A Parametric Analytical Diffusion Model for Indoor Ultra-Wideband Received Signal

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***Abstract*— A parametric model for indoor Ultra-wideband (UWB) impulse response at the receiver is derived analytically based on diffusion phenomenon and stochastic differential equations (SDE). This novel approach considers the channel in continuous space instead of characterizing the discrete multipath components. It was hypothesized that the multiple wave reflections in the rich scatter indoor environment cause diffusion-like behavior in the received signal. Our analysis suggested a geometric Brownian motion with exponential decaying factor as the channel stochastic impulse response. The simple closed form of the model has parameters that can be adjusted for different indoor channel behaviors, i.e., match the power delay profile and channel statistics. The model parameters were estimated for an office building, and its statistics were compared to the statistics of a set of data from experiment. The IEEE channel model (CM3) was used for comparison purposes. The results have shown that our proposed model performs closely to dense multipath experimentally collected data.**

I. Introduction

Ultra-wideband (UWB) signal propagation in the indoor environment is a topic of great interest. The propagation complexities in the indoor environments with dense multipath make the analytical characterization of the channel impulse response properties more challenging. There are valuable documented resources based on extensive measurements (e.g., [1] [2] [3] [4]) for modelling the channel path loss, the frequency domain response and analysis of the statistics of the related channel parameters. Some previous studies has provided a thorough statistical description of multipath components (i.e., wave reflections, angles-of-arrival and times-of-arrival, e.g., [1]). Frequency domain channel sounding is considered in [2].

Some channel models that have been proposed for the indoor UWB environments are complicated or have too many parameters which should be estimated. Therefore, the use of these models are limited. The experimentally proposed IEEE channel models [3] are widely used for simulation purposes but it's not apparent how different parameters of the model can be matched to different buildings and environments. Complete characterization of the UWB channel can help engineers to design optimum UWB system components like receivers and optimum codes.

Most of the proposed channel models are based on characterizing the discrete multipath components. Although this is a standard approach for narrowband applications, UWB signals typically occupy multi-Gigahertz bandwidth which result a fine time resolution and therefore, hundreds or thousands of multipath components will be generated. The large number of multipath components usually lead to large number of parameters in the channel model which should be estimated for different environments. Alternatively, we can consider the channel model in the continuous signal space. By this, we imply that the large number of specular and diffused reflections in a rich scattering environment can be better approximated with a continuous received impulse response model. Therefore, the analysis will be done in the continuous space and the derived closed form solution of the statistical model will contain only a few parameters and thus, the model will be reduced in complexity.

In this paper, first we hypothesized properties of the UWB channel and then analytically derived the stochastic channel impulse response from a set of stochastic differential equations (SDE). In the absence of precise information about the channel (e.g., object locations and materials in the building) the exact analytical solution from Maxwell equations or ray tracing methods [5] can not be determined. As an alternate approach, we can develop simplified tractable stochastic models to describe the channel behavior. Note that it is impossible to reconstruct the continuous channel impulse response from the channel pulse response, since the pulse waveform has limited bandwidth and performs lowpass filtering. The primary objective of our analysis was to derive analytically a simple and accurate stochastic impulse response for the indoor UWB channel with few adjustable parameters.

The organization of the paper is as follows: Section II discusses the UWB signal estimation and formulates the corresponding stochastic differential equation (SDE). From basic physical assumptions and channel properties, in section III, we adjust the mathematical assumptions to physical realities and determine the corresponding SDE parameters. The solution to the proposed SDE will be also presented in section III. In section IV, the stochastic impulse response of the channel is derived. Finally, in section V, the accuracy of the new proposed model is verified by comparing the model statistics to that of experimentally collected data.

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II. UWB Signal Estimation and SDE Formulation

