

# Waveform Optimizations for Ultrawideband Radio Systems

David M. Pozar, *Fellow, IEEE*

**Abstract**—Solutions are presented for various optimizations of transient waveforms and signals used in ultrawideband radio systems. These include the transmit antenna generator waveform required to maximize receive antenna voltage amplitude (with bounded input energy), the transmit antenna generator waveform that provides the “sharpest” received antenna voltage waveform, and the transmit antenna generator waveform that maximizes received energy with an inequality constraint on the radiated power spectral density. Using variational methods, general optimization results are derived for arbitrary antennas, including the effects of generator and load impedances, and numerical examples are provided for lossless dipoles and resistively loaded dipoles using moment method solutions. Closed-form results are provided for short dipole antennas for some special cases.

**Index Terms**—Broadband, optimization, time domain, transient, ultrawideband (UWB).

## I. INTRODUCTION

ULTRAWIDEBAND (UWB) radio is characterized by a wide system bandwidth and a low radiated power spectral density. Typical 3-dB bandwidths of UWB systems are 20% or greater, over frequency bands generally ranging from several hundred megahertz to a few gigahertz [new Federal Communications Commission (FCC) regulations restrict UWB operation primarily to the 3.1–10.6 GHz band]. UWB radio does not employ a carrier, relying instead upon the radiation and propagation of bandlimited baseband transient pulses. Wide bandwidth provides potentially fine time resolution, while significant low-frequency components allow propagation through a wide variety of materials and structures. The following features of UWB radios are of interest in a number of future applications involving short-range communications, data networking, ranging, and location [1], [2].

- 1) UWB radio maximizes the utility of underused spectrum segments.
- 2) Fine time resolution helps to mitigate indoor fading and multipath effects.
- 3) Low radiated power levels provide an inherent measure of covertness.
- 4) Wide bandwidth allows ranging accuracy of 1 ft or less.
- 5) Low frequency content allows operation through walls, foliage, etc.

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The author is with the Electrical and Computer Engineering Department, University of Massachusetts at Amherst, Amherst, MA 01003 USA (e-mail: pozar@ecs.umass.edu).

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- 6) Low power densities allow high levels of multiuser scaling.

The UWB radio concept is not without potential drawbacks, however—notably the risk of interference with licensed spectrum users. Of particular concern is the potential for interference with GPS systems, cellular telephones, and public safety communications. Since power decreases with separation between an interfering transmitter and a legitimate receiver, the maximum allowable power spectral density radiated at a particular distance from a UWB transmitter should be specified to minimize the possibility of interference. The FCC is presently considering this issue [3].

The radiated power spectral density from a UWB transmitter should ideally be as flat as possible, but the frequency characteristics of practical transmit and receive antennas are seldom conducive to this goal. In fact, antenna performance is often the primary limiting factor on the overall performance of UWB systems. Since the transmit and receive antennas invariably distort the shape of the transmitted and received waveforms [4], [5], it is worthwhile to consider the question of finding the best waveforms to optimize overall system performance in a particular sense.

This paper extends the variational methods developed in [6], [7] for optimizing transient radiation to derive solutions for input and output waveforms in a UWB radio system that optimize received signal amplitude, received signal energy, or received signal duration. As such, these solutions represent upper bounds on the performance that could be expected from practical UWB systems. The resulting fields satisfy Maxwell’s equations, along with additional constraints on input energy and signal bandwidth. General solutions are derived for arbitrary antennas, and numerical examples are provided for specific cases of lossless and resistively loaded dipole antennas. Frequency-domain moment method solutions are used for this purpose; the necessary transfer functions and input impedances can be obtained from the integral equation analysis. Finally, simple closed-form results for short dipole antennas are presented for some special cases

## II. ANALYSIS

The general configuration of a UWB radio is shown in Fig. 1. The transmit antenna is driven with a voltage source  $V_G(\omega)$  having an internal impedance  $Z_G(\omega) = R_G(\omega) + jX_G(\omega)$ , while the receive antenna is terminated with load impedance  $Z_L(\omega) = R_L(\omega) + jX_L(\omega)$  and has a terminal voltage  $V_L(\omega)$ . The input impedance of the transmit and receive antennas are  $Z_T(\omega) = R_T(\omega) + jX_T(\omega)$  and  $Z_R(\omega) = R_R(\omega) + jX_R(\omega)$ ,

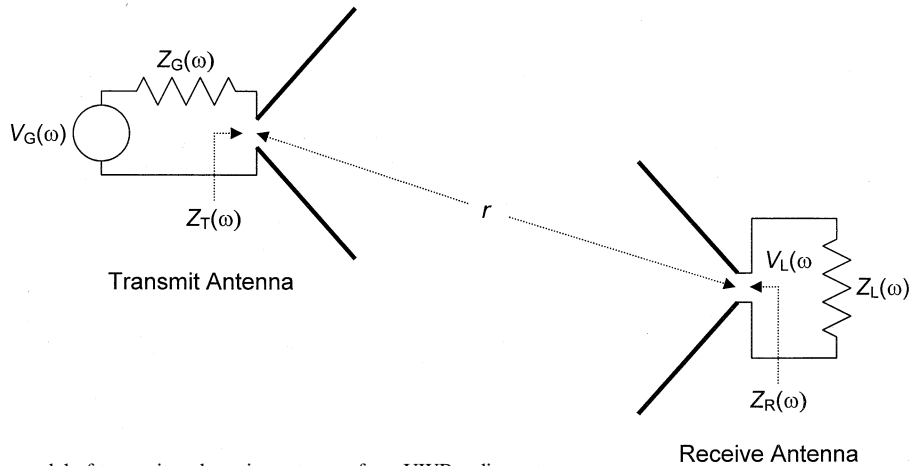


Fig. 1. Frequency-domain model of transmit and receive antennas for a UWB radio system.

respectively. The antennas are separated by a distance  $r$ , assumed to be large enough so that each antenna is in the far-field region of the other over the operating bandwidth. The transmit antenna radiates an electric field  $\bar{E}(\omega)$  incident at the position of the receive antenna.

The corresponding time domain quantities are given in terms of the inverse Fourier transforms

$$v_G(t) = \frac{1}{2\pi} \int_{\text{BW}} V_G(\omega) e^{j\omega t} d\omega \quad (1a)$$

$$v_L(t) = \frac{1}{2\pi} \int_{\text{BW}} V_L(\omega) e^{j\omega t} d\omega \quad (1b)$$

$$\bar{e}(t) = \frac{1}{2\pi} \int_{\text{BW}} \bar{E}(\omega) e^{j\omega t} d\omega. \quad (1c)$$

The integrations in (1a)–(1c) are over the bandlimited frequency range of  $-B$  to  $B$  Hz.

