

Optimal Integration Time for UWB Transmitted Reference Correlation Receivers

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Abstract—This paper derives the bit error probability (BEP) of the conventional and average correlation receivers of the ultra-wideband transmitted reference system by exploiting the concept of the orthogonal expansion, and the central limit theorem is not needed in the derivation. The effects of using different lengths of the integration time in the conventional correlation receiver in a single user multipath environment is also discussed. Three optimization criteria are evaluated, and the average BEPs adopted the optimal integration time based on these criteria do not differ significantly from each other. In the interested BEP range, about 2dB can be gained by using the optimal integration time instead of the channel delay spread in the correlator.

I. INTRODUCTION

Ultra-wideband (UWB) impulse radio systems transmit data by modulation of subnanosecond pulses. These narrow pulses are distorted by the channel, but often can resolve many distinct propagation paths (multipath) because of their fine time-resolution capability [1]. However, a Rake receiver that implements tens or even hundreds of correlation operations may be required to take full advantage of the available signal energy [2]. On the other hand, a receiver using a single correlator matched to one transmission path may be operating at a 10 - 15dB signal energy disadvantage relative to a full Rake receiver.

A Transmitted reference scheme, which can ease the stringent receiver requirement of the synchronization, channel estimation, and a Rake reception has attracted lots of attention [3]. In this TR modulated system, a reference waveform is transmitted before each data-modulated waveform for the purpose of determining the current multipath channel response. The proposed conventional correlation receiver correlates the data signal with the reference signal to acquire all the energy without requiring additional channel estimation and Rake reception. One major drawback of this simple receiver structure is the transmitted reference signal used as a correlator template is noisy. One method to clean the correlator template is to use the average correlation receiver, which average reference waveforms in one bit duration as the template with the penalty of increasing the receiver complexity. The bit error probabilities (BEPs) of these two receiver were evaluated with the help of the central limit theorem [3], [4]. This evaluation, although an approximation, can assess the BEPs accurately

when the noise bandwidth \times time dimension is large enough which is usually the case in a UWB system. The precise BEPs of correlation receivers with an ideal front-end bandpass filter are evaluated in this paper by exploiting the concept of the orthogonal expansion in section II.

The original idea of the conventional receiver is to capture all the energy available in a UWB multipath channel. But for the BEP performance, the correlator adopting the channel delay spread as the integration time is not the best choice. The received signal energy is small in the tail of the channel response, and the receiver can get more noise power than the signal energy through the excessive integration. This paper optimizes the integration time under three criteria which are minimizing the average BEP, maximizing the average decision signal energy to the noise power ratio (SNR), and maximizing the average decision SNR in which the received signal energy is replaced by its mean value (Section III). The results of these three criteria are compared in Section IV. Section V draws the conclusion.

II. UWB TR MODULATION AND CORRELATION RECEIVERS

The transmitted signal of a conventional UWB TR system with antipodal modulation is

$$s_{tr}(t) = \sum_{i=-\infty}^{\infty} g_{tr}(t - iT_f) + b_{\lfloor i/N_s \rfloor} g_{tr}(t - iT_f - T_d). \quad (1)$$

Here $g_{tr}(t)$ is a transmitted monocycle waveform that is non-zero only for $t \in (0, T_p)$, and T_f is the frame time. Each frame contains two monocycle waveforms. The first is a reference and the second, T_d seconds later, is a data-modulated waveform. The data bits $b_{\lfloor i/N_s \rfloor} \in \{1, -1\}$ with equal probability, and $\lfloor i/N_s \rfloor$ is the integer part of i/N_s . Hence each bit is transmitted in N_s successive frames to achieve an adequate bit energy in the receiver, and the channel is assumed invariant over this bit time.

In this TR system, T_d is greater than the multipath delay spread T_{mfs} to assure that no interference between reference signal and data signal exists. The frame time is designated to be $T_f = 2T_d > 2T_{mfs}$ so that no interframe interference exists. Because the single user case is considered here, the time-hopping and/or direct sequence modulation which is used to reduce multiuser interference is eliminated for simplicity, but without loss of generality.