Consider a building consisting of at least two rooms. An UWB transmitter exists in one room. We partition the entire building into two regions. Suppose that region 2 consists of the room with the transmitter in it. Assume there are enough objects in the building so that there exists a large number of multiple diffused and specular reflections which produce a diffusion-like propagation characteristic [6]. This is true for most buildings with objects in them. In the average sense, while the energy in region 2 diffuses to region 1, the energy in region 1 diffuses back to region 2. We showed [6] that the following differential equation holds for the spatial average power profile of the two regions

$$\dot{\mathbf{P}}(t) = \mathbf{A}\mathbf{P}(t), \tag{1}$$

where $\mathbf{P}(t) = \begin{bmatrix} P_1(t) & P_2(t) \end{bmatrix}^t$, and $P_1(t)$ and $P_2(t)$ are the spatial average power profiles of the impulse response in regions 1 and 2 respectively. $\dot{\mathbf{P}}(t) \triangleq \frac{d}{dt}\mathbf{P}(t)$ and $\mathbf{A}$ is a known matrix of constant coefficients which its elements depend on the building structure. The solution to (1) for region 1 is of the form

$$\mathbb{E}\{X(t)\} = P_1(t) = k_1 \left(e^{\alpha_1 t} - e^{\alpha_2 t}\right), \tag{2}$$

where $X(t)$ is the instantaneous received power profile of the impulse response at a sample point in region 1 and is proportional to the square of the received signal envelope, α_1, α_2 (functions of elements of matrix $\mathbf{A}$) and k_1 are parameters which are selected to adjust the model to the specific environment (constant values for each region of the building). $\mathbb{E}\{\cdot\}$ averages over the space of one region. For the purposes of this paper, the stochastic process $X(t)$ for one location is of the prime interest. The model (2) was proposed for the power profile of the impulse response, so that the infinite bandwidth of the transmitted impulse does not force any limitation on the shape of the average power profile.

The finite variance continuous process $X(t); t \geq 0$ consists of two parts: the average (deterministic) part as shown in (2), and the stochastic part, i.e., random variations. If $X(t)$ is considered as the received power at a specific location at time t, then at time $t + dt$ the power will be given by

$$X(t + dt) = X(t) + dX_t. \tag{3}$$

dX_t should be estimated at each time t, given the observation $X(t'); 0 \leq t' \leq t$. In other words, by observing the history of the process up to time t, the minimum mean square error (MMSE) estimate of future values for $X(\cdot)$ should be calculated. It is known that the MMSE estimation of dX_t is its the conditional mean. Denote the conditional mean random variable, which is a function of the path $\{X(t'); 0 \leq t' \leq t\}$, by dY_t, i.e.,

$$dY_t \triangleq \mathbb{E}\{dX_t | X(t'); 0 \leq t' \leq t\}. \tag{4}$$

dY_t is the predictable part of $X(t)$ (The orthogonal projection of $X(t)$ in the case of Gaussian, or in the case of linear estimation with a squared loss function definition). The difference

$$dU_t = dX_t - dY_t \tag{5}$$

represents the prediction error, or the mean square unpredictable part of the increment dX_t. By definition, dU_t is called the innovation part of the process $dX(t)$. The innovation process is unbiased, i.e., $\mathbb{E}\{dU_t\} = 0$. Integrating (5) along time results

$$X(t) = \int_0^t dY_t + \int_0^t dU_t. \tag{6}$$

Note that $X(0) = 0$ since the received power is a continuous physical phenomenon. Hence (6) can be written as the so called Doob-Mayer-Fiske decomposition

$$X(t) = Y(t) + U(t) \tag{7}$$

where $Y(t)$ ia a continuous physical process predictable with $X(t)$. It turns out that $U(t)$ is always a (Gaussian) Wiener process and also is a $\mathcal{F}_t^X-$Martingale with $U(0) = 0$ [7], i.e.,

$$\mathbb{E}\{U(t)|\mathcal{F}_s^X\} = U(s), s < t. \tag{8}$$

$\mathcal{F}_t^X$ is the $\sigma-$algebra generated by collections of random variables X, i.e., $\mathcal{F}_t^X = \sigma\{X(s); 0 \leq s \leq t\}$.

Alternatively, it is known that under certain regulatory conditions, a finite variance continuous process $X(t); t \geq 0$ can be decomposed in to a smooth signal component and a continuous but highly erratic noisy component which is a transformation of the standard Wiener process [8], i.e.,

$$X(t) = \int_0^t a(\tau, X(\tau))d\tau + \int_0^t b(\tau, X(\tau))dW_\tau \tag{9}$$

where $W(t)$ is the standard Wiener process and dW_t is the official differentiation of the Wiener process, i.e., white gaussian process. $a(t, X(t))$ and $b(t, X(t))$ are continuous memoryless transformations of the process $X(t)$. Taking the derivative of (9) we obtain

$$\begin{cases} dX_t = a(t, X(t))dt + b(t, X(t))dW_t \\ X(0) = 0 \end{cases} \tag{10}$$

dX_t, a process written as linear combination of ordinary finite variance process (Gaussian or non-Gaussian) and white gaussian noise, is called generalized process. Comparing (10) and (5), we find that the predictable part of dX_t, i.e., dY_t in (5) has the form of $a(t, X(t))dt$ in (10) and dU_t which is the unpredictable part of (5) holds the same statistics of $b(t, X(t))dW_t$.

