Generalization of Single-Carrier and Multicarrier Cyclic Prefixed Communication

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Abstract: **This paper describes a cyclic prefixed communication structure that is a generalization of both the orthogonal frequency-division multiplexing (OFDM) system and the cyclic prefixed single-carrier (CP-SC) system. By exploiting the fact that a circulant matrix (CM) is also a block circulant matrix (BCM), the proposed generalized cyclic prefixed (GCP) communication system provides an additional degree of freedom that allows trade-offs among peak-to-average power ratio (PAR), bit allocation capability, and frequency diversity. Instead of one large IDFT operation as in the OFDM transmitter, GCP time interleaves multiple smaller sized IDFT operations, so that PAR is correspondingly reduced. By appropriately selecting the block size in the BCM, the PAR value and the number of subcarriers for bit allocation can be chosen regardless of the length of the transmission block. When the block size in BCM is selected to be one or the entire size of the transmission block, the proposed GCP becomes OFDM or CP-SC, respectively.**

1 INTRODUCTION

Cyclic prefixed transmission has attracted extensive research interests because of its anti-multipath capabilities. There are two major types of cyclic prefixed systems -- orthogonal frequency-division multiplexing (OFDM) and cyclic prefixed single-carrier (CP-SC) [1]. In addition to its anti-multipath efficiency, OFDM is well known for its potential for achieving near channel capacity by adaptive modulation. The disadvantage of OFDM, however, is its large peak-to-average power ratio (PAR), which requires an expensive linear high-power amplifier that results in poor power efficiency. The PAR problem of OFDM has been recently reviewed in [2]. Researchers proposed many methods to alleviate the PAR problem by coding [3], tone reservation [4], trellis shaping [5], multiple signal representation [6], etc. Unlike in OFDM, CP-SC does not suffer from the large PAR problem, and as a result, cheap and power efficient power amplifier can be employed. A drawback of CP-SC, however, is its inability to perform bit allocation over frequencies to achieve high transmission rates.

In this paper, the single-carrier and multicarrier cyclic prefixed communication systems are generalized by recognizing that a circulant matrix (CM) can be viewed as a block circulant matrix (BCM) [7]. By viewing the CM as a BCM (with the block size being a design parameter), the CM can be decomposed as a diagonal matrix and four DFT related matrices, all of which can be realized by interleaving multiple smaller sized DFT/IDFT operations. By selecting the block size to be one, GCP becomes OFDM. Similarly, if the block size is chosen to be the size of the transmitted symbol block, GCP becomes CP-SC. By selecting a block size that is somewhere between these two extremes, a hybrid structure that combines the advantages of both OFDM and CP-SC systems can be realized.

GCP provides design choices that is not available in OFDM and CP-SC systems. Regardless of the transmission block length, the DFT size in GCP can be chosen to satisfy the desired PAR value. Moreover, unlike in OFDM or CP-SC where each information symbol is sent through one or all the carriers, respectively, the number of carriers (or the amount of frequency diversity) is a design parameter in GCP. As a result, GCP enjoys design flexibility in bit allocation capability and PAR control that is not available in OFDM and CP-SC.

The paper is organized as follows. The proposed decomposition scheme for CM is explained in Section II. The transceiver structure of the GCP and its performance analysis are provided in Section III. Section IV describes the simulation results. Conclusions are drawn in Section V.

2 DECOMPOSITION SCHEME

It is well known that the propagation channel of a cyclic prefixed transmission (OFDM or CP-SC) can be modeled as a CM matrix, which can be decomposed as

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DFT/IDFT and diagonal matrices. In this section, we introduce a novel decomposition of the CM matrix from which the proposed communication system is derived.

