

Multiple Access with Time-Hopping Impulse Modulation

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ABSTRACT A time-hopping modulation format employing impulse signal technology has several features which may make it attractive for multiple-access communications. These features are outlined, an estimate of the multiple-access capability of a communication system employing this format under ideal propagation conditions is presented, and emerging design issues are described.

Time Hopping with Impulses

The current emphasis on constant-envelope spread-spectrum modulations has caused engineers to ignore one design which has considerable potential, namely time-hopping. The technology for generating and receiving pulses on the order of a nanosecond or less in width, similar in shape to one cycle of a sine wave, is available. These *monocycles* can be received by correlation detection virtually at the antenna terminals, making a relatively low-cost receiver possible.

A typical hopping format with pulse-position data modulation (PPM) is given by

$$s^{(k)}(t) = \sum_j w(t - jT_f - c_j^{(k)}T_c - \delta d_{[j/N_s]}^{(k)}). \quad (1)$$

Here $w(t)$ represents the transmitted monocycle waveform that nominally begins at time zero on the transmitter's clock, and the quantities with superscript (k) indicate transmitter-dependent quantities. Hence the signal emitted by the k^{th} transmitter consists of a large number of monocycle waveforms shifted to different times, the j^{th} monocycle nominally beginning at time $jT_f + c_j^{(k)}T_c + \delta d_{[j/N_s]}^{(k)}$. Let's look at the structure of each component of time shift more carefully.

(A) Uniform Pulse Train Spacing: A pulse train of the form $\sum_{j=-\infty}^{\infty} w(t - jT_f)$ consists of monocycle pulses spaced T_f seconds apart in time. The *frame time* or *pulse repetition time* T_f typically may be a hundred to a thousand times the monocycle width, with its largest value constrained in part by the stability of the available clocks. The result is a signal with a very low duty cycle. Multiple-access signals composed of uniformly spaced pulses are vulnerable to occasional *catastrophic collisions* in which a large number of pulses from two signals are received at the same time instants, much as might occur in spread ALOHA systems [4].

(B) Pseudorandom Time-Hopping: To eliminate catastrophic collisions in multiple accessing, each link (indexed by k) is assigned a distinct pulse shift pattern $\{c_j^{(k)}\}$ which we refer to as a *time hopping code*. These hopping codes $\{c_j^{(k)}\}$ are periodic pseudorandom codes with period N_p , i.e., $c_{j+iN_p}^{(k)} = c_j^{(k)}$ for all integers j and i . Each code element is

an integer in the range

$$0 \leq c_j^{(k)} < N_h. \quad (2)$$

The time hopping code therefore provides an additional time shift to each pulse in the pulse train, with the j^{th} monocycle undergoing an added shift of $c_j^{(k)}T_c$ seconds. Hence the added time shifts caused by the code are discrete times between 0 and N_hT_c seconds.

We further assume that

$$N_hT_c \leq T_f, \quad (3)$$

and hence the ratio N_hT_c/T_f indicates the fraction of the frame time T_f over which time-hopping is allowed. Since a short time interval is required to read the output of a monocycle correlator and to reset the correlator, we assume that N_hT_c/T_f is strictly less than one. If N_hT_c is too small, then catastrophic collisions remain a significant possibility. Conversely, with a large enough value of N_hT_c and well designed codes, then the multiple-access interference in many situations can be modeled as a Gaussian random process.

Because the hopping code is periodic with period N_p , the waveform $\sum_j w(t - jT_f - c_j^{(k)}T_c)$ is periodic with period

$$T_p = N_pT_f. \quad (4)$$

One effect of the hopping code is to reduce the power-spectral density from the line spectral density ($1/T_f$ apart) of the uniformly spaced pulse train down to a spectral density with finer line spacing $1/T_p$ apart.