In (10), the white Gaussian component dW_t is independent of the past of X and W (i.e., independent of $X(t'); 0 \leq t' < t$ and $W(t'); 0 \leq t' < t$). Since $dX(t) = X(t + dt) - X(t)$ is a forward increment equation and as a result of (10), the conditional distribution of $X(t+dt)$, given the past and present $X(t'); 0 \leq t' \leq t$ depends only on the present value of $X(t)$. But this is the definition of Marcov property. Therefore, $X(t)$ is a Marcov process, meaning that the probability law of the entire future of the process is completely determined by the present value, and hence is independent of the past. The process $X(t)$ does not have independent increments since dX_t depends on $X(t)$ through $a(t, X(t))$ and $b(t, X(t))$. Not a surprise that $X(t)$ need not even be a Gaussian process.

$a(t, X(t))$ and $b(t, X(t))$ should be calculate
dense multipath channel and then (10) should
Before adjusting the above functions with the ph
problem, it should be mentioned that for measurab
$a(t, x)$ and $b(t, x)$, if there exist constants K_1 a
that the following conditions are satisfied for all

$$\|a(t,x) - a(t,y)\| + |b(t,x) - b(t,y)\| \leq K_1|$$
$$\|a(t,x)\| + \|b(t,x)\| \leq K_2(1 + \|x\|)$$

then a unique solution to SDE (10) exists [7] and has the following properties:

1) X is $\mathcal{F}_t^W-$adapted, i.e., $X(t) \in \mathcal{F}_t^W; \forall t \geq 0$ meaning that for each fixed t the process value $X(t)$ is a function of the Wiener trajectory on the interval $[0, t]$.
2) X is a Markov process with continuous trajectories.

Therefore SDE (10) is a transformation of the space $C[0, \infty)$ into itself, i.e., a Wiener trajectory $W(\cdot)$ is mapped into the corresponding trajectory $X(\cdot)$ as the solution.

III. UWB Channel Physics and SDE Solution A Probabilistic View

The physics of UWB signal propagation phenomenon should be used to find the corresponding functions $a(\cdot, \cdot)$ and $b(\cdot, \cdot)$ for UWB channel. To adjust (10) to the UWB channel, we consider that for the perfectly smooth reflection surfaces, specular reflection is the only type of existing reflection, while irregular surfaces (relative to wavelength, as it is for almost all surfaces for UWB signal) contribute in both diffused and specular reflections. As a result, in most cases a combination of specular reflections together with diffused reflections exist.

Based on different diffused and specular reflection coefficients (material dependent), multiple reflections of waves, different absorption coefficients and different angles of reflections, also considering the impulse signal spreading in time due to frequency dependent reflection coefficients and according to central limit theorem (CLT), the difference between the actual and predicted power at the receiver is Gaussian, that complies with the previously discussed $dU_t = b(t, X(t))dW_t$. In the weak form of CLT, the assumption of finite variance is the essential necessity for the convergence to normality. The CLT convergence in distribution gives us a useful general approximation, though the goodness of this approximation is a function of original distributions and must be checked case by case. In section V, the accuracy of this hypothesis will be investigated by comparing the analytical results with the results from the experiment.

dU_t also has independent increment property, since different reflections at different times are probably reflected from different objects and materials with unrelated reflection and absorption coefficients (Fig.1). In addition, the difference in power dU_t is proportional to the existing instantaneous power in the environment $X(t)$. The larger the instantaneous power in the environment, the higher the probability of stronger reflections from the objects, and therefore the probability for larger variations in the received signal is higher. Hence

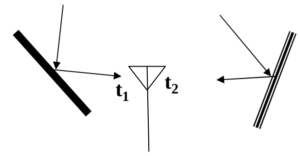

Fig. 1. Received signals at different times are reflected probably from different materials with unrelated reflection and absorption coefficients.