The propagation channel response at the transmitted data symbol rate is $h[i]$ $i = 0, \ldots, N_b - 1$. N_b is the number of data symbols in a transmission block (excluding CP). By applying CP to the transmitted symbol block, the channel can be modeled as a $N_b \times N_b$ CM $\mathbf{H}$.

$$\mathbf{H} = \begin{bmatrix} h[0] & h[N_b-1] & \ldots & h[1] \\ h[1] & h[0] & \ldots & h[2] \\ \ldots & \ldots & \ldots & \ldots \\ h[N_b-1] & \ldots & h[1] & h[0] \end{bmatrix} \quad (1)$$

The proposed decomposition is based on the fact that the CM $\mathbf{H}$ can be viewed as a BCM composed of N blocks, each of which has a block size M, where M and N satisfy $N_b = MN$. To make the BCM structure more explicit, the last M rows of $\mathbf{H}$ are denoted as $[\ \mathbf{H}_{N-1} \ldots \mathbf{H}_0\ ]$, where $\mathbf{H}_k$ $(k = 0, \ldots, N-1)$ are $M \times M$ Toeplitz matrices. The Toeplitz matrix $\mathbf{H}_k$ is described by

$$\mathbf{H}_k = \begin{bmatrix} h[kM] & h[kM-1] & \ldots & h[kM-M+1] \\ h[kM+1] & h[kM] & \ldots & \ldots \\ \ldots & \ldots & \ldots & h[kM-1] \\ h[kM+M-1] & \ldots & h[kM+1] & h[kM] \end{bmatrix} \quad (2)$$

where we define $h[-i] = h[N_b - i]$ for indices less than zero. $\mathbf{H}$ is then rewritten in a BCM form as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_0 & \mathbf{H}_{N-1} & \ldots & \mathbf{H}_1 \\ \mathbf{H}_1 & \mathbf{H}_0 & \ldots & \mathbf{H}_2 \\ \ldots & \ldots & \ldots & \ldots \\ \mathbf{H}_{N-2} & \ldots & \mathbf{H}_0 & \mathbf{H}_{N-1} \\ \mathbf{H}_{N-1} & \ldots & \mathbf{H}_1 & \mathbf{H}_0 \end{bmatrix} \quad (3)$$

The BCM $\mathbf{H}$ can be decomposed as [7]

$$\mathbf{H} = \mathbf{F}_B^H \mathbf{diag}(\mathbf{H}^{(0)}, \ldots, \mathbf{H}^{(N-1)}) \mathbf{F}_B \quad (4)$$

In (4), the DFT related matrices $\mathbf{F}_B = [W_N^{-kn} \mathbf{I}_M] / \sqrt{N}$ $(k, n = 0, \ldots, N-1)$ where $W_N^{-kn} = e^{-j2\pi kn/N}$, $\mathbf{I}_M$ is the M-dimensional identity matrix and **diag**( ) denotes a block diagonal matrix with the matrices inside the parenthesis on the diagonal. The superscript H stands for conjugate transpose. To build $\mathbf{F}_B$, $W_N^{-00}\mathbf{I}_M$ is at the upper left corner and $W_N^{-(N-1)(N-1)}\mathbf{I}_M$ is at the lower right corner. $\mathbf{F}_B$ is a unitary matrix so that $\mathbf{F}_B^{-1} = \mathbf{F}_B^H$. $\mathbf{H}^{(n)}$ is related to $\mathbf{H}_k$ by DFT operations

$$\mathbf{H}^{(n)} = \sum_{k=0}^{N-1} \mathbf{H}_k W_N^{-kn}, \ \mathbf{H}_k = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{H}^{(n)} W_N^{kn} \quad (5)$$

From (2) and (5),

$$\mathbf{H}^{(n)} = \begin{bmatrix} h^{(n)}[0] & \ldots & h^{(n)}[1] W_N^{-n} \\ \ldots & \ldots & \ldots \\ h^{(n)}[M-1] & \ldots & h^{(n)}[0] \end{bmatrix} \quad (6)$$

where

$$h^{(n)}[m] = \sum_{k=0}^{N-1} h[kM+m] W_N^{-nk} \quad (7)$$

for $m = 0, \ldots, M-1$. $\mathbf{H}^{(n)}$ is similar to a circulant matrix except that the uptriangle elements are multiplied by W_N^{-n}. Although the steps are omitted here, it can be proved that $\mathbf{H}^{(n)}$ has M constant eigenvectors, where the mth eigenvector is

$$\mathbf{e}_m^{(n)} = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 & e^{j2\pi\left(m + \frac{n}{N}\right)\frac{1}{M}} & \ldots & e^{j2\pi\left(m + \frac{n}{N}\right)\frac{M-1}{M}} \end{bmatrix}^T \quad (8)$$