We model the received TR signal $r(t)$ of bit b_0 in a stationary channel over a bit time by

$$r(t) = \sum_{i=0}^{N_s-1} [g(t-iT_f) + b_{\lfloor i/N_s \rfloor} g(t-iT_f-T_d)] + n(u, t), \quad (2)$$

where $n(u, t)$ represents band-limited white Gaussian receiver noise with one-sided power spectral density N_0 and bandwidth B_w , and $g(t)$ is the received waveform which is the convolution of a transmitted monocycle waveform $g_{tr}(t)$ and channel impulse responses including effects of antennas and the front-end bandpass filter. A conventional correlation receiver correlates the received data-modulated waveform with the reference waveform, which is received T_d seconds earlier, and sums the N_s correlator outputs that are affected by a single data bit to be the decision statistic. The decision rule of bit b_0 is

$$D_s = \sum_{j=0}^{N_s-1} \int_{jT_f+T_d}^{jT_f+T_d+T_{\text{corr}}} r(t-T_d)r(t)dt \stackrel{1}{\geq} 0, \quad (3)$$

where $T_{\text{corr}} \leq T_{\text{mids}}$ is the correlator's integration time.

With an ideal bandpass filter, the BEP can be evaluated by applying the orthogonal expansion technique as well as Appendix 9A in [5]. Without actually implementing it in the conventional receiver, we use the concept that both the bandpass signal and noise have complex lowpass equivalence. The energy in the lowpass equivalence of the received waveform $\hat{g}(t)$ is twice the energy in $g(t)$. The lowpass equivalence of the bandlimited noise $\hat{n}(u, t)$ has power spectral density $2N_0$ from $-B_w/2$ to $B_w/2$, and zero elsewhere. In addition to the lowpass equivalence, another useful theorem states that a time-limited (T_{corr}) band-limited (B_w) signal has dimension $B_w T_{\text{corr}}$ at most, and can be represented by $\{b_k(t)\}_{k=1}^{B_w T_{\text{corr}}}$, a complete orthonormal set with

$$\int_0^{T_{\text{corr}}} b_j(t)b_k^*(t)dt = \begin{cases} 0 & j \neq k \\ 1 & j = k, \end{cases} \quad (4)$$

where $b_k^*(t)$ is the complex conjugate of $b_k(t)$. Thus the lowpass equivalence of the filtered signal and noise with time duration T_{corr} can be represented by the complete set $\{b_k(t)\}_{k=1}^{B_w T_{\text{corr}}}$. The complex lowpass equivalence of the received waveform is now written as

$$\hat{g}(t) = \sum_{k=1}^{B_w T_{\text{corr}}} \hat{g}_k b_k(t), \quad (5)$$

where the weights are

$$\hat{g}_k = \int_0^{T_{\text{corr}}} \hat{g}(t)b_k^*(t)dt,$$

and

$$\eta E_p = \int_0^{T_{\text{corr}}} g^2(t)dt = \frac{1}{2} \sum_{k=1}^{B_w T_{\text{corr}}} \hat{g}_k^2 \quad (6)$$

with $E_p = \int_0^{T_{\text{mids}}} g^2(t)dt$ being the total energy in a receiver waveform, and η being the efficiency factor. The noise covariance function of $\hat{n}(u, t)$, $K_{\hat{n}\hat{n}^*}(t_1, t_2)$, satisfies

$$2N_0 b_k(t_1) = \int_0^{T_{\text{corr}}} K_{\hat{n}\hat{n}^*}(t_1, t_2)b_k(t_2)dt_2 \quad (7)$$

for $k = 1, 2, \dots, B_w T_{\text{corr}}$ and $t_1 \in [0, T_{\text{corr}}]$, namely $\{b_k(t)\}_{k=1}^{B_w T_{\text{corr}}}$ are eigenfunctions of $\hat{n}(u, t)$ with same eigenvalues $2N_0$. The noise for $t \in [0, T_{\text{corr}}]$ is represented as

