N-Orthogonal Time-Shift-Modulated Codes for Impulse Radio

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ABSTRACT— **N-orthogonal signal set consists of $M = NL$ equal energy, equal time duration signals that have the following two properties: (1) The signal set may be divided into L disjoint subsets, each subset containing N signals, and (2) Signals from different subsets are orthogonal. In this paper we describe the construction of N-orthogonal sets when L is a positive integer, using Multiple-Time-Shift-Keyed modulation. This signals are of interest in the Impulse Radio Channel, where the communication waveforms convey information exclusively in the time shift values and consist of trains of time-shifted ultra-narrow pulses.**

I. INTRODUCTION

Impulse Radio (IR) [Scholtz, 1993] is a novel modulation scheme potentially well suited for reliable simultaneous communications among multiple users exchanging information at rates of the order of Megabits/second over the indoor wireless channel. IR is a Spread Spectrum (SS) scheme which uses time hopping (TH) for the SS sequence modulation, and Pulse Position Modulation (PPM) for the data modulation. The communications waveforms convey information exclusively in the time shift values and consist of trains of time-shifted ultra-narrow pulses. IR is a non-constant envelope, ultra-wide-band, "carrier-less" modulation with bandwidth in excess of 1 G-Hz.

The analysis in [Scholtz,1993] focused on communications using *binary* data PPM over the IR channel disturbed with *Additive White Gaussian Noise* (AWGN). In order to both increase the data transmission rate and make efficient use of the Signal-to-Noise-Ratio (SNR) available, it is desirable to work with block-coded signals that have negative cross correlation values.

In this paper we describe the construction of N-orthogonal time-shift-modulated signals for IR. [1] A given N-orthogonal signal set consists of $M = NL$ equal energy, equal time duration signals that have the following two properties: (1) The signal set may be divided into L disjoint subsets, each subset containing N signals, and (2) Signals from different subsets are orthogonal.

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[1] N-orthogonal phase-modulated signals are the generalization of bi-orthogonal signal sets and were first introduced by Reed and Scholtz [Reed, 1966], and later studied by Viterbi and Stiffler [Viterbi, 1967].

In [Reed, 1966] they give a construction of N-orthogonal sets, when L is an integer power of N, using Multiple Phase-Shift-Keyed (M-PSK) modulation with phase values $\theta_i = \frac{2\pi}{M}(i-1), i = 1, 2, \ldots, N$. The matrix containing the normalized signal cross correlation values Λ can be partitioned in the following form

$$\hat{\Lambda}_{NO} = \begin{bmatrix} \Lambda_{M-PSK} & \mathbf{0} & \ldots & \mathbf{O} \\ \mathbf{0} & \Lambda_{M-PSK} & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \Lambda_{M-PSK} \end{bmatrix}$$

where

$$\Lambda_{M-PSK} = \begin{bmatrix} 1 & \cos(\theta_2-\theta_1) & \ldots & \cos(\theta_N-\theta_1) \\ \cos(\theta_2-\theta_1) & 1 & \ldots & \cos(\theta_N-\theta_2) \\ \vdots & \vdots & \ddots & \vdots \\ \cos(\theta_N-\theta_1) & \cos(\theta_N-\theta_2) & \ldots & 1 \end{bmatrix}$$

In this paper we give the construction of N-orthogonal sets, when L is a positive integer, using Multiple Time-Shift-Keyed (M-TSK) modulation with time shifts

$$(\tau_1, \tau_2, \ldots, \tau_N, \tau_{N+1}, \tau_{N+2}, \ldots, \tau_M) \tag{1}$$

The matrix containing the normalized signal correlation values Λ_{NO} can be partitioned in the following form

$$\tilde{\Lambda}_{NO} = \begin{bmatrix} \Lambda_{M-TSK} & \mathbf{0} & \ldots & \mathbf{O} \\ \mathbf{0} & \Lambda_{M-TSK} & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \Lambda_{M-TSK} \end{bmatrix} \tag{2}$$

where

$$\Lambda_{M-TSK} = \begin{bmatrix} 1 & \gamma_w(\tau_2-\tau_1) & \ldots & \gamma_w(\tau_N-\tau_1) \\ \gamma_w(\tau_2-\tau_1) & 1 & \ldots & \gamma_w(\tau_N-\tau_2) \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_w(\tau_N-\tau_1) & \gamma_w(\tau_N-\tau_2) & \ldots & 1 \end{bmatrix} \tag{3}$$

and $\gamma_w(\tau)$ is the signal correlation function of the pulse $w(t)$ used to transmit information.