(C) Data Modulation: The data sequence $\{d_i^{(k)}\}$ of transmitter k is a binary (0 or 1) symbol stream that conveys information in some form. Since this is an *oversampled* modulation system with N_s monocycles transmitted per symbol, the modulating data symbol changes only every N_s hops, and assuming that a new data symbol begins with pulse index $j = 0$, the index of the data symbol modulating pulse j is $[j/N_s]$. (Here the notation $[x]$ denotes the integer part of x .) In this modulation method, when the data symbol is 0, no additional time shift is modulated on the monocycle, but a time shift of δ is added to a monocycle when the symbol is 1. Other forms of data modulation can be employed to benefit the performance of the synchronization loops, interference rejection, implementation complexity, etc. Of course, the data modulation further smoothes the power spectral density of the pseudorandom time-hopping modulation.

In this modulation format, a single symbol has a duration $T_s = N_sT_f$. For a fixed frame (pulse repetition) time T_f , the *binary symbol rate* R_s determines the number N_s of

monocycles that are modulated by a given binary symbol, via the equation

$$R_s = \frac{1}{T_s} = \frac{1}{N_s T_f} \text{ sec.}^{-1} \quad (5)$$

Receiver Signal Processing

When N_u links are active in this multiple-access system, then the received signal $r(t)$ can be modeled as

$$r(t) = \sum_{k=1}^{N_u} A_k s^{(k)}(t - \tau_k) + n(t), \quad (6)$$

in which A_k models the attenuation of transmitter k 's signal over the propagation path to the receiver, and τ_k represents time asynchronisms between the clocks of transmitter k and the receiver. The waveform $n(t)$ represents white Gaussian receiver noise.

Let's assume that the receiver is interested in determining the data sent by transmitter 1. If only that signal is present, then

$$r(t) = A_1 s^{(1)}(t - \tau_1) + n(t). \quad (7)$$

When appropriately synchronized, e.g., having learned the value of τ_1 (or at least $\tau_1 \bmod N_p T_f$), the receiver can determine a sequence $\{\mathcal{T}_i\}$ of time intervals, with interval \mathcal{T}_i containing the waveform representing data bit $d_i^{(1)}$ (or $d_i^{(1)} \bmod N_p$). When perfectly synchronized to the first signal, the receiver is then confronted with a standard hypothesis testing problem,

$$\mathcal{H}_d: \quad r(t) = A_1 w_{\text{bit}}(t - \delta d) + n(t) \quad (8)$$

in which d is either 0 or 1, and the observation is over $t \in \mathcal{T}_i$. The bit waveform in this time interval is given by

$$w_{\text{bit}}(t) = \sum_{j=iN_s}^{(i+1)N_s-1} w(t - jT_f - c_j^{(1)}T_c - \tau_1) \quad (9)$$

The optimal receiver (e.g., see [1]) for this single signal in additive white Gaussian noise is simply a bit-duration correlator employing

$$v_{\text{bit}}(t) = w_{\text{bit}}(t) - w_{\text{bit}}(t - \delta) \quad (10)$$

$$= \sum_{j=iN_s}^{(i+1)N_s-1} v(t - jT_f - c_j^{(1)}T_c - \tau_1) \quad (11)$$

as a template signal, where the embedded one-pulse template signal is

$$v(t) = w(t) - w(t - \delta). \quad (12)$$

The corresponding optimal decision rule is

$$\text{say } \mathcal{H}_0 \text{ is true} \iff \underbrace{\int_{t \in \mathcal{T}_i} r(t) v_{\text{bit}}(t) dt}_{\alpha} > 0. \quad (13)$$

When more than one link is communicating in this multiple-access system, the optimal processor for receiving the desired signal is not of the form (13), but is a complicated processing structure that takes advantage of all of the receiver's knowledge concerning the form of the interfering signals [2]. Here we will retain the easily implemented decision procedure of (13), even when many transmitters are active. One possible block diagram of the receiver is shown in Figure 1.