$$\hat{n}(u, t + iT_f) = \sum_{k=1}^{B_w T_{\text{corr}}} \hat{n}_{r,i,k} b_k(t), \quad (8)$$

$$\hat{n}(u, t + iT_f + T_d) = \sum_{k=1}^{B_w T_{\text{corr}}} \hat{n}_{d,i,k} b_k(t) \quad (9)$$

for $i = 0, 1, \dots, N_s - 1$, in which

$$\begin{aligned} \hat{n}_{r,i,k} &= \int_0^{T_{\text{corr}}} \hat{n}(u, t + iT_f) b_k^*(t) dt, \\ \hat{n}_{d,i,k} &= \int_0^{T_{\text{corr}}} \hat{n}(u, t + iT_f + T_d) b_k^*(t) dt \end{aligned}$$

It can be computed from (4) and (7) that $\mathbb{E}\{\hat{n}_{r,i,k}\} = \mathbb{E}\{\hat{n}_{d,i,k}\} = 0$, $\mathbb{E}\{\hat{n}_{r,i,k}\hat{n}_{r,i,k}^*\} = \mathbb{E}\{\hat{n}_{d,i,k}\hat{n}_{d,i,k}^*\} = 2N_0$ for any i, k , and any two of $\{\hat{n}_{r,i,k}, \hat{n}_{d,i,k}\}_{i,k}$ are uncorrelated.

By defining

$$\hat{\mathbf{g}} \triangleq [\hat{g}_1, \hat{g}_2, \dots, \hat{g}_{B_w T_{\text{corr}}}]^t,$$

the received signal for the 0th bit can be represented as

$$\begin{aligned} \hat{\mathbf{r}} = \hat{\mathbf{r}}_s + \hat{\mathbf{n}} &= [\hat{\mathbf{g}}^t, b_0 \hat{\mathbf{g}}^t, \hat{\mathbf{g}}^t, b_0 \hat{\mathbf{g}}^t, \dots, \hat{\mathbf{g}}^t, b_0 \hat{\mathbf{g}}^t]^t \\ &+ [\hat{\mathbf{n}}_{r,0}^t, \hat{\mathbf{n}}_{d,0}^t, \hat{\mathbf{n}}_{r,1}^t, \hat{\mathbf{n}}_{d,1}^t, \dots, \hat{\mathbf{n}}_{r,N_s-1}^t, \hat{\mathbf{n}}_{d,N_s-1}^t]^t, \end{aligned}$$

where

$$\begin{aligned} \hat{\mathbf{n}}_{r,i} &= [\hat{n}_{r,i,1}, \hat{n}_{r,i,2}, \hat{n}_{r,i,3}, \dots, \hat{n}_{r,i,B_w T_{\text{corr}}}]^t, \\ \hat{\mathbf{n}}_{d,i} &= [\hat{n}_{d,i,1}, \hat{n}_{d,i,2}, \hat{n}_{d,i,3}, \dots, \hat{n}_{d,i,B_w T_{\text{corr}}}]^t. \end{aligned}$$

By denoting

$$\mathbf{X} = [\mathbf{X}_0^t, \mathbf{X}_1^t, \dots, \mathbf{X}_{N_s-1}^t]^t \quad \text{with} \quad \mathbf{X}_i = \hat{\mathbf{g}} + \hat{\mathbf{n}}_{r,i}, \quad (10)$$

$$\mathbf{Y} = [\mathbf{Y}_0^t, \mathbf{Y}_1^t, \dots, \mathbf{Y}_{N_s-1}^t]^t \quad \text{with} \quad \mathbf{Y}_i = b_0 \hat{\mathbf{g}} + \hat{\mathbf{n}}_{d,i}, \quad (11)$$

the decision rule in (3) is equivalent to

$$D_s = \frac{1}{2} (\mathbf{X}^H \mathbf{Y} + \mathbf{Y}^H \mathbf{X}) \stackrel{1}{\geq} 0, \quad (12)$$