The current at the input to the transmit antenna is given by

$$I_T(\omega) = \frac{V_G(\omega)}{Z_G(\omega) + Z_T(\omega)} \quad (2)$$

and the receiver load voltage in terms of  $V_{\text{oc}}(\omega)$ , the open-circuit voltage of the receive antenna, is given by

$$V_L(\omega) = \frac{V_{\text{oc}}(\omega) Z_L(\omega)}{Z_L(\omega) + Z_R(\omega)}. \quad (3)$$

We also define two transfer functions. Let  $H_{\text{LG}}(\omega)$  be the voltage transfer function that relates the receive antenna load voltage to the generator voltage at the transmit antenna

$$V_L(\omega) = H_{\text{LG}}(\omega) V_G(\omega) e^{-j\omega r/c} \quad (4)$$

where  $c$  is the speed of light. This definition thus excludes the time delay between the transmit and receive antenna. Also, let  $\bar{F}_{\text{EG}}(\omega)$  be a vector transfer function that relates the radiated electric field at the receive antenna to the transmit antenna generator voltage

$$\bar{E}(\omega) = \bar{F}_{\text{EG}}(\omega) V_G(\omega) e^{-j\omega r/c}. \quad (5)$$

Although not explicitly shown, it should be understood that both of these transfer functions are functions of range as well as the elevation and azimuth angles at each antenna.

The open-circuit voltage at the receive antenna can be found from the vector effective height  $\bar{h}(\omega)$  of the receive antenna [8]

$$V_{\text{oc}}(\omega) = \bar{h}(\omega) \cdot \bar{E}(\omega). \quad (6)$$

The time-domain voltage waveform at the receive antenna can now be written in terms of the generator voltage using (1b) and (4)

$$v_L(t') = \frac{1}{2\pi} \int_{\text{BW}} H_{\text{LG}}(\omega) V_G(\omega) e^{j\omega t'} d\omega \quad (7)$$

where  $t' = t - r/c$  is the retarded time variable.

We also need to define various energy quantities. The energy available from the generator is given as [9]

$$W_{\text{avail}} = \frac{1}{8\pi} \int_{\text{BW}} \frac{|V_G(\omega)|^2}{R_G(\omega)} d\omega \quad (8a)$$

where  $R_G(\omega)$  is the real part of the generator impedance. The energy delivered to the transmit antenna is given by

$$W_{\text{in}} = \frac{1}{2\pi} \int_{\text{BW}} \frac{|V_G(\omega)|^2 R_T(\omega)}{|Z_T(\omega) + Z_G(\omega)|^2} d\omega. \quad (8b)$$

The energy received by the load at the receive antenna is given by

$$W_{\text{rec}} = \frac{1}{2\pi} \int_{\text{BW}} \frac{|V_L(\omega)|^2}{Z_L^*(\omega)} d\omega. \quad (8c)$$

Again, the integrations in (8a)–(8c) are over the bandwidth of  $-B$  to  $B$  Hz. Use of the fact that the imaginary part of a physically realizable input impedance is an odd function of frequency has been used to simplify (8b).

The following results are useful for extremizing linear and quadratic functionals of the function  $V(\omega)$  [10]:

$$\begin{aligned} \nabla \int V(\omega) H(\omega) d\omega &= H^*(\omega) \\ \nabla \int V(\omega) H(\omega) V^*(\omega) d\omega &= [H(\omega) + H^*(\omega)] V(\omega). \end{aligned}$$

In these results,  $H(\omega)$  is a potentially non-self-adjoint operator, but the adjoint is conveniently given by the conjugate func-

tion for the problems considered here.  $\nabla$  is the gradient operator, defined in the context of variational calculus as in [10].

#### A. Maximization of Received Voltage Amplitude

We first consider the maximization of the received voltage amplitude at the receive antenna, for bandlimited signals, with a constraint on the energy delivered to the transmit antenna. Following the variational calculus procedures used in [6], [7], and [10], we define the functional

$$J = -v_L(t' = 0) + \lambda W_{\text{in}} \quad (9)$$

relative to the independent function  $V_G(\omega)$  and where  $\lambda$  is a Lagrange multiplier. (The negative sign on  $v_L$  ensures maximization for the functionals being used in this work.) The constraint that  $W_{\text{in}} = 1$  J must also be enforced by using (8b). We choose the maximization time as  $t' = 0$  with no loss of generality.

Then the functional of (9) can be extremized with the following result:

$$\nabla J = 0 = \frac{-1}{2\pi} H_{\text{LG}}^*(\omega) + \frac{\lambda R_T(\omega) V_G(\omega)}{\pi |Z_T(\omega) + Z_G(\omega)|^2}. \quad (10)$$

Solving for  $V_G(\omega)$  gives

$$V_G(\omega) = \frac{H_{\text{LG}}^*(\omega) |Z_T(\omega) + Z_G(\omega)|^2}{2\lambda R_T(\omega)}. \quad (11)$$

The Lagrange multiplier is found by using (11) in (8b) and setting  $W_{\text{in}} = 1$

$$\lambda^2 = \frac{1}{2\pi} \int_{\text{BW}} \frac{|H_{\text{LG}}(\omega)|^2 |Z_T(\omega) + Z_G(\omega)|^2}{4R_T(\omega)} d\omega. \quad (12)$$

The solution given in (11) and (12) is essentially the matched filter solution [10] for the linear system that consists of the transmit and receive antennas, along with their termination impedances.

A slightly different solution can be obtained by constraining the available energy from the generator, as opposed to the energy delivered to the transmit antenna. The required functional then becomes

$$J = -v_L(t' = 0) + \lambda W_{\text{avail}} \quad (13)$$

where the available energy is given by (8a). Then the optimum solution is

$$V_G(\omega) = \frac{2R_G(\omega) H_{\text{LG}}^*(\omega)}{\lambda} \quad (14)$$

with

$$\lambda^2 = \frac{1}{2\pi} \int_{\text{BW}} R_G(\omega) |H_{\text{LG}}(\omega)|^2 d\omega. \quad (15)$$

This solution accounts for power dissipated in the internal generator impedance, and so will generally result in a lower maximum receive voltage amplitude at  $t' = 0$  than the solution of (11) and (12). But presumably less energy will be lost in the generator impedance.

#### B. Optimizing Received Waveform ‘‘Sharpness’’

A technique proposed in [10] suggests that minimizing the received energy while constraining the received voltage amplitude to a fixed value at a specific point in time, as well as constraining the input energy, will enhance the sharpness of the output voltage waveform. The output voltage amplitude should be chosen less than the maximum that can be obtained from the matched filter solution given in (11) and (12)—this then introduces additional freedom that may allow the optimization process to reduce the effective duration of the output pulse.