The corresponding eigenvalue is

$$\lambda_m^{(n)} = \frac{1}{\sqrt{M}} \sum_{m'=0}^{M-1} h^{(n)}[m'] e^{j2\pi\left(m + \frac{n}{N}\right)\frac{m'}{M}} \quad (9)$$

Define an eigenvector matrix $\mathbf{E}^{(n)}$ and eigenvalue matrix $\Lambda^{(n)}$.

$$\mathbf{E}^{(n)} = \begin{bmatrix} \mathbf{e}_0^{(n)} & \ldots & \mathbf{e}_{M-1}^{(n)} \end{bmatrix}^H \quad (10)$$

$$\Lambda^{(n)} = diag(\lambda_0^{(n)}, \ldots, \lambda_{M-1}^{(n)}) \quad (11)$$

Because $(\mathbf{E}^{(n)})^{-1} = (\mathbf{E}^{(n)})^H$, $\mathbf{H}^{(n)}$ can be decomposed as

$$\mathbf{H}^{(n)} = (\mathbf{E}^{(n)})^H \Lambda^{(n)} \mathbf{E}^{(n)} \quad (12)$$

As a result, the channel matrix $\mathbf{H}$ in (4) can be expressed as a product of a diagonal matrix Λ and four fixed DFT related matrices.

$$\mathbf{H} = \mathbf{F}_B^H \mathbf{E}^H \Lambda \mathbf{E} \mathbf{F}_B \quad (13)$$

where

$$\mathbf{E} = \mathbf{diag}(\mathbf{E}^{(0)}, \ldots, \mathbf{E}^{(N-1)}) \quad (14)$$

$$\Lambda = \mathbf{diag}(\Lambda^{(0)}, \ldots, \Lambda^{(N-1)}) \quad (15)$$

Based on this decomposition, a novel communication structure can be derived as described in the following section.

3 GENERALIZED CYCLIC PREFIXED SYSTEM

3.1 Communication Scheme

A block of $N_b = MN$ information symbols a_i, $i = 0, \ldots, N_b - 1$, is put into a vector $\mathbf{a} = [a_0, a_1, \ldots, a_{N_b-1}]^T$. The transmitted signal $\mathbf{s}$ is obtained by applying $\mathbf{F}_B^H$ to the data symbol vector $\mathbf{a}$.

$$\mathbf{s} = \mathbf{F}_B^H \mathbf{a} \tag{16}$$

This operation is described as follows. The data symbol block is divided into M sub-blocks. Each N-symbol sub-block is obtained by decimating $\mathbf{a}$ by M starting at the mth time index, where $m \in \{0, 1, ... M\text{-}1\}$. The mth sub-block is defined as $\mathbf{a}_m = [a_m, a_{m+M}, \ldots, a_{m+nM}, \ldots, a_{m+(N-1)M}]^T$, $n = 0, \ldots, N-1$. Applying N-point IDFT to each sub-block, the output is $\mathbf{b}_m = [b_m^0, b_m^1, \ldots, b_m^{N-1}]^T$. The IDFT operation corresponding the mth sub-block is $\mathbf{b}_m = \mathbf{F}_N^H \mathbf{a}_m$, where $\mathbf{F}_N^H = [W_N^{nn'}]/\sqrt{N}$ $(n, n' = 0, \ldots, N-1)$ is an IDFT matrix. The IDFT outputs are interleaved to form the transmitted sequence $\mathbf{s} = [b_0^0, \ldots, b_{M-1}^0, b_0^1, \ldots, b_{M-1}^1, \ldots, b_0^{N-1}, \ldots, b_{M-1}^{N-1}]^T$ Fig. 1 shows the IDFT structure at the transmitter.