where \mathbf{X}^H denotes the complex conjugate transpose of \mathbf{X} . Now, (12) can be equated to (9A.1) and (9A.2) in Appendix 9A in [5] by letting $A = 0$, $B = 0$, $C = \frac{1}{2}$ and $L = N_s B_w T_{\text{corr}}$. The BEP of this conventional correlation receiver is $\Pr\{D_s < 0 | b_0 = 1\}$ due to the symmetry of the transmitted data and receiver noise, and can be computed by utilizing (9A.15) in [5]. The value of a and b needed in computing the BEP can be calculated using (6) as well as (9A.4) and (9A.5) in [5], which result in $a = 0$ and $b = \sqrt{\frac{\eta E_b}{N_0}}$ with $E_b = 2N_s E_p$. The BEP of a conventional correlation receiver conditioned on the channel response $g(t)$ is

$$\begin{aligned} P_{\text{bit}}^s &= \frac{1}{2} + \frac{1}{2^{2N_s B_w T_{\text{corr}} - 1}} \sum_{l=1}^{N_s B_w T_{\text{corr}}} \binom{2N_s B_w T_{\text{corr}} - 1}{N_s B_w T_{\text{corr}} - l} \\ &\times \left[Q_l \left(0, \sqrt{\frac{\eta E_b}{N_0}} \right) - Q_l \left(\sqrt{\frac{\eta E_b}{N_0}}, 0 \right) \right], \end{aligned} \quad (13)$$

where $g(t)$ is implicitly imbedded in E_b . It is obvious now that the orthogonal expansion only helps to calculate the precise BEP without really implementing it in the receiver. Because $Q_l(b, 0) = 1$ for all l, b , and

$$Q_l(0, b) = \sum_{n=0}^{l-1} \exp\left(-\frac{b^2}{2}\right) \frac{(b^2/2)^n}{n!}$$

if l is an integer, (13) is further simplified to

$$P_{\text{bit}}^s = \frac{1}{2^{2N_s B_w T_{\text{corr}} - 1}} \sum_{l=1}^{N_s B_w T_{\text{corr}}} \binom{2N_s B_w T_{\text{corr}} - 1}{N_s B_w T_{\text{corr}} - l} \times \sum_{n=0}^{l-1} \frac{1}{n!} \exp\left(-\frac{\eta E_b}{2N_0}\right) \left(\frac{\eta E_b}{2N_0}\right)^n. \quad (14)$$

One method to improve the BEP performance of the conventional correlation receiver is to average the N_s reference waveforms in one bit time to be a noise reduced correlator template. The decision statistic of this average correlation receiver is

$$D_a = \sum_{j=0}^{N_s-1} \int_{jT_f+T_d}^{jT_f+T_d+T_{\text{corr}}} \hat{r}(t) \left[\frac{1}{N_s} \sum_{i=-j}^{N_s-1-j} \hat{r}(t+iT_f-T_d) \right] dt$$

with $D_a \geq 0$. The BEP conditioned on each channel realization $g(t)$ can be computed by employing the same method as for the conventional correlation receiver, but now $\mathbf{X} = \sum_{i=0}^{N_s-1} \mathbf{X}_i$ and $\mathbf{Y} = \frac{1}{N_s} \sum_{i=0}^{N_s-1} \mathbf{Y}_i$ instead of the definitions in (10) and (11). Equation (9A.1) and (9A.2) in [5] are equated to D_a by letting $A = 0$, $B = 0$, $C = \frac{1}{2}$ and $L = B_w T_{\text{corr}}$. The BEP of this average correlation receiver conditioned on the channel realization $g(t)$ by substituting $a = 0$ and $b = \sqrt{\frac{\eta E_b}{N_0}}$ into (9A.15) in [5] after simplification is

$$P_{\text{bit}}^a = \frac{1}{2^{2B_w T_{\text{corr}} - 1}} \sum_{l=1}^{B_w T_{\text{corr}}} \binom{2B_w T_{\text{corr}} - 1}{B_w T_{\text{corr}} - l} \times \sum_{n=0}^{l-1} \frac{1}{n!} \exp\left(-\frac{\eta E_b}{2N_0}\right) \left(\frac{\eta E_b}{2N_0}\right)^n. \quad (15)$$

The average BEP of the conventional and average correlation receivers over channels can be obtained if $\mathbb{E}\{\exp(-\frac{\eta E_b}{2N_0})(\frac{\eta E_b}{2N_0})^n\}$ exists, i.e., the moment generating function of $\frac{\eta E_b}{2N_0}$ exists.