Multiple-Access System Performance

When N_u transmitters are active and the receiver wishes to determine the data modulating transmitter 1, then the received signal $r(t)$ of (6) can be viewed as

$$r(t) = A_1 s^{(1)}(t - \tau_1) + n_{\text{tot}}(t), \quad (14)$$

where

$$n_{\text{tot}}(t) = \sum_{k=2}^{N_u} A_k s^{(k)}(t - \tau_k) + n(t) \quad (15)$$

is assumed to be a mean-zero Gaussian random process. Standard techniques [1] can then be used to show that the probability of error P_{error} , when using the decision procedure of (13), is given by

$$P_{\text{error}}(N_u) = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{S_{\text{out}}(N_u)}}^{\infty} \exp(-x^2/2) dx, \quad (16)$$

where

$$S_{\text{out}}(N_u) = m^2 / \sigma_{\text{tot}}^2(N_u), \quad (17)$$

$$m = \int_{t \in \mathcal{T}_i} A_1 w_{\text{bit}}(t) v_{\text{bit}}(t) dt = A_1 N_s m_p, \quad (18)$$

$$m_p = \int_{-\infty}^{\infty} w(t)[w(t) - w(t - \delta)] dt, \quad (19)$$

$$\sigma_{\text{tot}}^2(N_u) = \mathbf{E} \left\{ \left[\int_{t \in \mathcal{T}_i} n_{\text{tot}}(t) v_{\text{bit}}(t) dt \right]^2 \right\} \quad (20)$$

Furthermore, when only the desired transmitter is active, then

$$\sigma_{\text{tot}}^2(1) = \sigma_{\text{rec}}^2 \triangleq \mathbf{E} \left\{ \left[\int_{t \in \mathcal{T}_i} n(t) v_{\text{bit}}(t) dt \right]^2 \right\}, \quad (21)$$

and

$$S_{\text{out}}(1) = \frac{(A_1 N_s m_p)^2}{\sigma_{\text{rec}}^2}. \quad (22)$$

Thus, $S_{\text{out}}(1)$ is equivalent to the output signal-to-noise ratio that one might observe in single link experiments. This is a convenient parameter because it absorbs all of the scaling problems that one must confront in handling receiver noise and non-monocycle forms of interference.

To complete the calculation of $S_{\text{out}}(N_u)$, we must quantify the total noise variance of (20) and confirm that the mean of the total noise $n_{\text{tot}}(t)$ is zero. The following assumptions were made for this purpose.

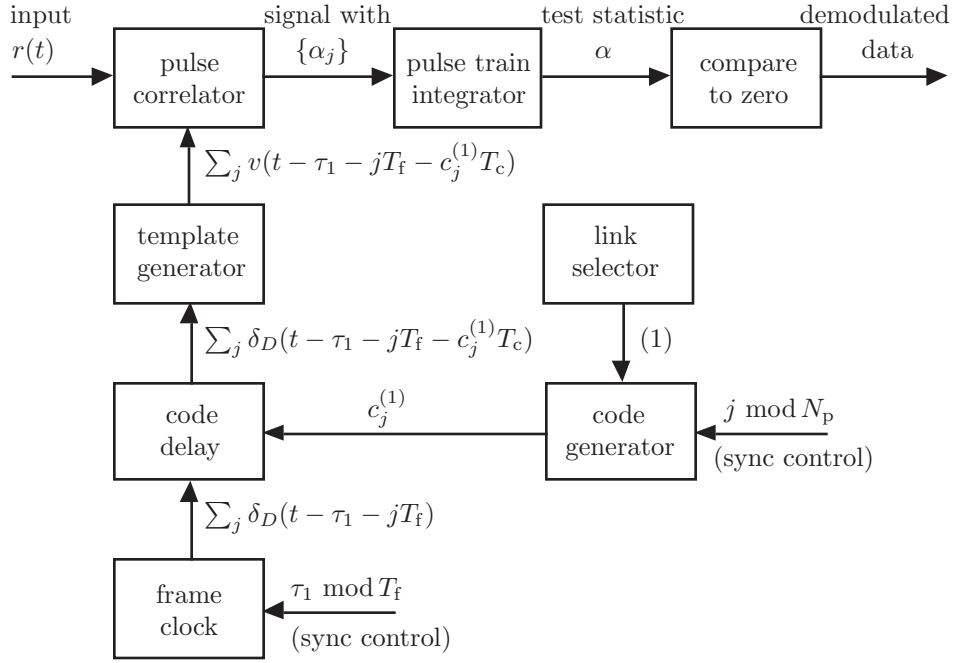


Figure 1: Receiver block diagram for the reception of the first user's signal. Clock pulses are denoted by Dirac delta functions $\delta_D(\cdot)$.