$$dU_t = \sigma X(t) dW_t \tag{12}$$

where σ is a constant which depends on the environment. Fixing $X(t)$ in the environment, the larger values of σ correspond to the stronger (specular) reflected waves and produce larger unpredictable variations dU_t. Taking the expected values from (5) results

$$\mathbb{E}\{dX_t\} = \mathbb{E}\{dY_t\} + \mathbb{E}\{dU_t\}. \tag{13}$$

As mentioned earlier, by definition the innovation process has zero mean. In fact, the mean of the process dX_t is considered in its predictable part. From (4), for any $t_2 \geq t_1$:

$$\begin{aligned} dY_{t_1} &\triangleq Y_{t_2} - Y_{t_1} = \mathbb{E}\{dX_{t_1} \mid X(t); 0 \leq t \leq t_1\} \\ &= \mathbb{E}\{X(t_2) - X(t_1) \mid X(t); 0 \leq t \leq t_1\} \end{aligned} \tag{14}$$

Given $X(t); 0 \leq t \leq t_1$, $X(t_1)$ is a constant, therefore

$$dY_{t_1} = \mathbb{E}\{X(t_2) \mid X(t); 0 \leq t \leq t_1\} - X(t_1) \tag{15}$$

Considering the Marcov property of $X(t)$, all the information in $X(t); 0 \leq t \leq t_1$ for estimating $\mathbb{E}\{X(t_2) \mid X(t); 0 \leq t \leq t_1\}$ is in $X(t_1)$. In other words, since $\mathbb{E}\{X(t_2) \mid X(t); 0 \leq t \leq t_1\} = \mathbb{E}\{X(t_2) \mid X(t_1)\}$, $X(t_1)$ is sufficient statistic to estimate $\mathbb{E}\{X(t_2) \mid X(t); 0 \leq t \leq t_1\}$. For simplicity purposes, $\mathbb{E}\{X(t_2) \mid X(t_1)\}$ which should be estimated was denoted by θ. Apparently θ is a function of $X(t_1)$. The pdf $f(X(t_1)|\theta)$ can be written as

$$f(X(t_1)|\theta) = \frac{f(\theta|X(t_1))f(X(t_1))}{f(\theta)} = f(X(t_1)) \tag{16}$$

which is a delta function when $X(t_1)$ is known. Since no function of X except the one that is identically zero with probability one for all θ satisfies $\mathbb{E}_\theta\{g[X(t_1)]\} = 0; \forall\theta$, $X(t_1)$ is a complete sufficient statistics for θ [9]. $\mathbb{E}_\theta\{\cdot\}$ is the expected value of its argument with the known parameter θ. Reference [9] has shown that if T is a complete sufficient statistic for a parameter θ, and $\varphi(T)$ is any estimator based only on T, then $\varphi(T)$ is the unique best unbiased estimator of its expected value. Consequently, for the Markov process $Z(t)$ with $\mathbb{E}\{Z(t)\} = Ke^{at}$, when $Z(t_0); t_0 \leq t$ is known, the best unbiased estimator for $\mathbb{E}\{Z(t)|Z(t_0)\}$ would be $Z(t_0)e^{a(t-t_0)}$.

$$\begin{aligned} \mathbb{E}\{Z(t)\} &= \mathbb{E}\{\mathbb{E}\{Z(t)|Z(t_0)\}\} = \mathbb{E}\{Z(t_0)\}e^{a(t-t_0)} \\ &= Ke^{at_0}e^{a(t-t_0)} = Ke^{at} \end{aligned} \tag{17}$$

specifically this is true for a constant coefficient equation, i.e., when K is not time varying.

$\mathbb{E}\{X(t)\}$ in (2) consists of two exponential terms, each with constant initial value $\pm k_1$. We consider X(t) as the summation of two processes; $X(t) = X_1(t) + X_2(t)$ so that $\mathbb{E}\{X_1(t)\} = k_1 e^{a_1 t}$ and $\mathbb{E}\{X_2(t)\} = -k_1 e^{a_2 t}$. If the values of $X_1(t_1)$ and $X_2(t_1)$ are known, the best unbiased estimator of the conditional expected value $\mathbb{E}\{X(t_2)|X_1(t_1), X_2(t_1)\}$ would be

