Receiver Sites for Accurate Indoor Position Location Systems

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Abstract

Hyperbolic radio-location systems possess intrinsic uncertainty that depends on the sensor's geometry and the modulation's time resolution. In this paper the best locations for the receivers to minimize the uncertainty in 2-dimensional (2-D) and 3-dimensional (3-D) models for indoor systems are found. For this purpose the minimum number of receivers with clear line-of-sight (LOS) to the emitter, required for locating a source, are three and four in 2-D and 3-D respectively. For 2-D, a triangular area and for 3-D a tetrahedron volume are considered the building blocks that are used to extend results to more arbitrarily shaped areas. The criteria for minimization of location uncertainty is uncertainty area or the largest diagonal in 2-D and the multiplication of line-of-position (LOP) errors in 3-D. Finally a theorem is given for the number of sensors that is sufficient to cover the entire area in 2-D and 3-D cases.

Keywords

Localization, Art Gallery Theorem, UWB, Receiver Siting, Arrays, Hyperbolic Navigation

I. Introduction

Emitter position-fix uncertainty in radio navigation systems is a function of hyperbolic line-of-position (LOP) crossing angles and so depends on the geometry of the sensors. The other source of error in such systems is a phenomenon that is called fix ambiguity [1]. Source localization has been of considerable interest in radar, sonar,navigation, geophysics and tracking. In most of these applications the source is located using the time delay of arrival or direction-of-arrival. While there is a rich literature of source localization methods, most of them have dealt with measurement error models, improvement of processing accuracy, and development of efficient algorithms by different techniques, including linear approximation/direct numerical optimization, likelihood/least-squares and iterative versus closed-form algorithms [2], [3], [4], [5], [6]. None of them have focused on decreasing the intrinsic uncertainty in hyperbolic systems by finding an appropriate geometry for the receiving stations.

In this paper we find the locations for the receivers in 2-D and 3-D that minimize location uncertainty for indoor systems. For this purpose, at first a triangular area and a tetrahedron volume are considered as the building block in 2-D and 3-D respectively. For each case, we use the minimum number of sensors necessary for receiving clear LOS signals, i.e., three and four for the 2-D and 3-D cases respectively. Then we use the results to position the receivers in more complex areas and volumes. This method is similar to that used for determining the minimum number of guards sufficient to cover the interior of an n-wall art gallery room in the "Art Gallery Theorem" [7]. The receivers are considered to be omni-directional sensors that are appropriate for UWB transmissions.

II. Hyperbolas of uncertainty area and their features in 2-D and 3-D

Assuming the receivers have a common clock available, and they all receive LOS signal, the source can be localized by the hyperbolic method. In this section, the features of the uncertainty area of the hyperbolic method are investigated.

The tangent line to hyperbola at a point is the bisector of the angle between lines from the point to two foci. This fact at point x for two sensors A and B is shown in Fig. 1. A parametric equation for point x on the hyperbola is

$$d(x, A) - d(x, B) = t, \tag{1}$$

in which t is the difference between propagation times of the received signals at sensors A and B. Here distances are normalized by propagation velocity and hence are measured in time units. If the propagation time difference t increases by the time resolution $\triangle t$, typically of the order of the reciprocal of the signal bandwidth, the emitter location x is now on point x' on the next hyperbola with equation

$$d(x', A) - d(x', B) = t + \triangle t. \tag{2}$$

For the rest of the paper we assume that $\triangle t$ is very small. With this assumption, tangent lines at point x and x' are parallel and the orthogonal distance between them can be obtained by subtracting (2) from (1) and using approximation as

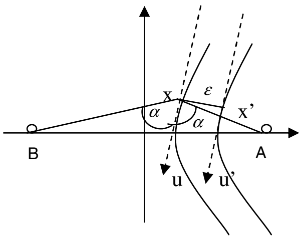

Fig. 1. hyperbola of LOP line for two receiving station at A and B and nominal source at x