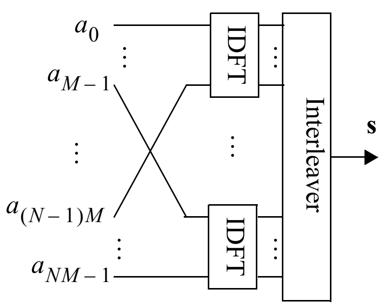

Fig. 1 The proposed transmitter

The transmitted signal is distorted by channel $\mathbf{H}$ and corrupted by additive white Gaussian noise $\mathbf{v}$. The received signal is

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{v} = \mathbf{F}_B^H \mathbf{E}^H \Lambda \mathbf{E} \mathbf{a} + \mathbf{v} \tag{17}$$

The information symbols can be recovered at the receiver by

$$\hat{\mathbf{a}} = \mathbf{a} + \mathbf{E}^H \Lambda^{-1} \mathbf{E} \mathbf{F}_B \mathbf{v} \tag{18}$$

The proposed Generalized Cyclic Prefix (GCP) communication system is shown in Fig. 2. OFDM and CP-SC are two special cases of GCP. When $M = 1$ and $N = N_b$, $\mathbf{F}_B^H$ and $\mathbf{F}_B$ are IDFT and DFT matrices, respectively; whereas, both $\mathbf{E}^H$ and $\mathbf{E}$ are identity matrices. The resulting system is an OFDM system. When $N = 1$ and $M = N_b$, $\mathbf{E}^H$ and $\mathbf{E}$ are IDFT and DFT matrices, respectively; whereas, $\mathbf{F}_B^H$ and $\mathbf{F}_B$ are identity matrices. This is a CP-SC system. Similar to OFDM and CP-SC, the inverse of the diagonal matrix in GCP is a set of one-tap equalizers. In practice, it is usually replaced by adaptive equalizers.

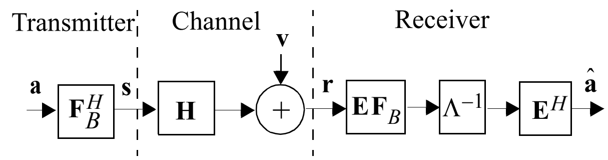

Fig. 2 The GCP system

Although MN data symbols are transmitted in each transmission block, only N-point IDFT is applied in the GCP transmitter. By comparison in an OFDM system, MN-point IDFT is required for the same block. The smaller size IDFT results in a smaller PAR, which in turn relaxes the power amplifier design and improves the power efficiency. The reduced dynamic range also relaxes the ADC resolution requirement in the receiver.

3.2 Transmission in Frequency Domain

An important advantage of OFDM is that bit allocation techniques can be employed to achieve high data rates. This is possible because each information symbol in OFDM is transmitted through a single carrier. To determine the bit allocation capability of GCP, we investigate how the data symbols are transmitted in the frequency domain. The frequency mapping of the data symbols is determined by applying a MN dimensional DFT matrix $\mathbf{F}_{MN}$ to the transmitted signal $\mathbf{F}_B^H \mathbf{a}$. The data symbols are transformed into frequency domain by $\mathbf{G} = \mathbf{F}_{MN} \mathbf{F}_B^H$.

Define $g_{(mN+n')(nM+m')}$ as the element of $\mathbf{G}$ in $(mN+n')$th row and $(nM+m')$th column. Further calculation shows

$$g_{(mN+n')(nM+m')} = \frac{1}{\sqrt{M}} W_{MN}^{-(mN+n')m'} \delta[n'-n] \tag{19}$$

The $MN \times MN$ transformation matrix $\mathbf{G}$ is expressed as

$$\mathbf{G} = \begin{bmatrix} \tilde{\mathbf{E}}_0 \\ \ldots \\ \tilde{\mathbf{E}}_{M-1} \end{bmatrix} \tag{20}$$

In (20), $\tilde{\mathbf{E}}_m$, the mth block of $\mathbf{G}$, is a $M \times MN$ block diagonal matrix

$$\tilde{\mathbf{E}}_m = \begin{bmatrix} \mathbf{e}_m^{(0)H} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{e}_m^{(1)H} & \ldots & \ldots \\ \ldots & \ldots & \ldots & \mathbf{0} \\ \mathbf{0} & \ldots & \mathbf{0} & \mathbf{e}_m^{(N-1)H} \end{bmatrix} \tag{21}$$

where $\mathbf{0}$ is $1 \times M$ zero vector.

