# An Adaptive Maximally Decimated Channelized UWB Receiver with Cyclic Prefix

Lei Feng, Won Namgoong Department of Electrical Engineering University of Southern California leifeng@usc.edu, namgoong@usc.edu

Abstract - The frequency channelized receiver based on hybrid filter bank is a promising receiver structure for ultra-wideband (UWB) radio because of its relaxed circuit requirements and robustness to narrowband interference. Maximally decimated channelizer requires the fewest number of ADCs, but it suffers from poor convergence speed, making it ill-suited for UWB systems. By applying cvclic prefix (CP) to the transmitted data, the channelizer and the propagation channel can be decomposed as a cascade of three fixed DFT related matrices and two diagonal matrices. This decomposition allows the rapidly varying propagation channel and the slowly varying channelizer to be updated at vastly different rates. An adaptive algorithm based on minimizing the data block mean squared error (MSE) of the cascaded equalizers is also proposed. The performance is comparable to an ideal full band receiver after initial convergence.

## **1 INTRODUCTION**

UWB system is characterized by its wide signal bandwidth of generally several gigahertz. Since digitizing such a wideband signal at least at the signal Nyquist rate is difficult using a single ADC, parallel ADCs need to be employed. Maximally decimated frequency channelized receivers based on hybrid filter banks (Fig. 1) achieve an effective sampling frequency that is *M* times the ADC sampling frequency, where *M* is the number of parallel ADCs [1]. Among the advantages of the frequency channelized receiver compared to the more conventional time channelized (i.e., time-interleaved ADC) receiver are the ease of designing the sample/hold circuitries, greater robustness to jitter/phase noise, and reduced ADC dynamic range requirements.

The uncertainties in the analog analysis filters and the time varying nature of the propagation channels necessitate adaptive methods in practical frequency channelized receivers. Adaptive synthesis filters can be employed but the drawbacks are slow convergence speed and large computational load [2][3]. Although the analog analysis filters drift slowly and can be modeled as being approximately constant, fast adaptive filter banks are required to quickly track variations in the propagation channel. Consequently, the slow convergence speed of the existing adaptive maximally decimated filter bank techniques are ill-suited in UWB systems.



Fig. 1  $M_s$  subband frequency channelized receiver.

To improve the convergence speed, our UWB transmitter sends blocks of data each with cyclic prefix (CP). The CP limits the matrix form of the propagation channel to a circulant matrix (CM) subspace instead of the space for all possible channel models [4]. The CM subspace matrices can be decomposed into the multiplication of two fixed DFT/IDFT matrices and a diagonal matrix, which can be compensated by a set of one-tap equalizers. Examples of communication systems that exploit the structural properties of the CM are cyclic prefixed single carrier (CP-SC) and orthogonal frequency division multiplexing (OFDM).

By appending the CP, the propagation channel and the frequency channelizer together can be modeled as a block circulant matrix (BCM). In the BCM subspace, all matrices can be decomposed into two fixed DFT related matrices and a block diagonal matrix [5]. Because the impulse response of the subband filters are modulated versions of a prototype filter, the BCM corresponding to the combined responses of the propagation channel and the channelizer can be further decomposed into three fixed DFT related matrices and two diagonal matrices. The diagonal matrix corresponding to the channelizer

This work was supported in part by the Army Research Office under contract number DAAD19-01-1-0477 and National Science Foundation under contract number ECS-0134629.

equalizer is adapted as a set of one-tap equalizers. The diagonal matrix corresponding to the propagation channel is adapted as a set of two-tap equalizers since the sampling rate is assumed to be twice the symbol rate. Two-tap equalizers are also needed in a full band receiver if fractionally spaced equalizers are employed to achieve high performance [6]. Detection can then be achieved by adapting the cascaded equalizers and performing several DFT/IDFT related operations.

