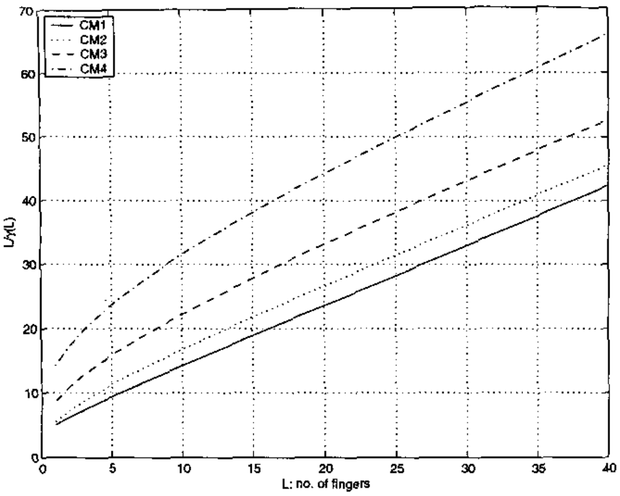
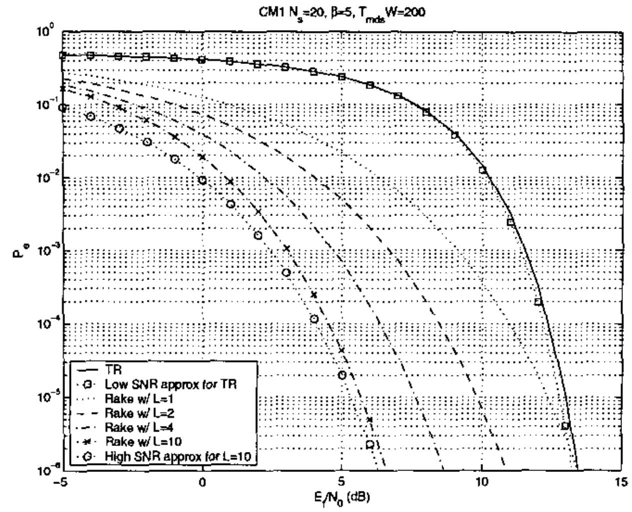
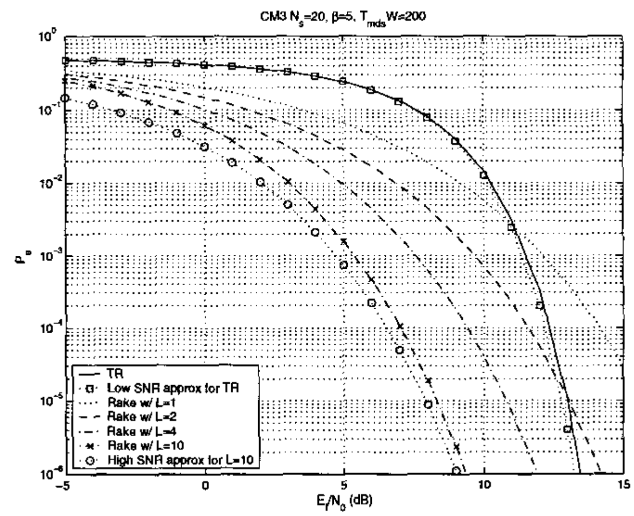


Fig. 2. $L/\gamma(L)$ of multipath channel models suggested in [5]



(a) CM1



(b) CM3

Fig. 3. Performance of UWB TR and SR systems in the absence of inter-pulse interference

proaches infinity, the channel estimation error vanishes as can be seen in (17). In that case, the detection SNR is given by

$$\lim_{\beta \to \infty} SNR_{\text{Rake}} = 2N_s\gamma(L)\left(\frac{E_f}{N_0}\right) \qquad (28)$$