This paper is organized as follows. In Section II representation of sequences of PPM signals and their cross correlation properties are established. In section III definition of M-TSK signals is given and some of their properties are described. In Section IV the construction of the N-Orthogonal sets is described.

II. PPM SIGNALS IN IR

The PPM signal conveying the user's information can be written as

$$S_i(t) = \sum_{k=1}^{N_s} w(t - kT_f - \delta_i^k), \quad i = 1, 2, \ldots, M \tag{4}$$

Here $S_i(t)$ represents the i-th signal in an ensemble of signals, each signal completely identified by the sequence of time shifts $\{\delta_i^k; k = 1, 2, \ldots, N_s\}$.

In IR, the signal $w(t)$ is an ultra-narrow pulse with duration T_w nanoseconds; $T_f >> T_w$ is the time shift value corresponding to the frame period; and the time shift corresponding to the data modulation is $\delta_i^k \in \{\tau_1 < \tau_2 < \ldots < \tau_M\}$, $0 \le \tau_i \le kT_w < T_f$, k a positive integer.

The complete ensemble of signals $S_i(t), i = 1, 2, \ldots, M$ will be represented by a matrix where each row corresponds to the time shifts $\{\delta_i^k; k = 1, 2, \ldots, N_s\}$ defining the i-th signal

$$\Delta = \begin{bmatrix} \delta_1^1 & \delta_1^2 & \ldots & \delta_1^k & \ldots & \delta_1^{N_s} \\ \delta_2^1 & \delta_2^2 & \ldots & \delta_2^k & \ldots & \delta_2^{N_s} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \delta_i^1 & \delta_i^2 & \ldots & \delta_i^k & \ldots & \delta_i^{N_s} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \delta_M^1 & \delta_M^2 & \ldots & \delta_M^k & \ldots & \delta_M^{N_s} \end{bmatrix} \tag{5}$$

The correlation between signal $S_i(t)$ and $S_j(t)$ is defined as

$$\begin{aligned} R_{ij} &= \int_{-\infty}^{\infty} S_i(t) S_j(t) dt \\ &= \int_{-\infty}^{\infty} \left[\textstyle\sum_{k=1}^{N_s} w(t - kT_f - \delta_i^k)\right] \left[\textstyle\sum_{l=1}^{N_s} w(t - lT_f - \delta_j^l)\right] dt \\ &= \int_{-\infty}^{\infty} \textstyle\sum_{k=1}^{N_s} w(t - kT_f - \delta_i^k) \quad w(t - kT_f - \delta_j^k) \quad dt \\ &\quad + \int_{-\infty}^{\infty} \textstyle\sum_{\substack{k=1 \\ k \neq l}}^{N_s} \sum_{l=1}^{N_s} \; w(t - kT_f - \delta_i^k) \quad w(t - kT_f - \delta_j^k) \quad dt \\ &= \sum_{k=1}^{N_s} \int_{-\infty}^{\infty} w(t - \delta_i^k) \quad w(t - \delta_j^k) \quad dt \end{aligned}$$

since for $k \neq l$ the pulses are non overlapping. Therefore

$$R_{ij} = E_w \sum_{k=1}^{N_s} \gamma_w(\delta_i^k - \delta_j^k) \tag{6}$$

where E_w is the energy in the pulse $w(t)$ and $\gamma_w(\tau)$ is the signal correlation function defined by

$$\gamma_w(\tau) \triangleq \int_{-\infty}^{\infty} w(t) w(t - \tau) dt \tag{7}$$

The energy in the i-th signal is

$$E_S = R_{ii} = \int_{-\infty}^{\infty} [S_i(t)]^2 = N_s E_w$$

and the normalized correlation value is

$$\alpha_{ij} = \frac{R_{ii}}{E_S} = \frac{1}{N_s} \sum_{k=1}^{N_s} \gamma_w(\delta_i^k - \delta_j^k) \tag{8}$$