Based on the minimum mean squared error (MSE) criterion, an adaptive algorithm is derived by taking advantage of the BCM structure. All constant matrices in the adaptive algorithm can be efficiently realized using FFT/IFFT as they are all related to the DFT/IDFT operation. Since the receiver can adaptively compensate the propagation channel and the channelizer separately, each of which vary at vastly different rates, the one-tap equalizers for the frequency channelizer can be fixed or updated very slowly to track variations in the analog analysis filters after initial convergence. The adaptive receiver then operates as if its input is from an ideal single ADC, resulting in fast tracking to channel variations as in a CP-SC system.

The paper is organized as follows. Section 2 describes the BCM model for the channelizer. Section 3 presents the adaptive algorithm. Simulation and conclusions are provided in sections 4 and 5, respectively.

#### **2 FREQUENCY CHANNELIZER MODEL**

A block of received UWB signal r(t) carrying K symbols (excluding CP) is

$$r(t) = \sum_{k=0}^{K-1} a_k p(t - kT) + v(t)$$
(1)

where  $a_k$  is the *k*th transmitted antipodal symbol, *T* is the symbol period, p(t) is the received signal pulse, and v(t) is the additive white Gaussian noise (AWGN).

#### 2.1 Modulated Filters

The maximally decimated frequency channelizer with  $M_s$  subbands is shown in Fig. 1. A set of  $M_s$  equally spaced mixers  $e^{-j2\pi m f_s t}$  ( $m = 0, ..., M_s - 1, f_s = 1/T_s$ ) downconverts the received signal r(t). Each of the  $M_s$ downconverted signals is passed through a low pass filter h(t) and digitized using an ADC operating at the sampling rate of  $1/T_s$ . The effective sampling frequency of the receiver is  $1/T_e$ , which is related to the ADC sampling frequency  $1/T_s$  by  $T_e = T_s/M$ , where  $M = 2M_s - 1$ . Also we assume the effective sampling frequency is twice the data rate, i.e.,  $T_e = T/2$ . The *m*th subband sampled signal is

$$x_{m}[l] = e^{j\phi_{m}}e^{-j2\pi m f_{s}t}r(t) \otimes h(t)|_{t = lT_{s}}$$
  
=  $e^{j\phi_{m}}\sum_{n'}r[Ml - n']h[n']e^{j2\pi m n'/M}$  (2)

where  $\varphi_m$  is the initial phase of the mixer,  $\otimes$  denotes convolution,  $r[n'] = r(n'T_e)$ , and  $h[n'] = T_e h(n'T_e)$ . We assume for simplicity that the mixers are designed so that  $\varphi_0 = \varphi_1 = \dots = \varphi_{M_s-1} = 0$ , although the proposed reception approach can be readily modified to account for the more general case with arbitrary initial phase values.

The discrete equivalent model is shown in Fig. 2. ADCs are replaced by downsamplers with rate M.  $h_m[n']$ ,  $m = 0, ..., M_s - 1$  are modulated versions of the lowpass prototype filter h[n'], i.e.,

$$h_m[n'] = h[n']e^{j2\pi mn'/M}$$
 (3)

The  $M_s$  subbands only cover part of the discrete spectrum between 0 and  $2\pi$ . To form a complete representation of the received signal, the remaining spectrum is obtained by conjugating all the subband signals except for the zeroth subband. Thus, for subbands  $m = M_s, ..., 2M_s - 2$ , the conjugate samples are  $x_m[l] = x_{M-1-m}^*[l]$  and the filters are still described by (3). We will subsequently use M subbands instead of  $M_s$  subbands to be consistent with discrete filter bank systems.



Fig. 2 Discrete equivalent model.

The subband samples at time  $(N-1)T_s$  can be represented as a vector  $\mathbf{x}[N-1] = [x_0[N-1]], ..., x_{M-1}[N-1]]^T$ , where *T* denotes transpose. Denoting the received signal in vector form as  $\mathbf{r} = [r[0], ..., r[MN-1]]^T$ ,

$$\mathbf{x}[N-1] = \begin{bmatrix} \mathbf{H}_{N-1} \dots \mathbf{H}_1 \mathbf{H}_0 \end{bmatrix} \mathbf{r}$$
(4)  
where  $\mathbf{H}_n$  ( $n = 0, ..., N-1$ ) are  $M \times M$  matrices.