The necessary function thus becomes

$$J = W_{\text{rec}} + \lambda_1 v_L(t' = 0) + \lambda_2 W_{\text{in}} \quad (16)$$

where we now have two Lagrange multipliers  $\lambda_1$  and  $\lambda_2$ . The constraints are applied to the receive voltage amplitude at  $t' = 0$

$$v_L(t' = 0) = v_0 \leq v_{\text{max}} \quad (17)$$

and to the input energy  $W_{\text{in}} = 1$  J. We define  $v_{\text{max}}$  as the value of the maximum voltage amplitude as obtained from the matched filter solution of (11) and (12). Extremizing the functional of (16) gives

$$\begin{aligned} \nabla J = 0 = & |H_{\text{LG}}(\omega)|^2 \left[ \frac{1}{Z_L(\omega)} + \frac{1}{Z_L^*(\omega)} \right] \\ & \times V_G(\omega) + \lambda_1 H_{\text{LG}}^*(\omega) + \frac{2\lambda_2 R_T(\omega)}{|Z_T(\omega) + Z_G(\omega)|^2} V_G(\omega). \end{aligned} \quad (18)$$

Solving for  $V_G(\omega)$  gives

$$V_G(\omega) = \frac{-\lambda_1 H_{\text{LG}}^*(\omega)}{\frac{2R_L(\omega) |H_{\text{LG}}(\omega)|^2}{|Z_L(\omega)|^2} + \frac{2\lambda_2 R_T(\omega)}{|Z_T(\omega) + Z_G(\omega)|^2}}. \quad (19)$$

The normalizations are given by using (19) in (7) and (17), and in (8b), as shown in (20) and (21) at the bottom of the page.

Observe that for specified values of  $v_0$  and  $W_{\text{in}}$ , (20) and (21) represent coupled equations for  $\lambda_1$  and  $\lambda_2$ . Numerical root-finding techniques are generally required for solution.

$$v_0 = \frac{-\lambda_1}{2\pi} \int_{\text{BW}} \frac{|H_{\text{LG}}(\omega)|^2}{\frac{2R_L(\omega) |H_{\text{LG}}(\omega)|^2}{|Z_L(\omega)|^2} + \frac{2\lambda_2 R_T(\omega)}{|Z_T(\omega) + Z_G(\omega)|^2}} d\omega \quad (20)$$

$$W_{\text{in}} = 1 \text{ Joule} = \frac{\lambda_1^2}{2\pi} \int_{\text{BW}} \frac{|H_{\text{LG}}(\omega)|^2 R_T(\omega)}{|Z_T(\omega) + Z_G(\omega)|^2 \left[ \frac{2R_L(\omega) |H_{\text{LG}}(\omega)|^2}{|Z_L(\omega)|^2} + \frac{2\lambda_2 R_T(\omega)}{|Z_T(\omega) + Z_G(\omega)|^2} \right]} d\omega \quad (21)$$

### C. Maximizing Received Energy With an Inequality Constraint on the Radiated Field

The expected FCC regulation that the radiated power spectral density at a specified distance from a UWB transmitter be less than a fixed value can be accommodated in optimization solutions through the use of an inequality constraint. Mathematically, we can attempt to maximize received voltage amplitude, or received energy, subject to the constraint that

$$|\overline{E}(\omega)| \leq E_0 \quad (22)$$

where  $E_0$  is the maximum allowable (peak) electric field intensity at a specified distance from the receiver. In the far field of the transmit antenna, this can be translated to a maximum power density of  $E_0^2/2\eta_0$ , where  $\eta_0 = 377 \Omega$ .

Inequality constraints of the form in (22) can be treated analytically in some cases by applying nonlinear programming techniques such as the Kuhn–Tucker theorem [11]. An alternative that can be applied more directly for the present case is the technique of slack functions, whereby the functional is modified with a nonnegative auxiliary function [11]. Thus, we can rewrite (22) as

$$|\overline{E}(\omega)|^2 = E_0^2 - u^2(\omega) \quad (23)$$

where  $u^2(\omega)$  is the slack function. Since  $u^2(\omega)$  is real and nonnegative, it is clear that (22) will always be satisfied. If we desire to maximize received energy, we can form the functional

$$\begin{aligned} J &= W_{\text{rec}} \\ &= \frac{1}{2\pi} \int_{\text{BW}} \frac{|\overline{h}(\omega) \cdot \overline{E}(\omega)|^2 R_L(\omega)}{|Z_R(\omega) + Z_L(\omega)|^2} d\omega \\ &= \frac{1}{2\pi} \int_{\text{BW}} \frac{|\overline{h}(\omega)|^2 R_L(\omega) [E_0^2 - u^2(\omega)]}{|Z_R(\omega) + Z_L(\omega)|^2} d\omega \end{aligned} \quad (24)$$

and extremize relative to  $u(\omega)$ , and thus indirectly relative to  $\overline{E}(\omega)$ . The result is

$$\nabla J = 0 = \frac{-2 |\overline{h}(\omega)|^2 R_L(\omega)}{|Z_R(\omega) + Z_L(\omega)|^2} u(\omega) \quad (25)$$

which implies that  $u(\omega) = 0$ , and thus  $|\overline{E}(\omega)| = E_0$ . Once  $\overline{E}(\omega)$  is found, we can work backward to find the necessary  $V_G(\omega)$  that will produce this radiated electric field. Note that the phase of  $\overline{E}(\omega)$  is not specified in this solution, as a result of the fact that the energy functional is independent of phase. Otherwise the result is not surprising, as it says that the available spectrum should be filled with the maximum allowable power density in order to maximize received energy. Maximizing the received voltage amplitude at a particular time would, in principle, define a phase distribution for the electric field, but maximizing voltage amplitude alone does not involve a quadratic functional, and so does not lead to an analytic solution using these techniques.

### D. Maximizing Received Energy With a Constraint on the Generator Voltage Amplitude

In this case, the required functional is given by

$$J = -W_{\text{rec}} + \lambda v_G(t = 0). \quad (26)$$

Note that the constraint only applies to the generator voltage at a specific instant of time, and so does not provide an overall limit on the generator amplitude. This might seem to limit the utility of this case, but there may be some situations where this is a useful approach. In addition, this result essentially completes the possible permutations of UWB antenna optimizations that can be carried out with a variational approach.

The optimum generator voltage is found to be

$$V_G(\omega) = \frac{\lambda}{|H_{\text{LG}}(\omega)|^2 \left[ \frac{1}{Z_R(\omega)} + \frac{1}{Z_R^*(\omega)} \right]} \quad (27)$$

with the normalization

$$\lambda = \frac{2\pi v_0}{\int_{\text{BW}} \frac{1}{|H_{\text{LG}}(\omega)|^2 \left[ \frac{1}{Z_R(\omega)} + \frac{1}{Z_R^*(\omega)} \right]} d\omega} \quad (28)$$

where  $v_0$  is the constrained value of the generator voltage at  $t = 0$ .

## III. NUMERICAL EXAMPLES FOR DIPOLE ANTENNAS

The above results have been derived for arbitrary transmit and receive antennas and will be demonstrated here for several cases involving lossless and resistively loaded wire dipole antennas. The piecewise sinusoidal moment method is used to obtain the necessary quantities to implement the above optimization results. These include the input impedances of the antennas and the voltage transfer function  $H_{\text{LG}}(\omega)$  between the antennas. Since moment method solutions for dipole antennas are well established, we refer the reader to the literature for details of the calculation of these quantities [12], [13]. For simplicity, we assume both dipoles are identical, with length  $L$ , radius  $a$ , and conductivity  $\sigma$ . We also assume the dipoles are parallel and radiating in the broadside directions. The range dependence of the received antenna voltage is removed.