The structure of $\mathbf{G}$ indicates that M data symbols $(a_{nM}, a_{nM+1}, \ldots, a_{nM+M-1})$ are combined and sent through M carriers at frequencies

$(W_{MN}^{n}, W_{MN}^{N+n}, \ldots, W_{MN}^{(M-1)N+n})$. The MN total data symbols are divided into N groups. Each group are sent through an independent sub-channel and each sub-channel has M equally spaced carriers. Given M, such carrier grouping maximizes the minimum carrier separation so that it is optimal in terms of maximizing frequency diversity [8]. The amount of frequency diversity is set by properly selecting M. If $M = 1$, there is no frequency diversity as the GCP system reduces to an OFDM system.

3.3 Achievable Bit Rate

Given a channel with additive Gaussian noise, the achievable bit rate per data symbol can be calculated from the signal-to-noise ratio (SNR). The detection noise is $\mathbf{E}^H\Lambda^{-1}\mathbf{E}\mathbf{F}_B\mathbf{v}$ as given in (18). Assuming $\mathbf{v}$ is a white noise with variance N_0, $\mathbf{F}_B\mathbf{v}$ is also a white noise vector with the same variance. Since $\mathbf{E}^H$ and $\mathbf{E}$ are block diagonal matrices and $\mathbf{v}$ is white noise, the detection noise covariance matrix is a block diagonal matrix

$$\mathbf{R}_v = \mathbf{diag}\left(\mathbf{R}_0 \ \ldots \ \mathbf{R}_n \ \ldots \ \mathbf{R}_{N-1} \right) \tag{22}$$

where

$$\mathbf{R}_n = N_0(\mathbf{E}^{(n)})^H(\Lambda^{(n)})^{-1}((\Lambda^{(n)})^{-1})^H\mathbf{E}^{(n)} \tag{23}$$

The block diagonal form of $\mathbf{R}_v$ indicates that the detection is performed on N independent sub-channels. The data symbols in the nth sub-channel is corrupted by noise associated with $\mathbf{R}_n$. Since the diagonal elements of $\mathbf{R}_n$ are equal, the data symbols in the same sub-channel are corrupted by noise with the same power. Assuming that all the data symbols have the same power E_s, the nth sub-channel SNR is

$$SNR_n = \frac{ME_s}{tr\{\mathbf{R}_n\}} = \frac{ME_s}{N_0}\left(\sum_{m=0}^{M-1}\frac{1}{\left|\lambda_m^{(n)}\right|^2}\right)^{-1} \tag{24}$$

In additive white Gaussian noise channel, the achievable bit rate per transmitted symbol is ([9])

$$C_n = \log_2(1 + SNR_n) \tag{25}$$

With high SNR approximation, the system achievable bit rate per transmitted symbol is

$$\begin{aligned} C &= \frac{1}{N}\sum_{n=0}^{N-1} C_n \\ &= \frac{1}{N}\log_2 \prod_{n=0}^{N-1}\left(\frac{ME_s}{N_0}\left(\sum_{m=0}^{M-1}\frac{1}{\left|\lambda_m^{(n)}\right|^2}\right)^{-1}\right) \end{aligned} \tag{26}$$

Since the geometrical mean is larger than harmonic mean for positive real value, the achievable bit rate of the proposed system is less than that of OFDM and greater than that of CP-SC [10].

4 SIMULATION RESULTS

UWB channels are considered in the simulation. The multipath model used is the CM1 channel model recommended by the IEEE P802.15-02/368r5-SG3a. The signal bandwidth is 500MHz. CP is assumed to exceed the channel length. GCP systems with M=2 and M=8 are simulated. The GCP system with M=2 reduces the PAR by 3dB compared to OFDM. More generally, the GCP system reduces the PAR by 10log$M$ dB, i.e., a 3dB reduction in PAR for every doubling of M.