$$\mathbf{H}_{n} = \begin{bmatrix} h_{0}[(n+1)M-1] & \dots & h_{0}[nM] \\ \dots & \dots & \dots \\ h_{M-1}[(n+1)M-1] & \dots & h_{M-1}[nM] \end{bmatrix}$$
(5)

From (3), each row of  $\mathbf{H}_n$  is a modulated version of its first row. Hence,  $\mathbf{H}_n$  can be decomposed as

$$\mathbf{H}_{n} = \begin{bmatrix} h_{0}[(n+1)M-1] & \dots & h_{0}[nM] \\ W_{M}^{M-1}h_{0}[(n+1)M-1] & \dots & h_{0}[nM] \\ & \dots & & \dots \\ W_{M}^{(M-1)^{2}}h_{0}[(n+1)M-1] & \dots & h_{0}[nM] \end{bmatrix}$$

$$= \mathbf{F}_{M} \mathbf{H}_{n}^{d} \tag{6}$$

where  $W_M^m = e^{j2\pi m/M}$  and

$$\mathbf{F}_{M} = \begin{bmatrix} 1 & \dots & 1 & 1 \\ W_{M}^{M-1} & \dots & W_{M}^{1} & 1 \\ \dots & \dots & \dots & \dots \\ W_{M}^{(M-1)^{2}} & \dots & W_{M}^{M-1} & 1 \end{bmatrix}$$
(7)

 $\mathbf{H}_{n}^{d} = diag(h_{0}[(n+1)M-1], ..., h_{0}[nM])$ (8) diag() denotes forming a diagonal matrix.

#### 2.2 Block Circulant Matrix Representation

To achieve faster adaptive performance in the frequency channelized receiver, CP is appended at the beginning of each transmitted signal block. After removing the samples corresponding to the CP at the receiver, N samples are collected in each subband. Since the effective sampling frequency is twice the data rate and each block includes K data symbols, we assume 2K = MN. The subband sample vector **x** is represented by the multiplication of a BCM **H** and the received signal vector **r**, i.e.,

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}[0] \\ \dots \\ \mathbf{x}[N-1] \end{bmatrix} = \begin{bmatrix} \mathbf{H}_0 & \mathbf{H}_{N-1} & \dots & \mathbf{H}_1 \\ \mathbf{H}_1 & \mathbf{H}_0 & \dots & \mathbf{H}_2 \\ \dots & \dots & \dots & \dots \\ \mathbf{H}_{N-1} & \dots & \mathbf{H}_1 & \mathbf{H}_0 \end{bmatrix} \mathbf{r} = \mathbf{H}\mathbf{r} (9)$$

Using the results on BCM from [5], H can be expressed as

 $\mathbf{H} = \mathbf{F}_{B}^{-1} \cdot \mathbf{diag}(\mathbf{H}^{(0)}, ..., \mathbf{H}^{(k)}, ..., \mathbf{H}^{(N-1)}) \cdot \mathbf{F}_{B} (10)$ where **diag**() denotes forming a block diagonal matrix. In (10),  $\mathbf{F}_{B} = [W_{N}^{-kn}\mathbf{I}_{M}], \mathbf{F}_{B}^{-1} = [W_{N}^{kn}\mathbf{I}_{M}]/N,$ (k, n = 0, ..., N-1).  $\mathbf{I}_{M}$  is an  $M \times M$  identity matrix.  $\mathbf{H}^{(k)}$  is related to  $\mathbf{H}_{n}$  by DFT operations

$$\mathbf{H}_{n} = \frac{1}{N} \sum_{k=0}^{N-1} \mathbf{H}^{(k)} W_{N}^{kn}, \quad \mathbf{H}^{(k)} = \sum_{n=0}^{N-1} \mathbf{H}_{n} W_{N}^{-kn} \quad (11)$$