The complete set of correlation values α_{ij} corresponding to the ensemble of signals Δ is given by the symmetric $M \times M$ matrix

$$\Lambda \triangleq \begin{bmatrix} 1 & \sum_{k=1}^{N_S} \frac{\gamma_w(\delta_{21}^k)}{N_S} & \ldots & \sum_{k=1}^{N_S} \frac{\gamma_w(\delta_{M1}^k)}{N_S} \\ \sum_{k=1}^{N_S} \frac{\gamma_w(\delta_{21}^k)}{N_S} & 1 & \ldots & \sum_{k=1}^{N_S} \frac{\gamma_w(\delta_{M2}^k)}{N_S} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{k=1}^{N_S} \frac{\gamma_w(\delta_{M1}^k)}{N_S} & \sum_{k=1}^{N_S} \frac{\gamma_w(\delta_{M2}^k)}{N_S} & \ldots & 1 \end{bmatrix} \tag{9}$$

where $\delta_{ij}^k = \delta_i^k - \delta_j^k$ and Λ is non negative definite.

III. M-TSK SIGNALS

Definition 1: Multiple time-shift-keyed (M-TSK) signals. The signal $w(t)$ is a pulse of duration T_w. The signal set $w(t-\tau_1), w(t-\tau_2), \ldots, w(t-\tau_N)$ with $\tau_1 = 0, \tau_(i-1) < \tau_i \le (i-1)T_w, i = 2, 3, \ldots, N$ form a set of coherent N-ary time-shit-keyed modulated signal set.

Definition 2: Correlation function of M-TSK signals. The signal correlation function of $w(t)$ is defined by

$$\gamma_w(\tau) \triangleq \int_{-\infty}^{\infty} w(t) w(t - \tau) dt$$

The minimum value of $\gamma_w(\tau)$ will be denoted γ_{min}, and τ_{min} will denote the smallest value of τ in $[0, T_w]$ such that $\gamma_{min} = \gamma_w(\tau_{min})$.

Definition 3: Cross correlation matrix of M-TSK signals. The cross correlation value between $w(t-\tau_i)$ and $w(t-\tau_j)$ is given by $\gamma_w(\tau_i - \tau_j)$ and the cross correlation matrix is Λ_{M-TSK} given in equation 3.

The following properties are straightforward to verify.

Proposition 1: Linear independence of M-TSK signals The signals in the set $w(t-\tau_1), w(t-\tau_2), \ldots, w(t-\tau_N)$ are linearly independent.

Proposition 2: Minimum cross correlation value of M-TSK signals.
The signal correlation function

$$\gamma_w(\tau) > -1 \forall \tau$$

hence, the signals $w(t-\tau_i), w(t-\tau_j), i \neq j$ can never be antipodal.

From these properties we can derive some properties of the signal set $S_i(t)$.

Proposition 3: Linear independence of PPM signals
The signals in the set $S_i(t)$ are linearly independent. Hence the dimensionality of the set $S_i(t)$ is always $M = NL$.

Proposition 4: Minimum cross correlation value of PPM signals.
The cross correlation value between signal $S_i(t)$ and $S_j(t)$

$$\alpha_{ij} = \frac{1}{E_S} \int_{-\infty}^{\infty} S_i(t) S_j(t) dt$$

$$
\begin{aligned}
&= \frac{1}{N_s}\sum_{k=1}^{N_s}\gamma_w(\delta_i^k-\delta_j^k)\\
&> -1
\end{aligned}
$$

hence, the signals $S_i(t), S_j(t), i \neq j$ can never be antipodal.