### A. Maximization of Received Voltage Amplitude

First consider the maximization of received voltage amplitude at  $t' = 0$ , with a constraint of 1 J for the available generator energy. The optimum generator voltage is given by (14) and (15). Applying these results to a lossless pair of dipoles with  $L = 15$  cm,  $a = 0.02$  cm,  $\sigma = \infty$ ,  $Z_G = 50 \Omega$ , and  $Z_L = \infty$  leads to the generator and receiver voltages shown in Fig. 2(a) and (b), where the peak receive voltage amplitude is 4.04E4 V, and the received energy is 1.53E-5 J (normalized by multiplying by  $r$ ). The solutions are bandlimited to 2 GHz.

Next consider the same set of dipoles, but with a resistive loading modeled by setting the dipole conductivity to 1000 S/m. The resulting generator and receiver voltages are shown in Fig. 3(a) and (b). The peak voltage has now dropped to about 2.3E4 V, and the received energy is 4.4 E-6 J (normalized by

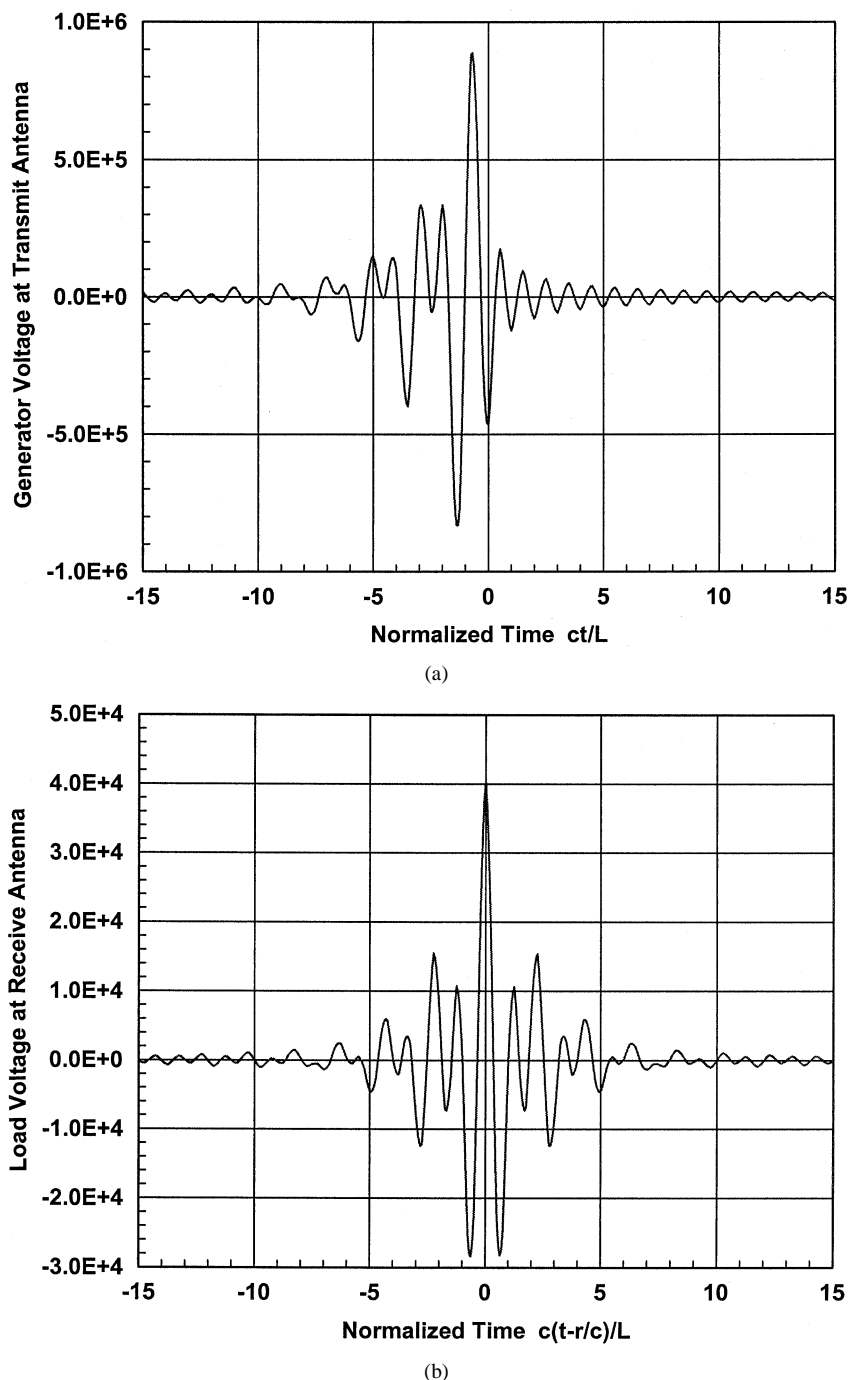


Fig. 2. (a) Generator voltage waveform to maximize receive voltage amplitude for lossless dipoles with  $L = 15$  cm,  $a = 0.02$  cm,  $\sigma = \infty$ ,  $Z_G = 50 \Omega$ ,  $Z_L = \infty$ . Available generator energy constrained to 1 J; bandwidth is 2 GHz. (b) Resulting optimized receive antenna voltage waveform for the antenna parameters of (a).

multiplying by  $r$ ). This represents a drop of about 4.9 dB in voltage and  $-5.4$  dB in energy. Also note that there is somewhat less overshoot and ringing in the response of the lossy dipoles, presumably due to the enhanced bandwidth introduced by the loading.

As a comparison with nonoptimized pulse excitation, the same antenna geometries of Figs. 2 and 3 were analyzed with a Gaussian pulse excitation. For an available energy of 1 J, and a Gaussian half-power width of about  $5E-11$  S, the resulting peak voltage amplitude for the lossless dipole was reduced by 6.3 dB from the optimum result of Fig. 2, while the resulting

peak voltage amplitude for the lossy dipole was reduced by 4.8 dB from the optimum result of Fig. 3.

To further understand the effect of resistive loading and termination impedance on the optimum solution, Fig. 4 shows the magnitude of the transfer function magnitude versus frequency for the dipoles used in the cases of Figs. 2 and 3, along with lossless dipoles with a receiver load impedance of  $Z_L = 50 \Omega$ . Observe that the receiver load impedance has a far greater effect on the transfer function than does the dipole conductivity.