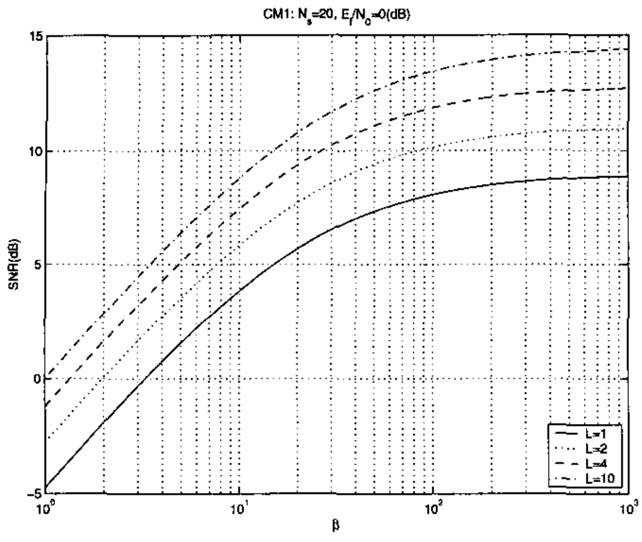
which is the classic result.

IV. NUMERICAL EXAMPLES AND COMPARISON OF SR AND TR SYSTEMS

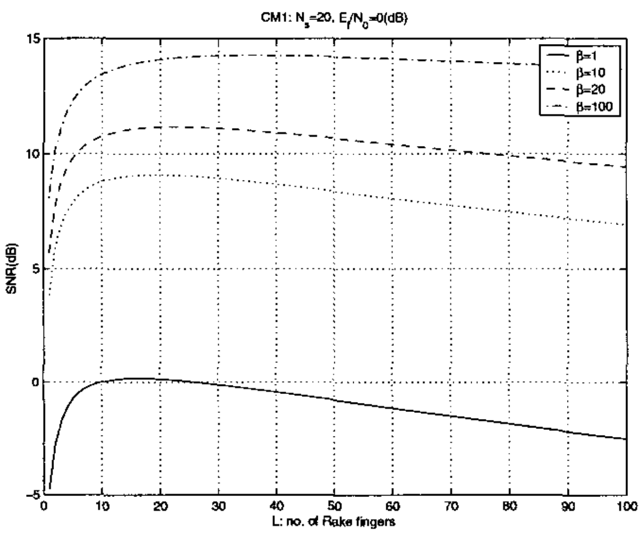
The probability of bit error P_e of the TR system and that of the SR system with Rake receivers $L = \{1, 2, 4, 10\}$, along with the low pulse SNR approximation for the TR system (12) and the high pulse SNR approximation for the Rake receiver (27) with $L = 10$ is plotted in Fig. 3 for channel models CM1 and CM3 from [5]. In this figure, inter-pulse interference has been ignored and the results are averaged over 100 channel realizations. We first note that the low pulse SNR approximation to the TR system and the high pulse SNR approximation to the SR system are fairly accurate for the P_e range and the system parameters of interest. Since these approximations are valid, SNR_{Rake} increases linearly with (E_f/N_0) while SNR_{TR} with $(E_f/N_0)^2$, which explains the different slopes of P_e with respect to (E_f/N_0) and indicates the better P_e performance for the TR systems in high pulse/frame SNR region. Also as expected, for the numbers L considered in this example, P_e is a decreasing function of L. In addition, the P_e cross point of the SR and the TR systems gets lower as the channel gets more diffusive (CM1→CM3). The inter-pulse interference uniformly degrades the performance in all cases. The results are not presented here due to space limitations.

We now look at the performance of the SR systems with a Rake receiver as a function of β and L. For $\beta \ll N_s$, as shown in the expression of the detection SNR (27), the denominator of the fraction is dominated by N_s, so SNR is a linear function of β. As $\beta \to \infty$, the detection SNR approaches the classic result with noiseless channel estimate (28) and eventually becomes independent of β. This behavior is illustrated in Fig. 4(a). Note that for small values of β, the overall detection SNR at the receiver output increases linearly with β (the energy spent on channel estimation), which justifies putting more energy into channel estimation at the expense of some spectral efficiency in this range.