III. INTEGRATION TIME ANALYSIS

The integration time of the correlator T_{corr} affects the BEP which can be seen in (14) and (15). Given a channel realization, the efficiency factor η increases as T_{corr} increases, therefore $\exp(-\eta E_b/2N_0)$ decreases but $(\eta E_b/2N_0)^n$ increases. In addition, the number of terms in the summation also increases as T_{corr} increases. Thus the BEP has its minimum at some value of T_{corr} , and starts to raise as T_{corr} diverges from this value. In the following, the optimal T_{corr} of a conventional correlation receiver is discussed, but can be easily generalized to different kinds of correlation receivers.

Due to the simple receiver constraint and that it is difficult to implement an adaptive algorithm using analog devices, the value of T_{corr} is fixed once the receiver is implemented. The best choice of T_{corr} is to minimize the average BEP which is immediately seen a difficult task from (14) because frequency selective UWB channels with random path arrival times make finding the distribution of $\frac{\eta E_b}{2N_0}$ difficult. By defining the decision SNR

$$f_s(T_{\text{corr}}) = \frac{N_s(\eta E_p)^2}{N_0 \eta E_p + \frac{B_w T_{\text{corr}} N_0^2}{2}},$$

which is the ratio of the signal energy to the noise power in the decision statistic D_s , another choice to optimize the integration time is to maximize $f_s(T_{\text{corr}})$. This criterion is equivalent to minimizing $N_s f_s^{-1}(T_{\text{corr}})$

$$N_s f_s^{-1}(T_{\text{corr}}) = \frac{N_0}{\eta E_p} + \frac{B_w T_{\text{corr}}}{2} \left(\frac{N_0}{\eta E_p} \right)^2, \quad (16)$$

which indicates that the optimal integration time based on this criterion depends on the energy per pulse E_p instead of energy per bit E_b . Minimizing $N_s f_s^{-1}(T_{\text{corr}})$ is still difficult to manage theoretically. By exploiting the average power profile of the received signal which is assumed exponential decays here without loss of generality [6], then

$$\mathbb{E}\{g^2(t)\} = \Omega a \exp(-at)$$

where $\Omega = \mathbb{E}\{E_p\}$ and $\frac{1}{a}$ is the power decay time constant. Under the exponential power decay profile assumption,

$$\mathbb{E}\{\eta E_p\} = \Omega[1 - \exp(-aT_{\text{corr}})]. \quad (17)$$

In the following three subsections, we are going to replace ηE_p in $f_s(T_{\text{corr}})$ by $\mathbb{E}\{\eta E_p\}$, and investigate the effects of choosing different values of T_{corr} . Note that $f_s(T_{\text{corr}})$ with this substitution does not equal the average decision SNR over channel statistics, and the observations we obtain will be justified in the next section by evaluating the average decision SNR and average BEP numerically.

A. Minimal integration time

A special case in which the noise power is extremely large is considered. Under this condition,

$$\left(\frac{N_0}{\eta E_p} \right)^2 \gg \frac{N_0}{\eta E_p},$$

and the quantity we want to minimize is approximate

$$N_s f_s^{-1}(T_{\text{corr}}) \cong \frac{B_w T_{\text{corr}}}{2} \left(\frac{N_0}{\eta E_p} \right)^2. \quad (18)$$