Fig. 3 shows the bit error rate performance as a function of the received E_s/N_0 when LMS algorithm is applied to update the equalizers. Each transmission block includes 128 data symbols modulated with QPSK. The channel is assumed to be constant. The first 20 blocks are used for training. Four systems are simulated: OFDM, CP-SC and GCP with M=2 and M=8. No coding and bit allocation is employed. OFDM performs worst due to the lack of frequency diversity. CP-SC performs the best since it suffers the least from the notches of the channel frequency response. The performance of the proposed system is between the OFDM and CP-SC systems, since M can be adjusted to obtain different levels of frequency diversity. As the frequency diversity increases, the uncoded system performance becomes better.

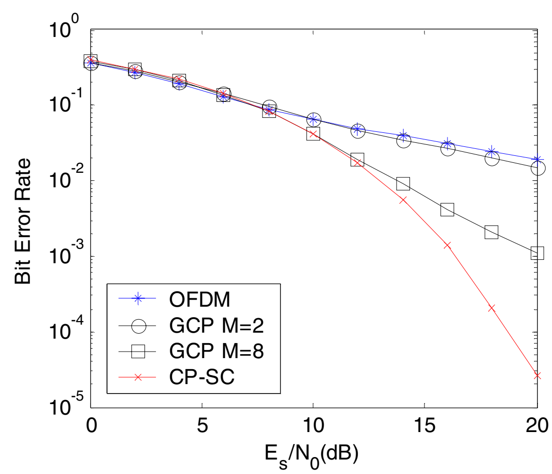

Fig. 3 Uncoded bit error rate

Using (26), the achievable bit rate per transmitted symbol versus $\log_2(MN)$ for the same UWB channel is plotted in Fig. 4 for OFDM, CP-SC and GCP systems with M=2 and M=8. The received E_s/N_0 is 20dB. As M increases, the bit allocation freedom reduces. As a result, the achievable bit rate is in the following descending order: OFDM, GCP M=2, GCP M=8 and CP-SC.

A new design trade off is present that increasing M results not only a reduced PAR and increased frequency diversity but also a decreased achievable bit rate.

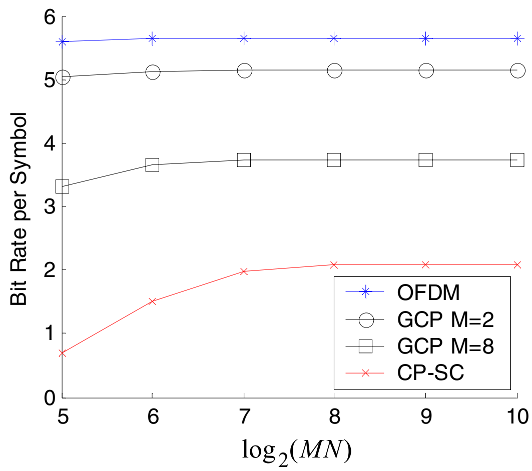

Fig. 4 Achievable bit rate per transmitted data symbol

5 CONCLUSIONS

By viewing the circulant matrix as a block circulant matrix, a generalized cyclic prefixed communication structure is proposed. By selecting the block size of the block circulant matrix to be one or the entire transmission block, the proposed system becomes OFDM or CP-SC, respectively. By selecting a block size that is somewhere between these two extremes, a hybrid structure that combines the advantages of both OFDM and CP-SC systems can be realized. PAR problem is alleviated since the one large IDFT in the OFDM transmitter is replaced with multiple smaller size IDFT. When a block size of M is selected, the proposed system reduces the PAR by $10\log M$ dB compared to an OFDM system. Since the number of subcarriers can also be controlled, the bit allocation capability and frequency diversity can be traded. As expected, simulations show that the bit error rate and the achievable bit rate of the proposed system are between that of OFDM and CP-SC.

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