$$\begin{aligned} &\hat{\mathbb{E}}\{X(t_2)|X_1(t_1), X_2(t_1)\} = \varphi(X_1(t_1), X_2(t_1)) \qquad (18)\\ =\;& X_1(t_1)e^{a_1(t_2-t_1)} + X_2(t_1)e^{a_2(t_2-t_1)} \quad ;\forall t_1, t_2 \ni t_1 < t_2 \end{aligned}$$

where $\hat{\mathbb{E}}\{\cdot\}$ is the orthogonal projection. By defining $dt = t_2 - t_1 \to 0$, (15) gives

$$\begin{aligned} \hat{d}Y_{t1} &= X_1(t_1)e^{a_1 dt} + X_2(t_1)e^{a_2 dt} - \\ &\quad [X_1(t_1) + X_2(t_2)]. \end{aligned} \qquad (19)$$

$\hat{d}Y_{t1}$ is the estimated value of dY_{t1}. The Taylor expansion approximation will result

$$\hat{d}Y_{t1} \approx a_1 X_1(t_1)dt + a_2 X_2(t_1)dt \qquad (20)$$

Therefore, (5) can be rewritten as

$$dX_t = a_1 X_1(t)dt + a_2 X_2(t)dt + \sigma X(t)dW_t. \qquad (21)$$

Substituting $X(t) = X_1(t) + X_2(t)$ we get

$$\begin{aligned} dX_{1t} + dX_{2t} = a_1 X_1(t)dt + a_2 X_2(t)dt +\\ \sigma X_1(t)dW_t + \sigma X_2(t)dW_t \end{aligned} \qquad (22)$$

The first and the second terms in the right hand side of (22) come from the conditional expected values of Markov processes with exponential mean and are independent of each other. Terms 3 and 4 in the right hand side of (22) should have the same Gaussian process dW_t which has been divided into two parts. Equation (22) was considered as two separate equations to be solved:

$$\begin{cases} dX_1(t) = a_1 X_1(t)dt + \sigma X_1(t)dW_t \\ dX_2(t) = a_2 X_2(t)dt + \sigma X_2(t)dW_t \\ X_1(0) = -X_2(0) = x_0 \end{cases} \qquad (23)$$

The third equation comes from the fact that the process $X(t)$ always starts from zero. Equation (2) also results $\mathbb{E}\{X_1(0)\} = -\mathbb{E}\{X_2(0)\} = k_1$. The value of x_0 depends on the power of transmitter and physical characteristics of the channel, e.g., the distance between the transmitter and the receiver, objects in the room and their locations, etc. Assume that ,somehow, the value of x_0 can be estimated. The first equation in (23) can be considered as

$$\begin{cases} \dot{X}_1(t) = [a_1 + \sigma \dot{W}(t)]X_1(t) \\ X_1(0) = x_0 \end{cases} \qquad (24)$$

where $\dot{W}(\cdot)$ is the white Gaussian process.

Considering that the solution to the corresponding deterministic linear equation is an exponential function of time, we define the process Z where $Z(t) = \ln[X_1(t)]$. Since X is the received UWB power, it is always non-negative and $\ln(X)$ exists. For now, we assume that X is a strictly positive solution to (24). Taylor expansion and partial derivative rule gives

$$\begin{aligned} dZ_t &= \frac{\partial Z(t)}{\partial t}dt + \frac{\partial Z(t)}{\partial X_1}dX_1 \\ &+ \frac{1}{2}\left(\frac{\partial^2 Z(t)}{\partial t^2}dt^2 + \frac{\partial^2 Z(t)}{\partial X_1^2}dX_1^2\right) + \dots \end{aligned} \qquad (25)$$

Since$dt \to 0$, dt^2 can be disregarded as the second order infinitesimal value. But X_1 contains a white Gaussian process which may vary rapidly and hence $[dX_t]^2$ can not be neglected

$$\begin{aligned} [dX_t]^2 &= [aX_1(t)dt + \sigma X_1(t)dW_t]^2 \\ &\approx \sigma^2 X_1^2(t)(dW_t)^2 \end{aligned} \qquad (26)$$

$(dW_t)^2$ which is quadratic variation process of the Wiener process converges to dt in L^2 almost surely, therefore $[dX_t]^2 = \sigma^2 X_1^2(t)dt$, and hence (25) can be rewritten as

$$\begin{aligned} dZ_t &= \frac{1}{X(t)}dX_t - \frac{1}{2X^2(t)}[dX_t]^2 \\ &= adt + \sigma dW_t - \frac{1}{2}\sigma^2 dt \end{aligned} \qquad (27)$$

therefore, the following equations were obtained

$$\begin{cases} dZ_t = (a - \frac{1}{2}\sigma^2)dt + \sigma dW_t \\ z_0 = ln(x_0) \end{cases} \qquad (28)$$