By replacing ηE_p with $\mathbb{E}\{\eta E_p\}$, we now want to minimize

$$\frac{B_w T_{\text{corr}}}{2} \times \left(\frac{N_0}{\Omega} \right)^2 \times \left[\frac{1}{1 - \exp(-aT_{\text{corr}})} \right]^2, \quad (19)$$

which is a convex function of $T_{\text{corr}} \in (0, \infty)$, and has an unique minimum. After differentiating (19) with respect to T_{corr} and equating it 0, the equation which determines the optimal value of T_{corr} is

$$\ln(1 + 2aT_{\text{corr}}) = aT_{\text{corr}}, \quad (20)$$

which does not depend on the receiver bandwidth B_w and Ω/N_0 . In this extremely high noise power case, the value of T_{corr} only depends on the power decay time constant $\frac{1}{a}$. The solution of (20) is $aT_{\text{corr}} = 1.2564$, and

$$T_{\text{corr}} = \frac{1.2564}{a} = 1.2564 \times \text{time constant}. \quad (21)$$

This high noise power case represents the minimum value of T_{corr} for a conventional correlation receiver, and T_{corr} in a general situation should be larger than this value. For another special case that the received signal power is extremely high, it is not really meaningful because the integration time then should be as long as possible, i.e., the channel delay spread.

B. Optimal integration time

For the normal E_p/N_0 case, ηE_p in (16) is replaced by (17), and the quantity to be minimized is

$$N_s f_s^{-1}(T_{\text{corr}}) = \frac{N_0}{\Omega} \times \frac{1}{1 - \exp(-aT_{\text{corr}})} + \frac{B_w T_{\text{corr}}}{2} \left[\frac{N_0}{\Omega} \times \frac{1}{1 - \exp(-aT_{\text{corr}})} \right]^2. \quad (22)$$

The right hand side of (22) is differentiated with respect to T_{corr} to achieve

$$\left(\frac{2\Omega}{B_w N_0} + 1 + 2aT_{\text{corr}} \right) \exp(-aT_{\text{corr}}) - \frac{2\Omega}{B_w N_0} \exp(-2aT_{\text{corr}}) - 1 = 0. \quad (23)$$

Equation (23) shows that the optimal integration time depends on B_w , $1/a$, and Ω/N_0 . Given B_w and Ω/N_0 , the value of aT_{corr} which makes (23) sustained can be computed numerically. For a specific BEP, the required pulse energy increases and the optimal integration time decreases as B_w increases because of the increasing incoming noise.

C. Performance degradation V.S. excess or lack of integration

The optimal integration time changes according to application environments, but the value adopted by the receiver is difficult to change once the correlator is implemented. How much the performance degrades because of the excessive or short integration should be considered before choosing the adequate value. In (16) with ηE_p replaced by $\mathbb{E}\{\eta E_p\}$, $N_s f_s^{-1}(T_{\text{corr}})$ includes two portions

$$g_s(T_{\text{corr}}) = \frac{N_0}{\Omega} \times \frac{1}{1 - \exp(-aT_{\text{corr}})}, \quad (24)$$

$$h_s(T_{\text{corr}}) = \frac{B_w T_{\text{corr}}}{2} \left[\frac{N_0}{\Omega} \times \frac{1}{1 - \exp(-aT_{\text{corr}})} \right]^2. \quad (25)$$

Equation (24) indicates that $g_s(T_{\text{corr}})$ decreases as T_{corr} increases for $T_{\text{corr}} \in [0, \infty)$. Equation (25) shows that $h_s(T_{\text{corr}})$ also decreases as T_{corr} increases for $T_{\text{corr}} \in [0, t_B)$ with some value t_B , then starts to increase as T_{corr} increases for $T_{\text{corr}} \geq t_B$. The value of t_B is determined by B_w , N_0/Ω and $1/a$, and the optimal T_{corr} is greater than or equal to t_B . Another observation from (24) and (25) is that $g_s(T_{\text{corr}})$ for $T_{\text{corr}} \in (0, \infty)$ as well as $h_s(T_{\text{corr}})$ for $T_{\text{corr}} \in (0, t_B)$ decrease roughly exponentially

	cm1	cm2	cm3	cm4
$\frac{1}{a}$ (ns)	4	8	12	16
λ (1/ns)	3.5	3.5	3.5	3.5
η : mean	0.85908	0.83218	0.82051	0.81577
η : std	0.0432	0.041828	0.039349	0.038193

TABLE I
CHANNEL PARAMETERS AND THE EFFICIENCY FACTOR η .