Defining matrices  $\mathbf{F}_D$  and  $\mathbf{C}$  as

$$\mathbf{F}_D = \mathbf{diag}(\mathbf{F}_M, \dots, \mathbf{F}_M) \tag{12}$$

$$\mathbf{C} = \mathbf{diag} \left( \sum_{n=0}^{N-1} \mathbf{H}_{n}^{d}, \dots, \sum_{n=0}^{N-1} \mathbf{H}_{n}^{d} W_{N}^{-n(N-1)} \right)$$
(13)

and substituting (6) into (11), the BCM matrix **H** given in (10) is

$$\mathbf{H} = \mathbf{F}_B^{-1} \mathbf{F}_D \mathbf{C} \mathbf{F}_B \tag{14}$$

When CP is employed, it is well known that the propagation channel  $p[n] = p(nT_e)$  can be modeled as a circulant matrix **P**. Consequently, **P** can be decomposed as

$$\mathbf{P} = \mathbf{F}^{-1} \mathbf{\Lambda} \mathbf{F} \tag{15}$$

where  $\Lambda$  is a diagonal matrix and  $\mathbf{F} = [W_{MN}^{-kn}](k, n = 0, ..., MN - 1)$  is a DFT matrix.

Denoting vectors  $\mathbf{a} = [a_0, a_1, ..., a_{K-1}]^T$ ,  $\mathbf{s} = [a_0, 0, a_1, 0, ..., 0, a_{K-1}, 0]^T$ , and  $\mathbf{v} = [v[0], ..., v[MN-1]]^T$ , where  $v[n] = v(nT_e)$ , the received signal  $\mathbf{r}$  can be represented as

$$\mathbf{r} = \mathbf{P}\mathbf{s} + \mathbf{v} \tag{16}$$

From (9) and (16), the sampled signal is

ŝ

$$\mathbf{x} = \mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{H}\mathbf{v} \tag{17}$$

**s** can be estimated by multiplying **x** with  $\mathbf{P}^{-1}\mathbf{H}^{-1}$ . The inversion is readily obtained by substituting **P** and **H** with (15) and (14), respectively. To avoid noise enhancement,  $\mathbf{C}^{-1}$  and  $\Lambda^{-1}$  are replaced with diagonal matrix equalizers  $\mathbf{C}_{e}^{H}$  and  $\Lambda_{e}^{H}$ , respectively. The superscript *H* denotes conjugate transpose. The estimate of **s** is

$$= \mathbf{F}^{-1} \Lambda_e^H \mathbf{F} \mathbf{F}_B^{-1} \mathbf{C}_e^H \mathbf{F}_D^{-1} \mathbf{F}_B \mathbf{x}$$
(18)

The transmitted data **a** can be recovered by dropping the even rows of  $\mathbf{F}^{-1}$ . Defining  $\mathbf{F}_{1/2}^{-1} = [W_{MN/2}^{kn}]/(MN/2)$ , (k, n = 0, ..., MN/2-1) and  $\Lambda_e = \mathbf{diag}(\Lambda_{e0}, \Lambda_{e1})$ , where  $\Lambda_{e0}$  and  $\Lambda_{e1}$  are  $(MN/2 \times MN/2)$  diagonal matrices, the estimate of **a** is obtained from (18) as

$$\hat{\mathbf{a}} = \frac{1}{2} \begin{bmatrix} \mathbf{F}_{1/2}^{-1} & \mathbf{F}_{1/2}^{-1} \end{bmatrix} \begin{bmatrix} \Lambda_{e0}^{H} & \mathbf{0} \\ \mathbf{0} & \Lambda_{e1}^{H} \end{bmatrix} \mathbf{F} \mathbf{F}_{B}^{-1} \mathbf{C}_{e}^{H} \mathbf{F}_{D}^{-1} \mathbf{F}_{B} \mathbf{x}$$
$$= \frac{1}{2} \mathbf{F}_{1/2}^{-1} \begin{bmatrix} \Lambda_{e0}^{H} & \Lambda_{e1}^{H} \end{bmatrix} \mathbf{F} \mathbf{F}_{B}^{-1} \mathbf{C}_{e}^{H} \mathbf{F}_{D}^{-1} \mathbf{F}_{B} \mathbf{x}$$
(19)

Matrices in the bracket denote fractionally spaced equalizers [6], which are summation of two one-tap equalizers.