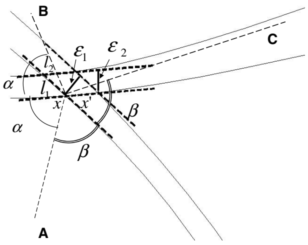

Fig. 2. Geometry of a hyperbolic location system with nominal emittert position x and receiving stations at A, B, and C. The uncertainty area approximated with a parallelogram

$$\epsilon = |\triangle t/2\sin\alpha|, \tag{3}$$

in which without loss of generality, we can ignore the absolute value. As indicated in Fig. 2, with two pairs of sensors AB and BC, the uncertainty area at point x is a curved parallelogram that its edges can be approximated with tangent lines at the point as below

$$l_i = \epsilon_i/\cos(90-(\beta+\alpha)) = \epsilon_i/\sin(\beta+\alpha), \tag{4}$$

in which ϵ_1 and ϵ_2 correspond to the hyperbolas of pairs of sensors AB and BC respectively. The area of the parallelogram is

$$S = l_1 l_2 \sin(\beta+\alpha). \tag{5}$$

Replacing (3) and (4) in (5) yields

$$S = \frac{\triangle t_1 \triangle t_2}{4\sin\alpha\sin\beta\sin(\beta+\alpha)}. \tag{6}$$

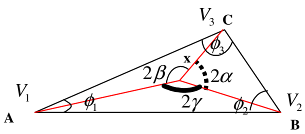

Fig. 3. Sensors located on the the vertices of the triangular area

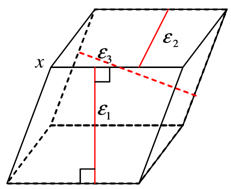

Fig. 4. Shape of uncertainty for every triplet of sensors for 3-D.

It can be shown that for the domain of ϕ_1, ϕ_2, and ϕ_3, function S for all the points inside and also for points outside of the triangle is monotone decreasing in the variables α and β. For the hyperbolas of line of positions for three sensors located on three corners of a rectangular area, with equal $\triangle t$ for all three pairs of sensors, when

$$\triangle t_1 = \triangle t_2 = \triangle t_3 = \triangle t, \tag{7}$$

three set of hyperbolas always cross at the same point. In the other words, at any point, the hyperbolas of the third pair of sensors is the diagonal of the parallelogram of the other two set of hyperbolas crossing at the point. This fact can be proved mathematically.

For the 3-D case we need at least four sensors (that are not located on a plane) to locate a source. For a differential delay $\triangle t$, the location of the transmitter is somewhere on a hyperboloid of two sheets for every pair of sensors [8]. Similar to the 2-D case it can be proved that the tangent plane at every point on these hyperboloid sheets is the bisector plane of the sensor pair. Assuming four sensors, there are $\binom{4}{2} = 6$ hyperboloid sets, however three of them are enough to locate the transmitter. Every three sets of hyperboloids at a point make a curved parallelepiped that can be approximated with a parallelepiped with tangent planes of the hyperboloid at that point as its faces as indicated in Fig. 4.

As in the 2-D case, the orthogonal distances of the faces of parallelepiped, i.e., orthogonal distances of the tangent planes at point x can be written as

$$\begin{aligned}\epsilon_1 &= |\triangle t/2\sin\alpha| \\ \epsilon_2 &= |\triangle t/2\sin\beta| \\ \epsilon_3 &= |\triangle t/2\sin\gamma|,\end{aligned} \tag{8}$$

where α,β and γ are the angles of the sensors triplet at point x.
Adding other hyperbolas decreases the parallelepiped of uncertainty. The form of uncertainty depends on the manner that sensor pairs are selected. For the Z configuration (e.g. AB, BC and CD) the form of uncertainty is different with the one for the Y configuration (e.g. AB, AC and AD) of selecting sensor pairs. The description and proof is omitted due to the space constraints.

III. Best locations of the receivers for 2-D

For the 2-D case, we consider two different criterions, the area of uncertainty and its largest diagonal, to be minimized.
Choosing the largest area of uncertainty as the criterion, we must choose the locations of the sensors in the triangular shape area $V_1V_2V_3$ in Fig. 3 in such a way that the area of the parallelogram of uncertainty for the worst point (point with the highest area of uncertainty) is minimized. It can be shown that the corners of the area are the best place for the sensors to be located.
Suppose we choose the largest diagonal of uncertainty as the criterion. Considering Fig. 2, the largest diagonal of the parallelogram is

$$d = \sqrt{l_1^2 + l_2^2 + 2l_1l_2\cos(\alpha+\beta)}. \tag{9}$$

Replacing (3) and (4) in (9) and manipulating (for equal $\triangle t$) gives

$$d = \frac{\triangle t}{2\sin(\alpha+\beta)}\sqrt{\frac{1}{\sin^2\alpha} + \frac{1}{\sin^2\beta} + \frac{2\cos(\alpha+\beta)}{\sin\alpha\sin\beta}}. \tag{10}$$

Similar to function S, this function is also decreasing versus α and β, therefore to minimize the diagonal of uncertainty for the worst point, the sensors must be located on the corners of the area.
Note that the other choices of sensors locations, i.e., locating them in such a way that some points of the area are on the baseline extension of two sensors, cause location fix ambiguity described in [1].