- (a) To estimate performance without choosing a hopping code, we assume that the elements $c_j^{(k)}$, $j = 1, \dots, N_p$, $k = 1, \dots, N_u$, are independent, identically distributed random variables on the interval $[0, N_h]$, and compute performance based on signal-to-noise ratios averaged over the code variables.
- (b) Asynchronous transmission dictates that the transmission time differences $\tau_k - \tau_1$, $k = 2, \dots, N_u$, are independent, identically distributed random variables, with $\tau_k - \tau_1 \bmod T_f$ being uniformly distributed on $[0, T_f]$.
- (c) To insure that no hopping code random variables occur more than once in a bit time, we assume that $N_s \leq N_p$.
- (d) We assume that the received monocycle waveform satisfies the relation

$$\int_{-\infty}^{\infty} w(t) dt = 0. \quad (23)$$

When the waveform $w(t)$ is averaged over uniformly distributed random time shifts as in (b), then (d) in most situations gives that the mean value of the average is zero. Hence this condition is sufficient for $\mathbf{E}\{n_{\text{tot}}(t)\} = 0$.

Assumptions (a) and (b) insure that the interference created by different monocycle transmitters is independent, and therefore

$$\sigma_{\text{tot}}^2 = \sigma_{\text{rec}}^2 + \sum_{k=2}^{N_u} A_k^2 \mathbf{E}\{n_k^2\}, \quad (24)$$

where

$$n_k = \int_{t \in \mathcal{T}_i} s^{(k)}(t - \tau_k) v_{\text{bit}}(t) dt. \quad (25)$$

For the particular waveforms and parameters that we have investigated, and caused in part by assumptions (c) and (d), we have found that

$$\mathbf{E}\{n_k^2\} \approx N_s \sigma_a^2 \quad (26)$$

where

$$\sigma_a^2 = T_f^{-1} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} w(t-s)v(t)dt \right]^2 ds. \quad (27)$$

Substituting (18),(24),(26),(27) into (17) gives the N_u user signal-to-noise ratio $S_{\text{out}}(N_u)$ for use in (16).

$$S_{\text{out}}(N_u) = \frac{1}{\frac{1}{S_{\text{out}}(1)} + \frac{1}{N_s} \frac{\sigma_a^2}{m_p^2} \sum_{k=2}^{N_u} \left(\frac{A_k}{A_1} \right)^2}. \quad (28)$$

An Example

We can illustrate the potential capacity of this system for a specific design. The signal design employed here is a function of $w(t)$ and δ . If δ is greater than the monocycle waveform's width, then the design corresponds to orthogonal signals. Although antipodal signals cannot be achieved in this pulse-position data modulation format, the optimal choice of δ for a given $w(t)$ in use on a single link can yield a negative cross-correlation of the two bit waveforms, and is given by

$$\delta_{\text{opt}} = \underset{\delta}{\text{argmin}} \int_{-\infty}^{\infty} w(t) w(t - \delta) dt. \quad (29)$$

The quantities $w(t)$ and δ affect the uncoded bit-error probability $P_{\text{error}}(N_u)$ only through the ratio σ_a^2/m_p^2 and

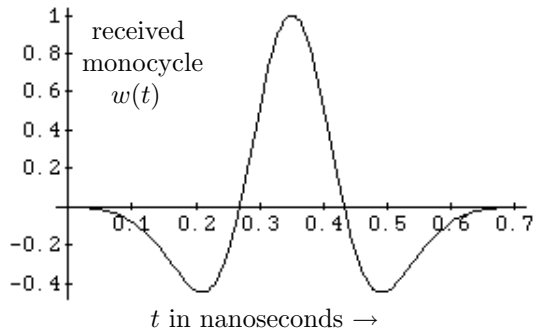


Figure 2: The received monocycle $w(t)$ as a function of time in nanoseconds.