As indicated in (26), for L small enough, the high pulse SNR approximation for the SR systems is valid and the detection SNR is an increasing function of L. However, when L gets larger, this approximation is no longer valid and there exists an optimal L beyond which the detection SNR decreases with L as can be seen from (25). The detection SNR



(a) The detection SNR as a function of β



(b) The detection SNR as a function of L

Fig. 4. The detection SNR of the SR systems as a function of β and L for the SR systems

for the SR systems with various values of β is plotted as a function of L in Fig. 4(b). Note that as β increases, the optimal L gradually increases as well. Also, the deteriorating performance of the detection SNR as a function L is more obvious for smaller values of β. This is because for smaller values of β, the receiver collects more estimation error for the same value of L and this estimations error builds up faster as L increases. However, when it comes to P_e, since $Q(\cdot)$ is a non-linear function, this phenomenon is more obvious for larger values of β instead.

From previous examples, the low pulse SNR approximation for the TR systems and the high pulse SNR approximation for the SR systems are shown to be valid. Therefore, we are able to do the following simple analysis and estimate the P_e cross point of the TR and SR systems.

$$\begin{aligned} &P_e(\mathrm{TR}) < P_e(\mathrm{Rake}) \\ \Leftrightarrow\quad &\left(\frac{E_f}{N_0}\right)^2 \frac{N_s}{2T_{mds}W} > 2N_s\gamma(L)\left(\frac{E_f}{N_0}\right)\left(\frac{\beta}{\beta+N_s}\right) \\ \Leftrightarrow\quad &\left(\frac{E_f}{N_0}\right) > \left(\frac{E_f}{N_0}\right)_{\mathrm{THR}} \triangleq \left(\frac{4\beta}{\beta+N_s}\right)\cdot(T_{mds}W)\cdot\gamma(L). \end{aligned} \tag{29}$$

As the threshold frame SNR $(E_f/N_0)_{\mathrm{THR}}$ gets smaller, it is more advantageous for the TR systems to outperform the SR systems. From (29), we can conclude that as the channel gets more diffusive ($\gamma(L)$ gets smaller for a fixed L) or when the number of fingers is not large enough, the TR system is more likely to outperform the SR system. Also obviously, as $(T_{mds}W)$ gets smaller, which results in reduced noise in the TR receiver, it also decreases the threshold. On the other hand, increasing β reduces the variance of channel estimation error on the reference signal in the SR systems, which leads to an increased threshold.

Interestingly, the threshold also depends on and is a decreasing function of N_s. For TR systems, the low pulse SNR approximation is used, so only one noise term (noise-cross-noise) N_3 (6) enters the equation. For SR systems with Rake receivers, it is the high pulse SNR approximation that is used, so two noise terms (signal-cross-noise) N_1 and N_2 (21) enter the equation. For N_3 in TR, the noise-cross-noise terms in each frame interval are independent, so the variance of N_3 increases with N_s linearly as shown in (7). On the other hand, for a Rake receiver, N_1 comes from the product of estimation noise and the desired signal. Since the estimation error is fixed during the N_s frames, the variance of N_1 increases linearly with N_s^2 (22). This can be explained by the following simple example. Let n_i and n be independent and identically distributed random variables, and likewise v_i and v. Also, let n_i and v_i be mutually independent and α a constant.

$$\begin{aligned} \mathbb{V}\mathrm{AR}\left\{\sum_{i=1}^{N_s} v_i n_i\right\} &= N_s \mathbb{V}\mathrm{AR}\{vn\} \propto N_s. \\ \mathbb{V}\mathrm{AR}\left\{\sum_{i=1}^{N_s} \alpha v\right\} &= \mathbb{V}\mathrm{AR}\{N_s\alpha v\} = N_s^2\alpha^2\mathbb{V}\mathrm{AR}\{v\} \propto N_s^2. \end{aligned} \tag{30}$$

This situation is similar to the coherent/non-coherent combining of signals. However, what is being combined here is the noise/estimation error, which explains why the noise power increases at different rates for the TR and SR systems and how N_s affects the threshold $(E_f/N_0)_{\mathrm{THR}}$. For the TR systems, the detection SNR is a linear function of N_s as shown in (10). For the SR systems with a Rake receiver, as N_s gets larger, N_1 dominates over N_2 and N_3 since the variance of N_1 increases with N_s^2 while that of N_2 or N_3 increases with N_s only. So the detection SNR becomes independent of N_s for N_s large enough because both the signal