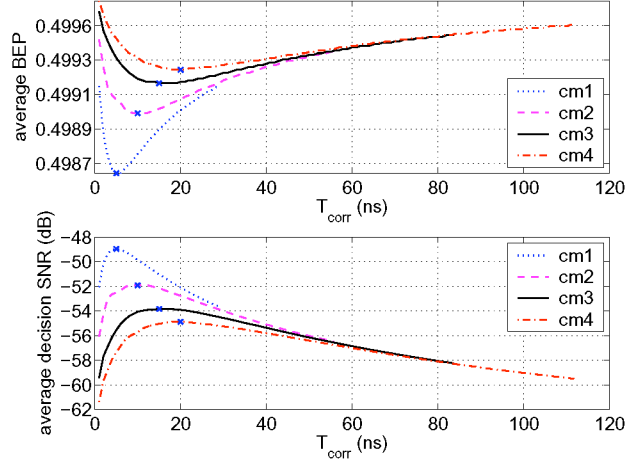


Fig. 1. Average BEP and average decision SNR for $E_p/N_0 = -23\text{dB}$ ($E_b/N_0 = -10\text{dB}$) with $N_s = 10$.

as T_{corr} increases, and $h_s(T_{\text{corr}})$ for $T_{\text{corr}} \in [t_B, \infty)$ increases roughly linearly. Therefore, under integration degrades BEP performance more than over integration.

IV. NUMERICAL EXAMPLES

This section uses a channel model in (26) to analyze the average BEP and average decision SNR versus different integration time numerically to verify the analysis and observations in Section III. The model is

$$h(t) = \sum_{l=0}^L \alpha_l \delta(t - T_l), \quad (26)$$

where α_l and T_l are the amplitude and arrival time of the l^{th} path. The magnitude of α_l has lognormal distribution, and the polarity of it can be +1 or -1 with equal probability. In addition, α_l and α_j are independent for $l \neq j$. The energy of a single transmitted pulse is normalized to 1, and $\mathbb{E}\{\alpha_l^2\} = c \exp(-aT_l)$ with some constant c such that $\sum_l \mathbb{E}\{\alpha_l^2\} = E_p$. The channel delay spread T_{mfs} is defined as the interval containing 99% of the energy in the average received waveform. The probability that a path arrives at time T_l has poisson distribution with the path arrival rate λ . The receiver bandwidth is equal to 4GHz, and 100 channel realizations are generated to get the numerically average BEP and decision SNR. The parameters used in this numerical analysis are listed in Table I. The resolution of searching the optimal T_{corr} is equal to 1ns.

Figure 1 shows the average decision SNR and BEP for the extremely large noise power case. Crosses in the figure, which mark the positions of the optimal integration time ($T_{\text{corr}}^{\text{opt}}$) for each channel model, indicate that $T_{\text{corr}}^{\text{opt}}$ acquired using these two criteria are the same and fit the results predicted by (21). This figure also shows that for a fixed N_s , B_w and E_b/N_0 , the value of $T_{\text{corr}}^{\text{opt}}$ increases as $\frac{1}{\alpha}$ increases but with worse performance because the incoming noise power also increases. This figure verifies that excessive integration harms the performance less than short integration.