IV. CONSTRUCTION OF N-ORTHOGONAL CODES

One method to generate N-orthogonal is illustrated in the following matrix

$$
\Delta_{NO} = \begin{bmatrix}
\delta_1^1 & \delta_1^2 & \cdots & \delta_1^{N_S}\\
\delta_2^1 & \delta_2^2 & \cdots & \delta_2^{N_S}\\
\vdots & \vdots & \vdots & \vdots\\
\delta_N^1 & \delta_N^2 & \cdots & \delta_N^{N_S}\\
\delta_1^1+T_w & \delta_1^2+T_w & \cdots & \delta_1^{N_S}+T_w\\
\delta_2^1+T_w & \delta_2^2+T_w & \cdots & \delta_2^{N_S}+T_w\\
\vdots & \vdots & \vdots & \vdots\\
\delta_N^1+T_w & \delta_N^2+T_w & \cdots & \delta_N^{N_S}+T_w\\
\delta_1^1+2T_w & \delta_1^2+2T_w & \cdots & \delta_1^{N_S}+2T_w\\
\delta_2^1+2T_w & \delta_2^2+2T_w & \cdots & \delta_2^{N_S}+2T_w\\
\vdots & \vdots & \vdots & \vdots\\
\delta_N^1+2T_w & \delta_N^2+2T_w & \cdots & \delta_N^{N_S}+2T_w\\
\vdots & \vdots & \vdots & \vdots\\
\delta_1^1+(L-1)T_w & \delta_1^2+(L-1)T_w & \cdots & \delta_1^{N_S}+(L-1)T_w\\
\delta_2^1+(L-1)T_w & \delta_2^2+(L-1)T_w & \cdots & \delta_2^{N_S}+(L-1)T_w\\
\vdots & \vdots & \vdots & \vdots\\
\delta_N^1+(L-1)T_w & \delta_N^2+(L-1)T_w & \cdots & \delta_N^{N_S}+(L-1)T_w
\end{bmatrix} \tag{10}
$$

In this case the PPM signals can be written as

$$
S_{nl}(t) = \sum_{k=1}^{N_s} w(t - kT_f - \delta_n^k - (l-1)T_w) \tag{11}
$$

$$
n = 1, 2, \ldots, N, \quad l = 1, 2, \ldots, L
$$

It can be easily verified that the $M \times M$ cross correlation matrix Λ_{NO} is given by

$$
\Lambda_{NO} = \begin{bmatrix}
\Lambda_{PPM} & \mathbf{0} & \ldots & \mathbf{O}\\
\mathbf{0} & \Lambda_{PPM} & \ldots & \mathbf{0}\\
\vdots & \vdots & \ddots & \vdots\\
\mathbf{0} & \mathbf{0} & \ldots & \Lambda_{PPM}
\end{bmatrix} \tag{12}
$$

where Λ_{PPM} is the $N \times N$ matrix given by

$$
\Lambda_{PPM} = \begin{bmatrix}
1 & \alpha_{21} & \ldots & \alpha_{N1}\\
\alpha_{21} & 1 & \ldots & \alpha_{N2}\\
\vdots & \vdots & \ddots & \vdots\\
\alpha_{N1} & \alpha_{N2} & \ldots & 1
\end{bmatrix} \tag{13}
$$

where $\alpha_{ij} = \frac{1}{N_s}\sum_{k=1}^{N_s}\gamma_w(\delta_i^k - \delta_j^k)$

V. SELECTION OF $\delta_n^k, \quad n=1,2,\ldots,N, l=1,2,\ldots,N_s$

In this section we describe two *ad-hoc* methods to select the time shift values $\delta_n^k, \quad n=1,2,\ldots,N, l=1,2,\ldots,N_s$.

A. METHOD 1

In this method

$$
\delta_n^k \stackrel{\triangle}{=} \tau_n, \quad k = 1, 2, \ldots, N_s \tag{14}
$$

and the matrix $\tilde{\Delta}_{NO}$ that represents the ensemble of signals is given by

$$
\tilde{\Delta}_{NO} = \begin{bmatrix}
\tau_1 & \tau_1 & \cdots & \tau_1\\
\tau_2 & \tau_2 & \cdots & \tau_2\\
\vdots & \vdots & \vdots & \vdots\\
\tau_N & \tau_N & \cdots & \tau_N\\
\tau_1+T_w & \tau_1+T_w & \cdots & \tau_1+T_w\\
\tau_2+T_w & \tau_2+T_w & \cdots & \tau_2+T_w\\
\vdots & \vdots & \vdots & \vdots\\
\tau_N+T_w & \tau_N+T_w & \cdots & \tau_N+T_w\\
\tau_1+2T_w & \tau_1+2T_w & \cdots & \tau_1+2T_w\\
\tau_2+2T_w & \tau_2+2T_w & \cdots & \tau_2+2T_w\\
\vdots & \vdots & \vdots & \vdots\\
\tau_N+2T_w & \tau_N+2T_w & \cdots & \tau_N+2T_w\\
\vdots & \vdots & \vdots & \vdots\\
\tau_1+(L-1)T_w & \tau_1+(L-1)T_w & \cdots & \tau_1+(L-1)T_w\\
\tau_2+(L-1)T_w & \tau_2+(L-1)T_w & \cdots & \tau_2+(L-1)T_w\\
\vdots & \vdots & \vdots & \vdots\\
\tau_N+(L-1)T_w & \tau_N+(L-1)T_w & \cdots & \tau_N+(L-1)T_w
\end{bmatrix} \tag{15}
$$