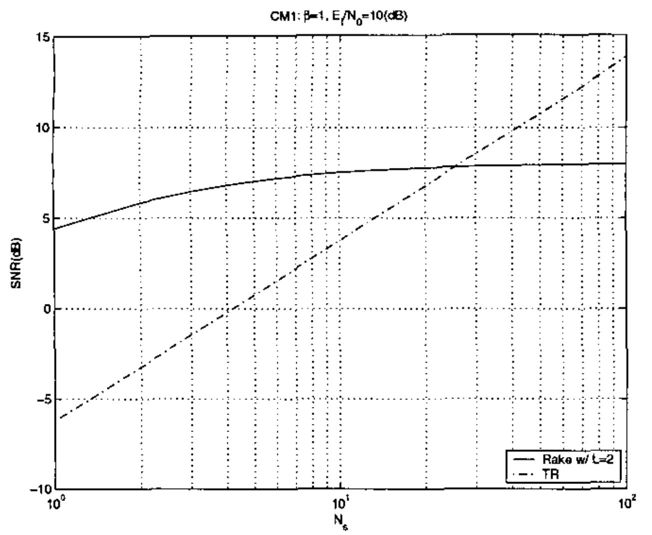


Fig. 5. The detection SNR for the TR and SR systems as a function N_s

power m_z^2 and the variance of N_3 increase linearly with N_s^2. This behavior is shown in Fig. 5, and it illustrates a quite surprising result that increasing the number of frames per bit N_s for the SR system beyond some point does not help reduce P_e at all. It should also be pointed out that the number of unmodulated pulses (β) in the SR system was fixed at one in the figure simply to illustrate the aforementioned fact that the noise power increases at different rates with N_s in SR and TR systems, and no effort was made to keep the unmodulated to modulated pulses ratio fixed in both SR and TR systems to provide a fair comparison of the output detection SNR.

Note that a similar analysis for Rake receivers with noisy channel estimates was performed in [7,8], where, instead of using central-limit arguments to approximate the probability density function of N_3, the result on product of Gaussian random variables from [9, Appendix B] was used to calculated the exact P_e of binary multi-carrier and direct sequence CDMA systems. We repeat the result of P_e for DS-CDMA systems (Eq. (3) in [8]) here for comparison.

$$P_e \approx -\left(\prod_{i=0}^{L-1} v_{1i}v_{2i}\right) \cdot \sum_{l=0}^{L-1} \lim_{v \to jv_{2l}} \left[\frac{v - jv_{2l}}{v} \prod_{d=0}^{L-1} \frac{1}{(v + jv_{1d})(v - jv_{2d})}\right] \quad (31)$$

where the variables v_{1i} and v_{2i} have to do with the means and covariances of m_Z and N_i's. The behavior of non-decreasing P_e as a function of L was also observed in [7,8]. In addition, the authors in [8] assumed that the overall received energy using L branches are fixed, which means as L increases, the energy per branch reduces accordingly. However, in our analysis, we generate a multipath channel independently of the number of fingers employed at the Rake. Also, to calculate P_e using (31), numerical calculations are required. On the other hand, using the closed-form formulation (25), we are able to estimate the optimal L or even the threshold frame SNR $(E_f/N_0)_{\text{THR}}$ to decide whether an SR or a TR system should be used based on either the channel statistics or realizations with much less complexity or computations. In addition, using our formulation, we are able to analytically identify how the parameters β and N_s affect the system performance.

V. Conclusion

In this paper, we analyzed and compared the performances of a TR system with a cross-correlation receiver and an SR system with a MRC-Rake receiver and noisy channel estimates. With the help of the low and high pulse SNR approximations, we are able to perform some simplified analyses and understand how the performance is influenced by the number of fingers used by the Rake receiver L, the power ratio for channel estimation β, the number of frames per bit N_s and the channel power decay profile which is related to $\gamma(L)$. Furthermore, with this analysis, we are able to determine when one system will outperform the other and how that is affected by the aforementioned system parameters, which provides insight into how to design an improved system with the advantages of both the TR and SR systems for multipath channels.

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