Figure 2 and 3 show $T_{\text{corr}}^{\text{opt}}$ s acquired through minimizing the average BEP, maximizing the average decision SNR, and finding the solution of (23), as well as the corresponding performance. In Figure 2, the values of $T_{\text{corr}}^{\text{opt}}$ obtained through different criteria are close at small E_f/N_0 , but could be different at large E_f/N_0 . Minimizing the average BEP produces larger $T_{\text{corr}}^{\text{opt}}$ than maximizing the average decision SNR, and as E_f/N_0 increases, the value of $T_{\text{corr}}^{\text{opt}}$ also increases. The value of $T_{\text{corr}}^{\text{opt}}$ obtained by solving (23) is the largest one among the three because the received waveform energy acquired by integrating an exponential function can be overestimated. Even divergence resulted from different criteria is demonstrated, Figure 2 and 3 display that the influence of this divergence on both the average BEP and the average decision SNR is small, which allows us to acquire $T_{\text{corr}}^{\text{opt}}$ easily through solving (23) or maximizing the average decision SNR instead of minimizing the average BEP. Figure 2 also shows that compared to integrating over the channel delay spread, the correlator adopting the optimal integration time can have approximate 2dB gain at $\text{BEP}=1\text{e-}4$. As E_f/N_0 increases, $T_{\text{corr}}^{\text{opt}}$ approaches T_{mfs} . Table I includes the mean and the standard deviation of the efficiency factor η over the 100 channel realizations with the optimal integration times for the average $\text{BEP}=1\text{e-}4$ and $N_s = 10$. The mean value of η decreases as $\frac{1}{\alpha}$ increases, and small standard deviations show the value of η for every channel realization is close to each other.

V. CONCLUSIONS

The BEP of UWB conventional and average correlation receivers are analyzed with the help of the orthogonal expansion theorem. The optimal integration time of a UWB conventional correlation receiver is also analyzed. It shows that the integration time has a minimal value which is not related to the receiver bandwidth and the bit repetition time. This minimal value corresponds to an extremely small SNR case, and the real operational value should be greater than this one. It also shows that excessive integration harms the performance less than short integration. With the parameters used in those numerical examples, about 2dB gain can be get by using optimal integration time instead of channel delay spread in the correlator in the interested BEP range.

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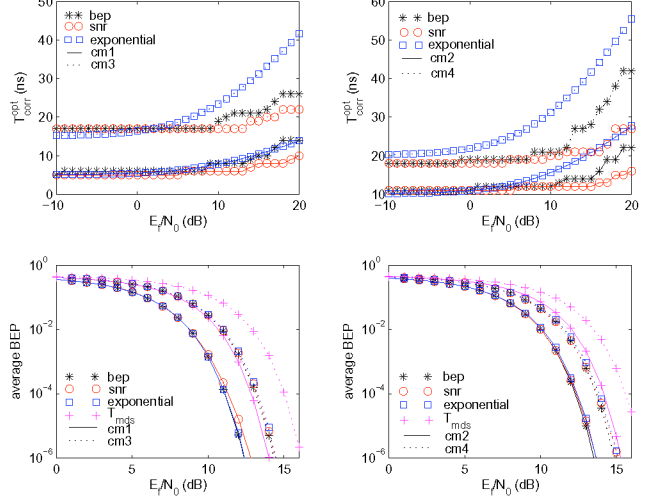


Fig. 2. The optimal integration times obtained by minimizing the average BEP (labelled bep), maximizing the average decision SNR (labelled snr), and solving (23) (labelled exponential) with $N_s = 10$, as well as the corresponding average BEP.

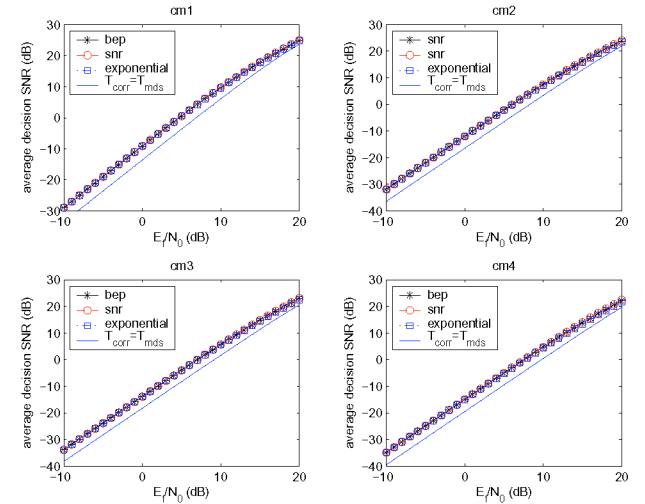


Fig. 3. The average decision SNR using optimal integration times obtained by minimizing the average BEP (labelled bep), maximizing the average decision SNR (labelled snr), and solving (23) (labelled exponential) with $N_s = 10$.

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