Solving (28) is not complicated, since the right hand side does not include $Z(\cdot)$ and can be integrated directly: $Z(t) = ln(x_0) + (a - \frac{1}{2}\sigma^2)t + \sigma W(t)$, which implies

$$X_1(t) = x_0 \exp\{(a_1 - \frac{1}{2}\sigma^2)t + \sigma W(t)\} \qquad (29)$$

Therefore

$$X(t) = x_0 e^{\sigma W(t)} e^{-\frac{1}{2}\sigma^2 t}\left(e^{a_1 t} - e^{a_2 t}\right) \qquad (30)$$

The expected value of $X(t)$ can be calculated to be $\mathbb{E}\{X(t)\} = x_0 \left(e^{a_1 t} - e^{a_2 t}\right)$. Equation (30) shows that $X(t)$, the UWB instantaneous power at one point, consists of three multiplicative components. The average term, $x_0(e^{a_1 t} - e^{a_2 t})$ which exists because of the average energy diffusion phenomenon in the environment [6], the exponential term $\exp[\sigma W(t)]$, which contains all the random variations in the form of log-normal process with increasing variance and an exponentially attenuation term with time, $\exp[-\frac{1}{2}\sigma^2 t]$. It is interesting to note that larger σ, which is corresponding to stronger specular reflections, results larger power variations and faster decay in power, as long as a_1 and a_2 are fixed. This happens with constant a_1, a_2, and x_0, which implies that the materials in the environment are the same but their surfaces are smoother. The reason may be that the specular reflections do not contribute in the average power diffusion process, and although these specular reflections cause larger variations in the instantaneous power according to the $e^{\sigma W(t)}$ term, but in the average, these strong reflections have higher possibility to leave the system more rapidly.

On the other hand, the reflection and absorption coefficients of the materials in the environment, the physical distances and

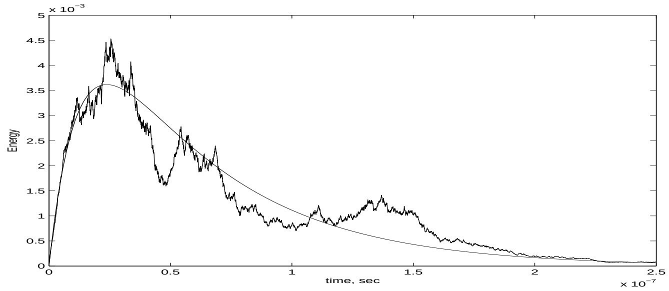

Fig. 2. Averaged received energy from UWB diffusion channel model together with a sample trajectory of its stochastic process.

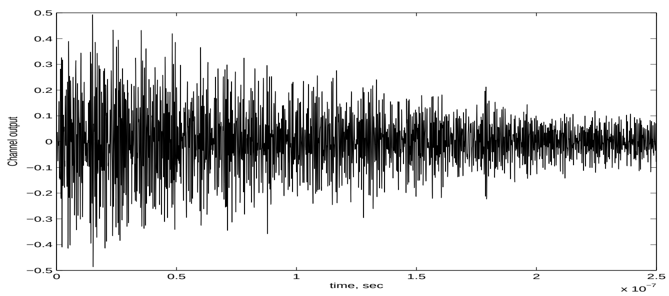

Fig. 3. UWB diffusion model channel response to second derivative of Gaussian waveform as the input template.

the structure of the environment affect a_1, a_2, x_0, and σ, and consequently change the average time of nonzero power in the environment, variations of the power, and the average and instantaneous power magnitude.