Effects of generator and load impedances, bandwidth, orientation angles, and other parameters can easily be studied

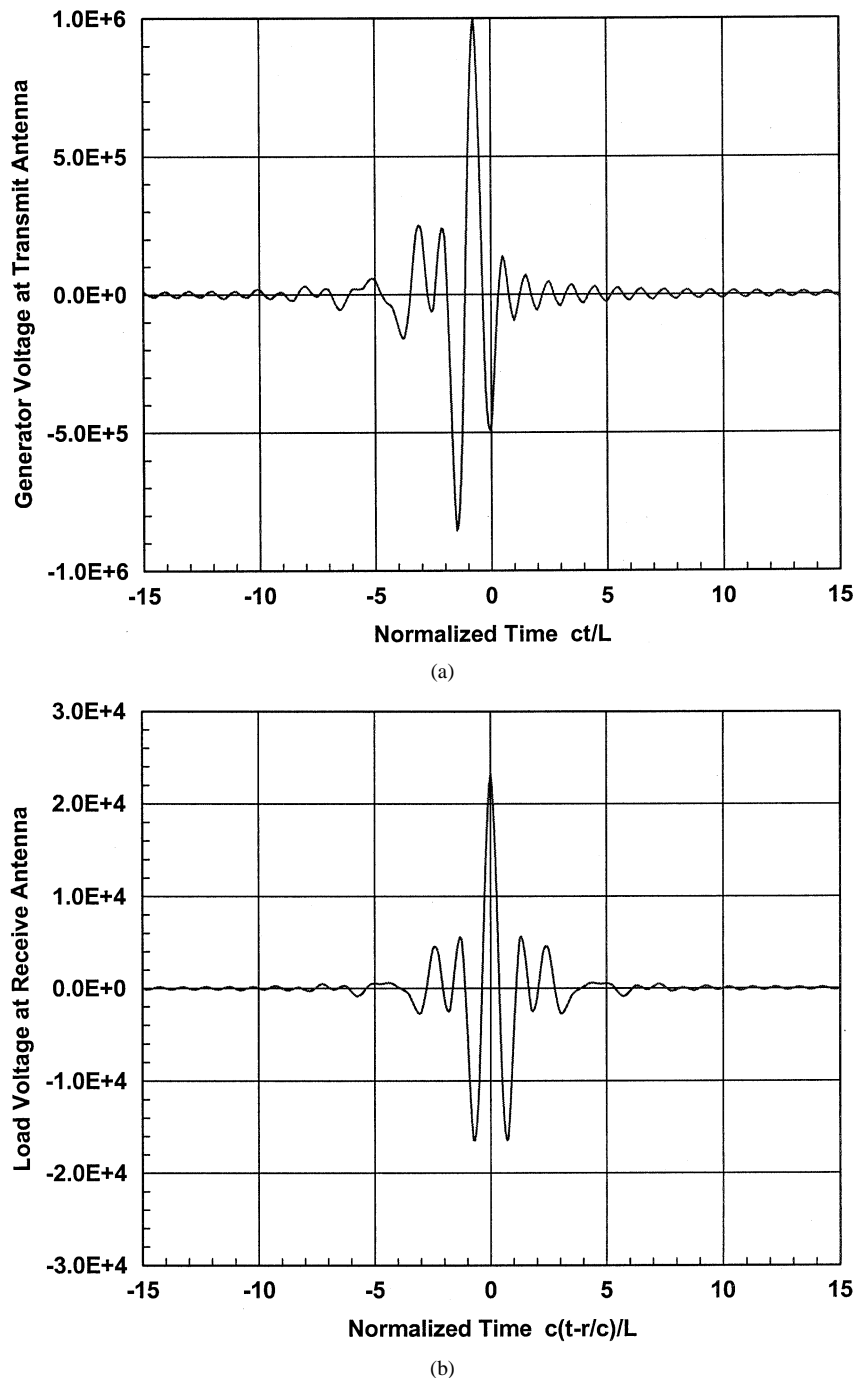


Fig. 3. (a) Generator voltage waveform to maximize receive voltage amplitude for dipoles with  $L = 15$  cm,  $a = 0.02$  cm,  $\sigma = 1000$  S/m,  $Z_G = 50 \Omega$ ,  $Z_L = \infty$ . Available generator energy constrained to 1 J; bandwidth is 2 GHz. (b) Resulting optimized receive antenna voltage waveform for the antenna parameters of (a).

with these solutions, but space limitations prevent us from presenting extensive data on these results. One important observation is that the use of complex termination impedances at either the generator or the receiver generally has the effect of greatly reducing the peak amplitude at the receiver and greatly increasing the ringing of the response. This is caused by the resulting resonant circuit introduced by reactive terminating impedances in conjunction with the resonant dipole response. Conjugate matching and other reactive matching networks should therefore be avoided in UWB antenna systems, even at the expense of lower efficiency. Another observation is that

increasing the signal bandwidth generally has minimal effect on the maximum amplitude, at least in the case of electrically large dipoles, since these antennas radiate effectively only over a relatively narrow band of frequencies near the first resonance.

#### B. Optimizing Received Waveform "Sharpness"

Next consider the optimization of the receive voltage waveform "sharpness" for a pair of lossy dipoles with  $L = 15$  cm,  $a = 0.02$  cm,  $\sigma = 1.0E4$  S/m,  $Z_G = 50 \Omega$ , and  $Z_L = \infty$  over a bandwidth of 4 GHz. The optimum solution is given by (19)–(21). A numerical root-finding method is used to solve (20)

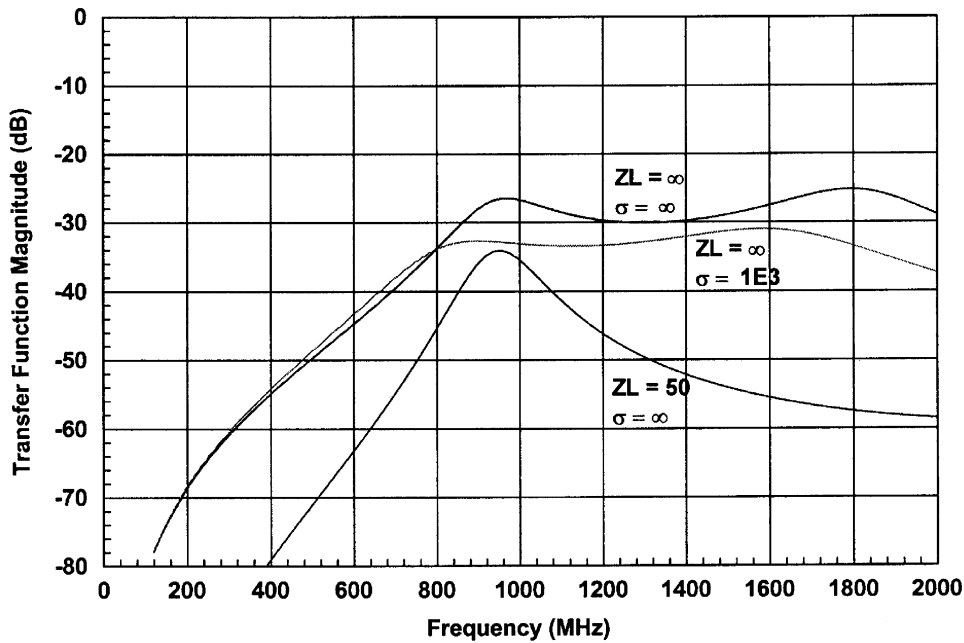


Fig. 4. Voltage transfer function magnitude versus frequency for a pair of dipoles with  $L = 15$  cm,  $a = 0.02$  cm,  $Z_G = 50 \Omega$  for various conductivities and receiver load impedance.