### **3 ADAPTIVE ALGORITHM**

Adapting the equalizer structure in (19) is not straightforward because the equalizers are cascaded. An adaptive algorithm for the cascaded equalizers is derived to minimize the MSE of a transmission block.

The recovered signal error is defined as  $\mathbf{e}_a = \mathbf{a} - \hat{\mathbf{a}}$ . The MSE of a block of signals is the expectation of  $\mathbf{e}_a^H \mathbf{e}_a$ . Since expectations are typically obtained by instantaneous estimations as in the LMS algorithm, the derivation is directly based on the squared error  $J = \mathbf{e}_a^H \mathbf{e}_a$ .

According to the method of steepest descent, the two equalizers can be updated with the corresponding gradients. Denote the diagonal elements of the equalizer  $C_e$  and  $\Lambda_e$  as column vectors **c** and  $\lambda$ , respectively. The gradient vectors are defined as

$$\nabla_{\lambda}J = 2\frac{\partial J}{\partial\lambda^*}, \quad \nabla_c J = 2\frac{\partial J}{\partial\mathbf{c}^*}$$
 (20)

The results are

$$\nabla_{\lambda} J = -\mathbf{u}_{\lambda} \bullet \begin{bmatrix} \mathbf{e}_{\lambda} \\ \mathbf{e}_{\lambda} \end{bmatrix}^{*}$$
(21)

$$\nabla_c J = -\mathbf{u}_c \bullet \mathbf{e}_c^* \tag{22}$$

where • denotes the inner product operation, i.e., element-wise multiplication.  $\mathbf{u}_{\lambda}$  and  $\mathbf{u}_{c}$  are inputs to the equalizer  $\Lambda_{e}$  and  $\mathbf{C}_{e}$ , respectively.  $\mathbf{e}_{\lambda}$  and  $\mathbf{e}_{c}$  are the corresponding output error. They are expressed as

$$\mathbf{e}_{\lambda} = (\mathbf{F}_{1/2}^{-1})^{H} \mathbf{a} - \frac{1}{MN} \left[ \Lambda_{e0}^{H} \ \Lambda_{e1}^{H} \right] \mathbf{F} \mathbf{F}_{B}^{-1} \mathbf{C}_{e}^{H} \mathbf{F}_{D}^{-1} \mathbf{F}_{B} \mathbf{x} (23)$$

$$\mathbf{e}_{c} = (\mathbf{F}_{B}^{-1})^{H} \mathbf{F}^{H} \Big[ \Lambda_{e0}^{H} \ \Lambda_{e1}^{H} \Big]^{H} \mathbf{e}_{\lambda}$$
(24)

$$\mathbf{u}_{\lambda} = \mathbf{F}\mathbf{F}_{B}^{-1}\mathbf{C}_{e}^{H}\mathbf{F}_{D}^{-1}\mathbf{F}_{B}\mathbf{x}$$
(25)

$$\mathbf{u}_c = \mathbf{F}_D^{-1} \mathbf{F}_B \mathbf{x} \tag{26}$$

The gradients can be applied to the adaptive algorithms by introducing data block index  $\tau$  as the iteration index. By appending  $\tau$  to all the signals (e.g.  $\mathbf{x}(\tau), \mathbf{C}_{e}^{H}(\tau), \mathbf{u}_{\lambda}(\tau), \mathbf{e}_{\lambda}(\tau)$ , etc.), the adaptive equations are

$$\lambda(\tau+1) = \lambda(\tau) + \mu_{\lambda} \mathbf{u}_{\lambda}(\tau) \bullet \begin{bmatrix} \mathbf{e}_{\lambda}(\tau) \\ \mathbf{e}_{\lambda}(\tau) \end{bmatrix}^{*}$$
(27)

$$\mathbf{c}(\tau+1) = \mathbf{c}(\tau) + \mu_c \mathbf{u}_c(\tau) \bullet \mathbf{e}_c(\tau)^*$$
(28)

The adaptive structure is shown in Fig. 5.