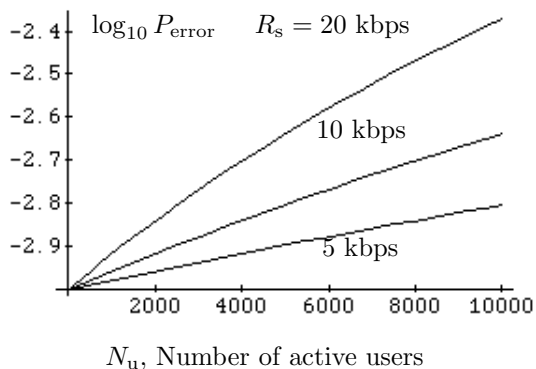


Figure 3: The base 10 logarithm of the probability of bit error, as a function of the number of simultaneous users N_u under perfect power control conditions. The one-user signal-to-noise ratio is set at $S_{\text{out}}(1) = 9.55 = 9.8$ dB, corresponding to a one-user error probability of 10^{-3} . The curves are parameterized by the bit transmission rate R_s , assuming a frame time T_f of 100 nanoseconds.

$S_{\text{out}}(1)$ (a function m_p^2) that appear in the signal-to-noise ratio $S_{\text{out}}(N_u)$. Hence, if $N_u = 1$ or if non-multiple-access noise dominates the signal-to-noise ratio computation, the optimum choice of δ is that which maximizes m_p^2 , that is, δ_{opt} . On the other hand, when multiple-access noise dominates the signal-to-noise ratio calculation, then σ_a^2/m_p^2 is the quantity that should be minimized by choice of δ .

For example, for the monocycle waveform of Figure 2, $\delta_{\text{opt}} \approx 0.156$ nanoseconds, while the ratio σ_a^2/m_p^2 is minimized by $\delta \approx 0.144$ nanoseconds. In this design, σ_a^2 is not particularly sensitive to δ and we evaluated $P_{\text{error}}(N_u)$ at δ_{opt} . The resulting uncoded bit-error probability is shown in Figure 3 for the case in which the single user signal-to-noise ratio (without multiple-access noise) is set so that $P_{\text{error}}(1) = .001$. Here perfect power control is assumed for the receiver in question, i.e., $A_k = A_1$ for all k . Similar results are readily calculated for other parameter choices using (16) and (28).

Conclusions and Caveats

The calculation just completed is quite similar to that for

code-division multiple-access receivers [3], and is based on the fact that both designs use single-channel correlation receivers as a phase-coherent means of bit-waveform detection. Time hopping has an edge in uncoded bit-error probability over a comparable fast-frequency hopping receiver because of this coherence. In comparison to code-division multiple access systems, time-hopping potentially has an edge because the near-far problem is not as acute — the near-far effect is only a factor when a strong pulse and a weak pulse happen to collide.

From a modulation viewpoint, the greatest potential for this time-hopping design comes from the excellent time-resolution that is provided by a monocycle waveform on the order of a nanosecond in duration. Propagation paths with differential delays on the order of this pulse-width or more can be resolved in a relatively simple appropriately designed receiver, and signal processing can be used to counter the normally degrading effects of multipath.

Excellent time resolution provides not only the promise of high performance, but also the technical challenge of dealing with small time-resolution cells. Five microseconds of multipath delay spread may require over 5,000 stages in a Rake multipath diversity combiner [5]. Similarly, a fifty-microsecond clock uncertainty at the beginning of clock acquisition in a receiver may require as many as 50,000 acquisition tests, each possibly on the order of a data bit time or more, before acquisition can be completed (e.g., see [6]).

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