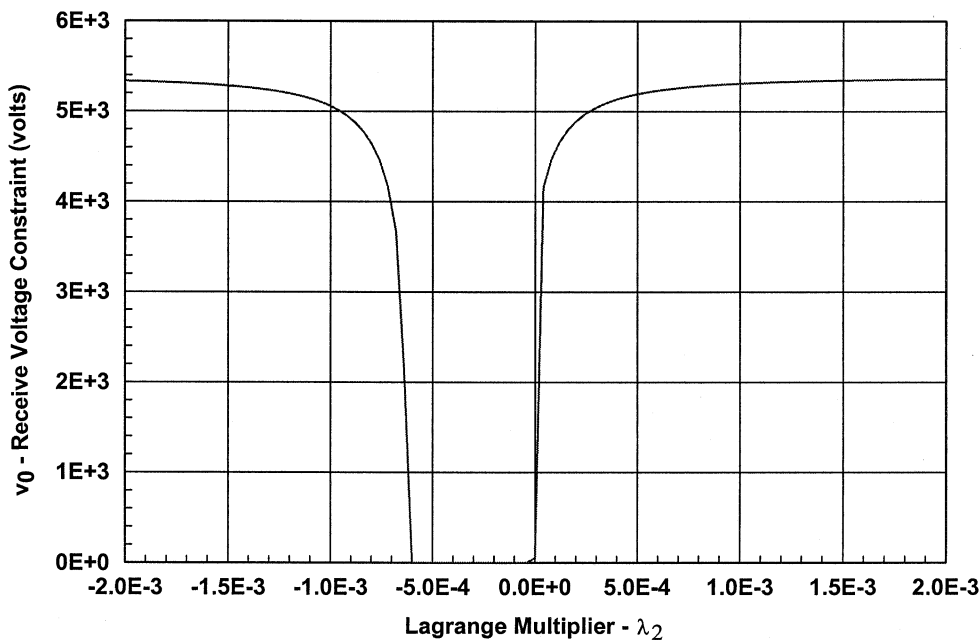


Fig. 5. Constrained receive voltage amplitude at  $t' = 0$  versus the Lagrange multiplier  $\lambda_2$  for lossy dipoles having  $L = 15$  cm,  $a = 0.02$  cm,  $\sigma = 1.0E4$  S/m,  $Z_G = 50 \Omega$ , and  $Z_L = \infty$ . The signal bandwidth is 2 GHz, and the constrained input energy is  $W_{in} = 1$  J.

and (21) for  $\lambda_1$  and  $\lambda_2$ , the Lagrange multipliers. This can be facilitated by first using (20) to eliminate  $\lambda_1$  from (21). Then, for a specified value of  $v_0$  (the constrained receive voltage amplitude at  $t' = 0$ ), (21) can be solved for  $\lambda_2$ . The required root-finding procedure is very sensitive, generally requiring double precision computation.

Fig. 5 shows the relation between these two quantities. Note that for a specified value of  $v_0$  there are two possible roots for  $\lambda_2$ . We have found that the negative root leads to a maximization of received pulse width, while the positive root leads to a min-

imum value. The maximum value of  $v_0$  is  $v_{max}$ —the matched filter solution of Section II-A for constrained input energy. In the present example,  $v_{max} = 5381$  V. Clearly, the solution for optimized “sharpness” cannot produce a larger receive voltage amplitude than the matched filter case—this can be demonstrated mathematically by manipulating the results of (19) and (20).

Setting  $v_0$  to values progressively less than  $v_{max}$  leads to output waveforms that show increasing compression, as evidenced by lower amplitudes away from the main pulse at  $t' = 0$ . The effect is demonstrated in Fig. 6. (The results in these figures

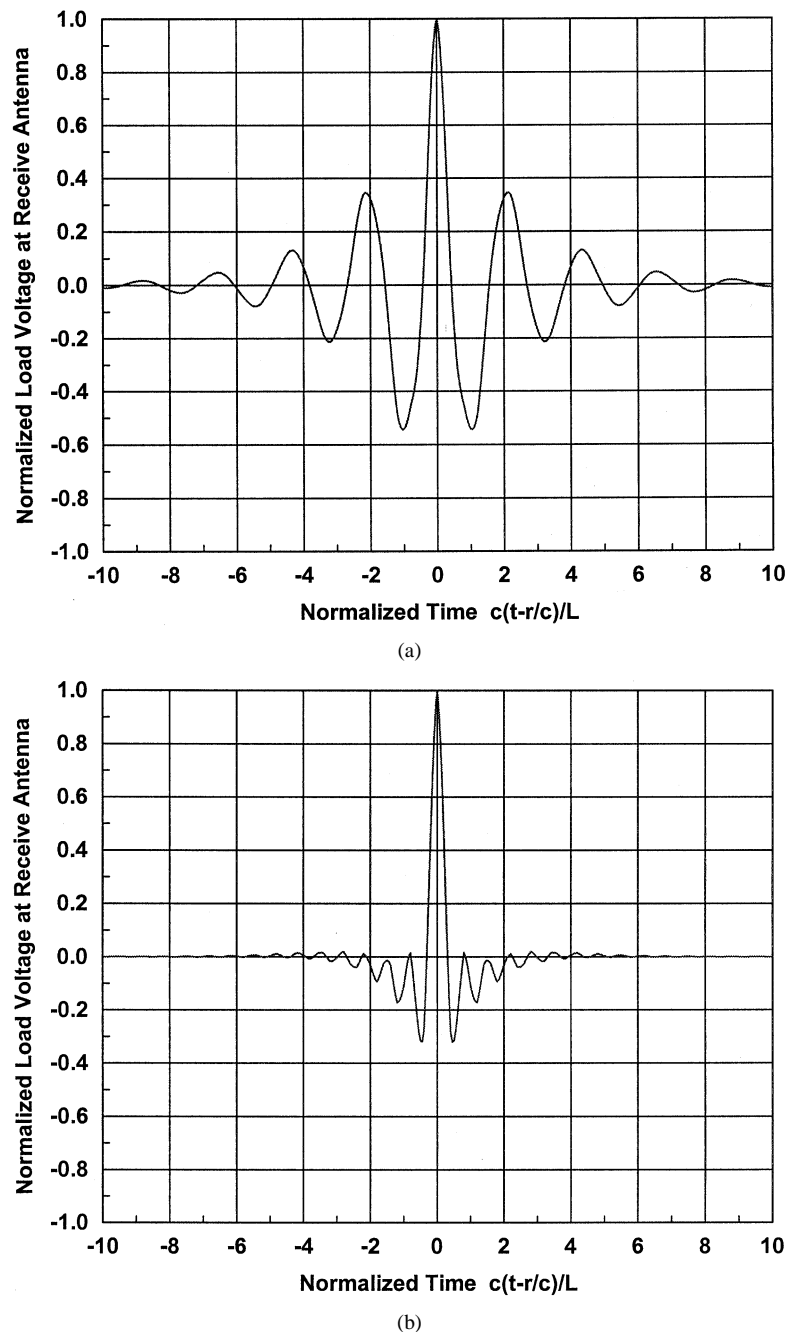


Fig. 6. Normalized receive voltage versus time for optimized waveform “sharpness” with (a)  $v_0 = 5381$  V, (b)  $v_0 = 3000$  V.

are normalized to a maximum value of unity in order to more easily compare the waveform shapes.) Observe that the response for  $v_0 = v_{\max} = 5381$  V in Fig. 6(a) has a  $\sin x/x$  form that exhibits a considerable amount of energy outside the region of the central pulse, continuing out to (normalized) time values of  $\pm 8$  or more. Lowering the constrained output voltage amplitude to 3000 V [see Fig. 6(b)] causes considerable sharpening of the response—the first overshoots are about half the values for the case of Fig. 6(a), and the ringing of the pulse is negligible for (normalized) time values of  $\pm 4$  or more. Further reduction of  $v_0$  to 500 V [see Fig. 6(c)] continues this trend, although with diminishing rewards. The first overshoots are reduced to about a third of the values in the response of Fig. 6(a), and the ringing is effectively stopped for (normalized) time values of  $\pm 3$ .