The cascaded equalizer structure enables the receiver to track the variations in the propagation channel and the channelizer at different speeds, each of which vary at vastly different rates. After the initial convergence of  $C_E$ , which is achieved after turning on the

receiver, the receiver basically adapts only equalizer  $\Lambda_E$ , resulting in tracking performance comparable to an ideal full band receiver for CP-SC system.

#### **4 SIMULATION RESULTS**

The transmitted data is modulated by a raised cosine pulse with roll off factor 0.8. The symbol period is 0.5ns. The effective sampling rate is 2 samples per symbol. The UWB channel model is CM1 model proposed by IEEE P802.15 working group.

The receiver is composed of  $M_s$ =4 subbands so that the effective sampling rate is 7 times the subband ADC sampling rate. The analog prototype filter is the fourth order Butterworth lowpass filter whose cutoff frequency is half of the ADC sampling frequency. Since most of the multipath energy is within the first 20ns, the CP period is selected to be approximately 20ns. Each data block includes 224 information bits and 42 CP bits. In this section, a full band receiver with same data format is used for comparison. The full band receiver is an ideal CP-SC receiver that samples the received signal with a single ADC and performs equalization with fractional spaced equalizers. The signal energy is  $E_b = \sum (p[n])^2$ , and the noise power  $N_0/2$  is the variance of v[n].

The convergence speeds are compared for two cases: a full band receiver and a channelized receiver with converged channelizer equalizer, where the channelizer equalizer  $C_e^H$  has already converged based on 5 different propagation channels. The MSE learning curves of the detected symbols when  $E_b/N_0 = 10dB$  are given in Fig. 3. The convergence speed of the channelized receiver is comparable to that of the full band receiver.



Fig. 3 MSE Learning Curves

The bit error rate (BER) is calculated after the initial channelizer equalizer convergence (Fig. 4). BER performance is simulated for a packet of 200 blocks when the first 1 and 10 blocks are used for training. In the channelized receiver, the step-sizes are set to 0.1 for the propagation channel equalizer and 0.01 for the channelizer equalizer. In the full band receiver, the step size is set to 0.1. The channelized receiver performance is similar to that of the full band receiver especially when  $E_b/N_0$  is low to moderate.



Fig. 4 BER Performance

## **5 CONCLUSIONS**

A computationally efficient and fast convergence adaptive algorithm for UWB channelized receivers is proposed. By applying CP to the transmitted data, the received channelized signal can be modeled as a BCM. By exploiting the modulation relations among the channelization filters, the effect of the propagation channel and the channelizer can be separately equalized. After initial convergence of the channelizer equalizer, the BER performance of the channelized receiver is comparable to an ideal full band receiver.

#### REFERENCE

- W. Namgoong, "A Channelized Digital Ultra-Wideband Receiver," IEEE Trans. Wireless Comm., vol. 2, pp. 502-510, May 2003
- [2] A. Gilloire and M. Vetterli, "Adaptive Filtering in Subbands with Critical Sampling: Analysis, Experiments, and Application to Acoustic Echo Cancellation," IEEE Trans. on Signal Proc. vol. 40, NO. 8, pp. 1862-1875, Aug. 1992
- [3] L. Feng and W. Namgoong, "A frequency channelized adaptive wideband receiver for high-speed links," Signal Processing Systems, 2003. SIPS 2003. IEEE Workshop on, pp. 24 -28, Aug. 2003
- [4] J. Louveaux, L. Vandendorpe and T. Sartenaer, "Cyclic Prefixed Single Carrier and Multicarrier Transmisson: Bit Rate Comparison," IEEE Comm. Letters, vol. 7, NO. 4, pp.180-182, Apr. 2003
- [5] R. Vescovo, "Inversion of Block-Circulant Matrices and Circular Array Approach," IEEE Trans. on Antennas and Propagation, vol. 45, NO. 10, pp.1565-1567, Oct. 1997
- [6] P.P. Vaidyanathan and B. Vrcelj, "Theory of Fractionally Spaced Cyclic-Prefix Equalizers," ICASSP'02, vol.2, pp.1277-1280, 2002



Fig. 5 Cascaded adaptive equalizers for UWB receivers