IV. UWB Channel Impulse Response

Equation (30) mathematically describes the statistics of the instantaneous power profile at each point in the building. In order to obtain the statistics of the channel impulse response, the received signal envelope was calculated. The received signal envelope is proportional to the square root of the power

$$e(t) = e_0 e^{\frac{1}{2}\sigma W(t)} e^{-\frac{1}{4}\sigma^2 t} \left(e^{a_1 t} - e^{a_2 t}\right)^{\frac{1}{2}} \quad (31)$$

where e_0 is proportional to square root of x_0. Fig. 2 shows a sample trajectory of the received energy $X(t)$ obtained from the UWB diffusion channel model, with its mean value $\mathbb{E}\{X(t)\}$. From (31) the impulse response of the channel is

$$C(t) = e_0 p(t) e^{\frac{1}{2}\sigma W(t)} e^{-\frac{1}{4}\sigma^2 t} \left(e^{a_1 t} - e^{a_2 t}\right)^{\frac{1}{2}} \quad (32)$$

In (32) $p(t)$ is a random process that can take values of +1 or -1 with equal probabilities based on the polarity of the reflections. To conceive more generality, we consider that the polarity $p(t)$ might be changed at Poisson times with probability 0.5. If the Poisson arrival rate λ is considered to be very big relative to the system resolution (e.g., sampling rate and UWB template bandwidth) $p(t)$ converges to independent ± 1 polarities at each time. Finally by applying the convolution integral, the received channel output when specific template $g(t)$ is used would be $r(t) = C(t) * g(t)$.

V. UWB Channel Statistics, Measurement Results and Model Validity

In order to verify the fidelity of the proposed model, statistics of the pulse response of the channel was compared to the statistics of a set of measured data taken by Win [1]. For reference the results were also compared to the statistics of the IEEE channel pulse response as a discrete channel model. Since it was not known how to match parameters of the model with the real environment, the typical values of IEEE-CM3 was used which was the closest to the experiment condition. For both models, the impulse response of the channel was simulated according to the model, then the response of the channel to the measured received pulse shape (obtained from clean LOS measurements [1]) was calculated. AWGN, with the same power level as measurements, was added to the signals. Where needed, the impulse responses were estimated by CLEAN algorithm from measured and simulated signals.

The parameters x_0, a_1 and a_2 of UWB diffusion channel model were estimated from measured data at one point for each room. The estimated values show that for the building where measurements were done, the standard deviation of a_1 and a_2 were 32% and 21% of their mean values respectively. This value was about 120% for x_0, but since the gain coefficient x_0 has a linear contribution, it can be estimated easily. σ and Poisson arrival rate of $p(t)$, λ, were estimated by minimizing the $\mathbb{L}_2$ distance of the averaged power profile of the model with the power profile of one point measurement.

The signal quality and Mean-excess-delay (MED) were calculated for 343 measured data set and correspondingly 343 simulated profiles from each model. The signal quality in dB was defined as $Q(u) = 10\log_{10} E_{tot}(u) - 10\log_{10} E_{ref}$, where u indexes the outcome of the stochastic process, $E_{tot}(u) = \int_0^T |r(u,t)|^2 dt$, T is the total observation time, $r(u,t)$ is the received signal (measured or resulted from the Models), and E_{ref} is a fixed reference quantity. Mean-excess-delay was defined as $\tau = \frac{\sum_i \hat{h}_i^2 t_i}{\sum_i \hat{h}_i^2}$, where $\hat{h}_i$ is the i^{th} channel impulse response estimate at time t_i.

Since the diffusion model parameters were estimated for the same type of channel and distance as measurements were done, the ensemble mean of the mean-excess-delays was expected to be almost the same for measured data and the model. However, since IEEE channel model could not be matched with the real environment, the corresponding results were quite different with measurement. Table I shows the statistics of the signal quality and Mean-excess-delay. To compare the performances of the correlator receivers, the quantity *energy capture* was defined as $EC \triangleq 1 - \frac{E_{min}(u,L)}{E_{tot}(u)}$, where $E_{\min} = \min_{c_i}\{\int_0^T |r(u,t) - \sum_{i=1}^L c_i w(t-\tau_i)|^2 dt\}$ [1]. Energy capture plots for measured signals and signals from each model is shown in Fig. 4, and Fig. 5. The results indicated that the proposed UWB diffusion channel model simulates the UWB signal behavior closely and the model parameters can be matched with any specific building. Pulse distortion effects due to antennas [10], channel frequency distortion and corresponding changes on the stochastic properties of

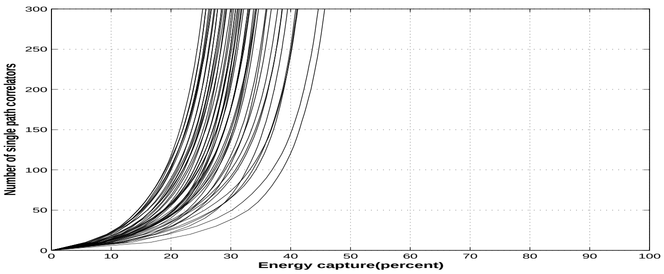

Fig. 4. Energy-capture for 49 measured signals.