A quantitative measure of the improvement in receive pulse “sharpness” can be defined as the following “compression ratio”:

$$\text{CR} = 10 \log \frac{W_{\text{rec-max}} v_0^2}{W_{\text{rec}} v_{\max}^2} \quad (29)$$

where  $W_{\text{rec-max}}$  is the receiver energy associated with the maximum received voltage amplitude (which occurs when  $v_0 = v_{\max}$ ). Thus, as  $v_0$  approaches  $v_{\max}$ ,  $W_{\text{rec}}$  approaches  $W_{\text{rec-max}}$  and the compression ratio approaches 0 dB (no compression, or improvement in “sharpness”). But as  $v_0$  is decreased, the received energy may decrease faster than the square of the receiver voltage (which is roughly proportional to the energy of the main pulse), resulting in an overall ratio



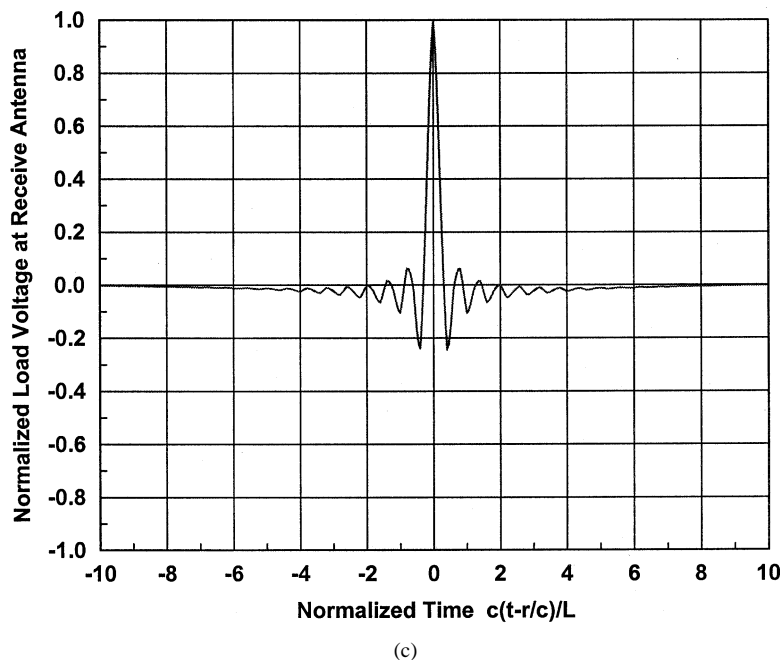


Fig. 6. (Continued.) Normalized receive voltage versus time for optimized waveform “sharpness” with (c)  $v_0 = 500$  V for the lossy dipoles defined in Fig. 5.

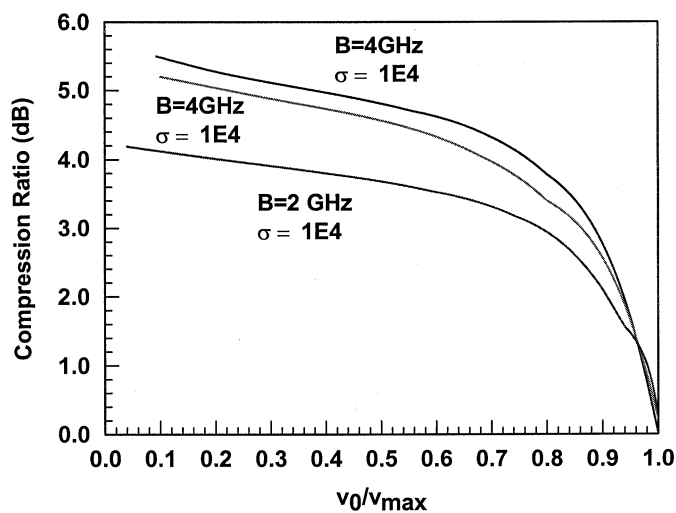


Fig. 7. Compression ratio versus normalized constrained receiver voltage for dipoles defined in Fig. 5.

greater than one. Fig. 7 shows the compression ratio for the dipoles of Figs. 5 and 6 and for solutions for the same dipoles but with two other values of bandwidth and conductivity. We see that compression ratios as high as 4–5 dB can be obtained. Such waveforms have very little ringing, and so can be advantageous for reducing intersymbol interference in UWB systems.

#### IV. ANALYTICAL RESULTS FOR SHORT DIPOLES

Essentially closed-form results for several of the optimizations of Section II can be obtained for electrically small antennas such as short dipoles and small loops, as long as the bandwidth is such that closed-form expressions can be found for the necessary input impedances and transfer functions. This section presents such results for short perfectly conducting transmit

and receive dipole antennas. In this case, the input impedance of each antenna (assumed identical) can be approximated as

$$Z_T(\omega) = Z_R(\omega) = R(\omega) - \frac{j}{\omega C_0} \quad (30)$$

where the radiation resistance is given by

$$R(\omega) = \frac{5L^2\omega^2}{c^2} = \alpha\omega^2 \quad (31)$$

where  $\alpha = 5L^2/c^2$  is a constant. The dipole capacitance is given by

$$C_0 = \frac{L}{240c \left( \ln \frac{L}{2a} - 1 \right)}. \quad (32)$$

The capacitance expression in (32) has been derived from the exact induced electromotive force result after using small argument approximations for the sine and cosine integrals—it has been compared to numerical moment method results and found to be more accurate than the usual expressions found in the literature.

The voltage transfer function between two short dipoles can be derived as

$$H_{LG}(\omega) = \frac{-j\omega\mu_0 h^2 Z_L(\omega)}{4\pi r [Z_G(\omega) + Z_T(\omega)] [Z_L(\omega) + Z_R(\omega)]} \quad (33)$$

where  $h = L/2$  is the dipole half-length. These expressions assume a piecewise sinusoidal current distribution on each dipole and generally give good results for frequencies such that  $L < \lambda/20$

A. Maximize Received Voltage Amplitude, Constrained Input Energy,  $Z_G = 0$ ,  $Z_L = \infty$

The solution for this problem is given by the general expressions in (11) and (12). For the special case of short dipoles with

$Z_G = 0$  and  $Z_L = \infty$ , the transfer function of (33) further reduces to

$$H_{LG}(\omega) = \frac{-j\omega\mu_0 h^2}{4\pi r Z_T(\omega)} \quad (34)$$

and then (11) can be evaluated as

$$V_G(\omega) = \frac{j\omega\mu_0 h^2 Z_T(\omega)}{8\pi r \lambda R_T(\omega)}. \quad (35)$$

The normalization of (12) can also be evaluated in closed form

$$\lambda^2 = \frac{\mu_0^2 h^4 B}{32\pi^2 r^2 \alpha} \quad (36)$$

where  $B$  is the bandwidth in Hertz and  $\alpha$  is a constant defined in (31). Combining these results and using the inverse transform of (7) provides the optimized time domain receiver voltage

$$v_L(t') = \frac{3h}{r} \sqrt{10B} \frac{\sin 2\pi B t'}{2\pi B t'} \quad (37)$$

showing that the response has a  $\sin x/x$  form, with a peak value that increases as the square root of bandwidth. The expected  $1/r$  range dependence is also apparent. The peak value of the receiver voltage response is  $3h\sqrt{10B}/r$ . The optimized amplitude moment method results of Section III-A were compared with these results for electrically short dipoles, with good agreement.