IV. Best locations of the receivers for 3-D

For 3-D case, since determining the volume or the edges of the parallelepiped of uncertainty is difficult, the multiplication of ϵ_i's, i.e., the multiplication of LOP errors, is considered as the criterion. So we locate four sensors in a tetrahedral shape area $V_1V_2V_3V_4$ (Fig. 5) in such a way that the following equation is minimized

$$S = \epsilon_1\epsilon_2\epsilon_3 = \frac{\triangle t^3}{8\sin\alpha\sin\beta\sin\gamma}. \tag{11}$$

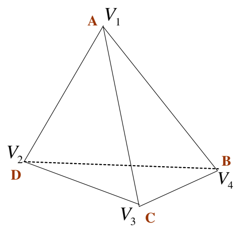

Fig. 5. Tetrahedral volume

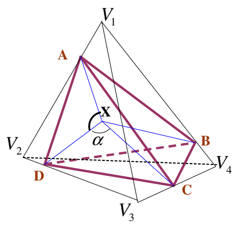

Fig. 6. Sensors located on the points other than the vertices of the volume

Suppose we locate the source with three pairs of sensors at each point and assume that we place four sensors A, B, C, and D on the vertices of the volume, as indicated in Fig. 5. Assuming the angles between the lines from the point to the sensors (i.e. $\angle AxB, \angle AxC, \ldots$), we use the following theorem:

Theorem: For the points inside the tetrahedron of four sensors, we can always find at least three obtuse angles correspond to independent hyperbolas.

(Independent hyperbolas are related to the independent pairs of sensors). The proof (omitted here) is given by considering five different cases for the angles.
By the above theorem, we prove that to have the best result, sensors must be placed on the corners of the volume so that all of the points of the volume locate inside the tetrahedron of the sensors.

V. Number of the receivers to cover an area in 2-D and 3-D

Now we can extend the results obtained in the previous sections to an arbitrary area. For 2-D case, suppose we have a $n - polygon$ area as shown in Fig. 7.
We triangulate the area as indicated and put the sensors on the corners. Since the sensors are assumed to be omni-directional, we can keep one sensor on every corner and remove duplicates. Therefore no matter how the area is triangulated, the sensors are located on the vertices as indicated in Fig. 7. But for locating a source at any point, we use the triplet of sensors that see the point and make a triangle with maximum value of minimum angle, i.e., a triangle that is closest to an equilateral triangle.

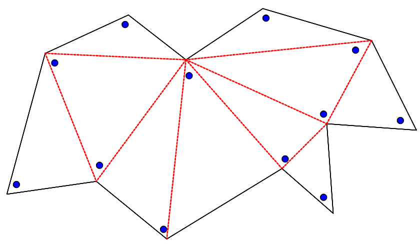

Fig. 7. The sensors are located on the vertices of the area

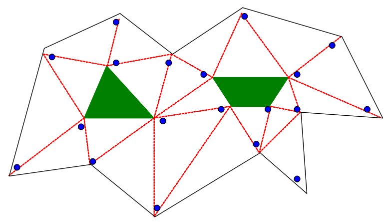

Fig. 8. n-polygon with h-holes as an arbitrary area. The sensors are located on the vertices of the area and of its holes

Therefore the number of the sensors to cover a $n - polygon$ is n. We can extend this conclusion to a *polygon* with h holes and n vertices in total as indicated in Fig. 8.
For the 3-D case, with a similar method we can conclude that for a *polyhedron* of n vertices and h holes, n sensors located on the vertices are sufficient to cover the volume. So we have the following theorem:
Theorem: n sensors are always sufficient to localize an emitter in entire interior of a $n - polygon$ in 2-D($n - polyhedron$ in 3-D) with LOS measurements. This result can be generalized to a polygon (polyhedron) with h holes and n vertices in total.

VI. Simulation Results

Simulation was done by MATLAB for the 2-D case for triangular-shaped areas with three sensors, with function S defined in (6) as the criterion. The result was compatible with theory and showed that the best place for the sensors is on the corners of the triangle.

VII. Acknowledgements

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