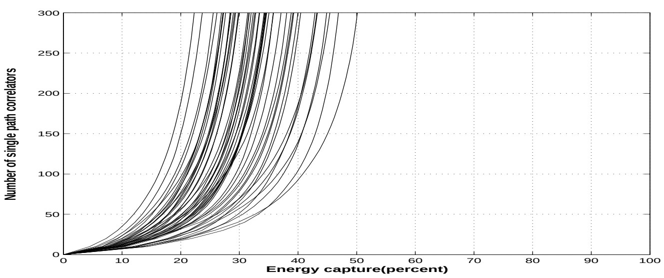

Fig. 5. Energy-capture for 49 simulated signals from UWB diffusion CM.

the received signal may be the most important sources of discrepancy between the simulation and experimental results. On the other hand Fig. 6 shows the corresponding results for IEEE-CM3. Since the IEEE channel model parameters are not easy to be matched with the real environment, the results might be quite different with that of the experimental data. The results also show that in the discrete channel models, the estimation of multipath components might be easier than in the real situations, and therefore we can capture more energy with less number of single path correlators.

Finally, we applied the proposed channel impulse response to the narrowband signals. Fig.7 and 8 shows the resulted histograms of amplitude and phase respectively. It is seen that the histograms fit to the well known results for the narrowband Rayleigh channels.

VI. Conclusion

We analytically derived an indoor parametric channel model for UWB communication signal. The proposed model parameters can be set for typical environments. The model also has the flexibility that the parameters be tuned to a specific building or location within the building. Analysis showed that the proposed UWB diffusion channel model performs reasonably close to the set of experimentally collected data.

TABLE I

Signal quality (Q) and mean-excess-delay (MED) statistics

	$std\{Q\}$	$\mathbb{E}\{MED\}$	$std\{MED\}$
Measurement	2.6 dB	65.0 nsed	0.178
Diffusion CM	2.75dB	68.1 nsec	0.194
IEEE CM3	1.4dB	54.0 nsec	0.0421

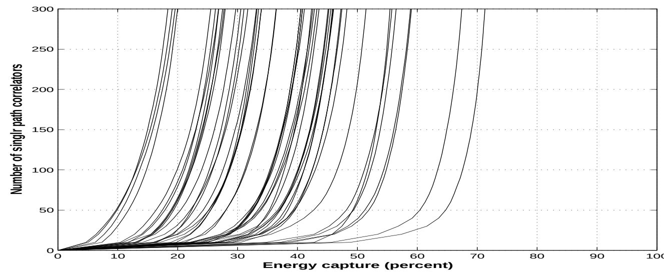

Fig. 6. Energy-capture for 49 simulated signals from UWB IEEE-CM3.

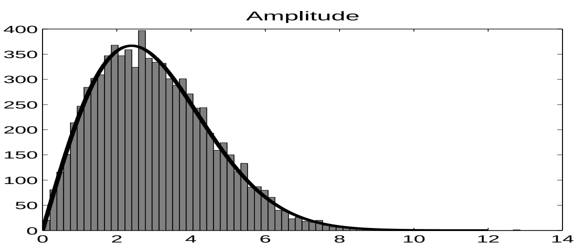

Fig. 7. Amplitude histogram of a narrowband signal at 900 MHz using the proposed channel impulse response, along with Nakagami m=1 distribution.

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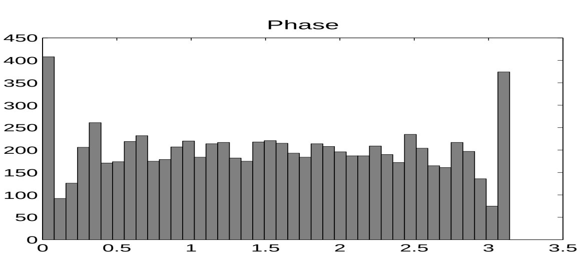

Fig. 8. Phase histogram of a narrowband signal at 900 MHz using the proposed channel impulse response.