It is probably logical to next derive the required generator voltage for this solution, but this is not possible because  $V_G(\omega)$  in (35) has a nonremovable singularity at  $\omega = 0$  (due to the fact that  $R_T(\omega) = 0$  at dc). The physical meaning of this result seems to be that the solution is trying to capture low-frequency energy at the source, even though low-frequency components will not propagate to the receive antenna and also do not contribute to the input energy (due to the very high reactive impedance of the transmit antenna). If we restrict the operating band to some minimum frequency (above dc), the inverse transform of (35) can then be obtained—the resulting receiver response is virtually unaffected by this lower limit, but it has a substantial effect on the transmit antenna waveform.

#### B. Maximize Received Voltage Amplitude, Constrained Input Energy, $Z_G(\omega) = Z_T^*(\omega)$ , $Z_L = \infty$

In this case we conjugate match the generator to the transmit antenna—in principal, this maximizes power transfer to the transmit antenna. Of course, this requires a generator reactance that is positive with a slope of  $1/\omega$ —conditions that are not possible for a physically realizable passive element. Nevertheless, the solution gives an upper bound on what can be achieved, and it may be possible to approximate the required frequency dependence with active circuit matching.

Setting  $Z_G(\omega) = Z_T^*(\omega)$  simplifies the transfer function of (33) to the following:

$$H_{LG}(\omega) = \frac{-j\omega\mu_0 h^2}{8\pi r R(\omega)} = \frac{-j\mu_0 h^2}{8\pi r \alpha \omega}. \quad (38)$$

Then applying the general solution of (11) and (12) gives the following optimization results:

$$V_G(\omega) = \frac{j\omega\mu_0 h^2}{4\pi \lambda r} \quad (39)$$

$$\lambda^2 = \frac{\mu_0^2 h^4 B}{64\pi^3 \alpha r^2}. \quad (40)$$

Then the time-domain receiver voltage is found as

$$v_L(t') = \frac{12\pi h}{r} \sqrt{5\pi B} \frac{\sin 2\pi B t'}{2\pi B t'} \quad (41)$$

which has a peak value of  $(12\pi h/r)\sqrt{5\pi B}$ . This value is larger than the previous case where  $Z_G = 0$  by a factor of  $4\pi\sqrt{\pi/2}$ , or about 24 dB. Again, the generator voltage has a singularity at dc.

#### C. Maximize Received Voltage Amplitude, Constrained Available Energy, $Z_G(\omega) = Z_T^*(\omega)$ , $Z_L = \infty$

In this case we maximize receiver voltage with a constraint on the available generator energy. The general solutions are given by (14) and (15). The transfer function is the same as in (38). The optimum generator voltage is then

$$V_G(\omega) = \frac{j\omega\mu_0 h^2}{4\pi \lambda r} \quad (42)$$

and the normalization condition is

$$\lambda^2 = \frac{\mu_0^2 h^4 B}{64\pi^3 \alpha r^2}. \quad (43)$$

Finally, the time-domain receiver voltage is found as

$$v_L(t') = \frac{30h}{r} \sqrt{\frac{\pi B}{5}} \frac{\sin 2\pi B t'}{2\pi B t'}. \quad (44)$$

The peak value of this response is  $(30h/r)\sqrt{\pi B/5}$ , which is about 4 dB larger than the case where the input energy was constrained and  $Z_G = 0$ .

Note that similar closed-form optimizations can be obtained for other small antennas such as electrically small loops, slots, and monopoles, although the required integrals can become very complicated. In fact, similar optimizations can be derived for any pair of antennas that can be represented with simple RC or RL equivalent circuits that are valid over the frequency band of interest.

## V. CONCLUSION

Several possible optimization solutions for bandlimited radiated waveforms for ultrawideband radio systems have been presented for general radiating elements, with arbitrary generator and load impedances. Constraints include input energy, available energy, received voltage amplitude, generator voltage amplitude, and radiated power spectral density. These solutions represent upper bounds on the performance of actual UWB systems. Examples of optimized results have been presented for lossless and resistively loaded wire dipole antennas. Closed-form results have been presented for optimization

solutions for short dipoles. These results can be applied to arbitrary UWB antenna elements, and to any set of antennas that can be represented with lumped element equivalent circuits over the frequency band of interest.

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**David M. Pozar** (S'74–M'80–SM'88–F'90) received the Ph.D. degree from The Ohio State University, Columbus, in 1980.

He joined the Faculty at the University of Massachusetts at Amherst in 1980. In 1988, he spent a sabbatical leave at Ecole Polytechnique Federale de Lausanne, Lausanne, Switzerland. He has published more than 100 papers on microstrip antennas and phased arrays. He is the author of *Microwave Engineering*, (New York: Wiley, 1997, 2nd ed.), *Microstrip Antennas* (New York: IEEE Press, 1995), and *Microwave and RF Design of Wireless Systems* (New York: Wiley, 2000). He is also the author of PCAAD, a software package for personal computer-aided antenna design.

Prof. Pozar received the Outstanding Professor for 1981 award from Eta Kappa Nu. In 1984, he received an NSF Presidential Young Investigator Award and the Keys to the Future Award from the IEEE AP-S. In 1985, he received the University of Massachusetts Engineering Alumni Association Outstanding Junior Faculty Award. In 1986, he received the R.W.P. King Best Paper Award from the IEEE AP-S. In 1987, he received the URSI Issac Koga Gold Medal for his work on printed antennas and phased arrays. He again received the R.W.P. King Best Paper Award in 1988. In 1989, he received the United Technologies Corporation Outstanding Teaching Award. He was a Distinguished Lecturer for the IEEE Antennas and Propagation Society in 1993–1995. In 1995, he received the College of Engineering Outstanding Senior Faculty Award. He received the College of Engineering College Outstanding Teacher Award in 1997. In 1998, he received the H. A. Wheeler Applications Prize Paper Award from IEEE AP-S. He received an IEEE Third Millennium Medal in 2000. He was Associate Editor of the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION (1983–1986 and 1989–1992), a Member of the IEEE Antennas and Propagation Society (AP-S) AdCom (1989–1991), and an Associate Editor of the IEEE AP-S NEWSLETTER (1982–1984).