It can be easily verified that the signal cross correlation coefficients are given by [2]

$$
\alpha_{ij} = \begin{cases}
0 & \left[\frac{i-1}{M}\right] \neq \left[\frac{j-1}{M}\right]\\
1 & i = j\\
\gamma_w(\tau_i - \tau_j) & \left[\frac{i-1}{M}\right] = \left[\frac{j-1}{M}\right]
\end{cases} \tag{16}
$$

and that the cross correlation matrix is given by Λ_{NO} in equation 2

A.1 SELECTION OF $(\tau_1, \tau_2, \ldots, \tau_N)$

The optimum single-user receiver for the IR channel consists of a TH despreading operation followed by a correlation receiver [Scholtz, 1993]. The symbol error probability $P_e(\frac{E_s}{N_o}, \Lambda)$ for this receiver depends only on the symbol SNR value $(\frac{E_s}{N_o})$ and the cross correlation properties Λ of the communications signal set [Weber, 1987]. [3] The same statement applies to the union bound on the symbol error probability $UBP_e(\frac{E_s}{N_o}, \Lambda)$.

One criterion to select the time shift values is to use $(\tau_1^{opt}, \tau_2^{opt}, \ldots, \tau_N^{opt})$ that minimize the probability of symbol error $P_e(\frac{E_S}{N_o}, \Lambda_{M-TSK})$. This is an optimization problem that can be stated as follows:

[2]Here $[\cdot]$ denotes integer part.

[3]The probability of error can be expressed as [Weber, 1987]

$$
P_e(\tfrac{E_S}{N_o}, \Lambda) = 1 - \tfrac{1}{M}\exp(-\tfrac{1}{2}\tfrac{E_S}{N_o})\Phi(\sqrt{\tfrac{E_S}{N_o}}, \Lambda)
$$

where $\Phi(\sqrt{\frac{E_S}{N_o}}, \Lambda) = E_\psi\left\{\exp\sqrt{\frac{E_S}{N_o}}\max_i \psi_i\right\}$ and ψ is a random vector with p.d.f $N(\mathbf{0}, \Lambda)$.

$$\underset{\{\Lambda_{M-TSK}\}}{minimize} \quad P_e(\tfrac{E_s}{N_o},\Lambda_{M-TSK})=1-\tfrac{1}{N}\exp(-\tfrac{E_S}{N_o})\Phi(\sqrt{\tfrac{E_S}{N_o}},\Lambda_{M-TSK})$$
$$\text{subject to } \Lambda_{M-TSK} \in \Upsilon \subset \{N \times N \text{matrices}\}.$$

where Υ is the class of admissible Λ defined by

1. $\Lambda = \Lambda^{\top}$;
2. $x\Lambda x^{\top} \geq 0, \forall x \in \Re^N$;
3. $\alpha_{i,i} = 1, i = 1, 2, \ldots, N$;
4. -1 ¡ $\gamma_{min} \leq \alpha_{i,j} = \gamma_w(\tau_i - \tau_j) \leq 1, i \neq j,\ i,j = 1, 2, \ldots, N$;
5. $\gamma_{min} \overset{\triangle}{=} \underset{\{\tau\}}{min} \quad \gamma_w(\tau)$,

In general, the optimal solution Λ_{M-TSK}^{opt} might depend on the SNR value $(\frac{E_S}{N_o})$ and has the form

$$\Lambda_{M-TSK}^{opt}(\tfrac{E_S}{N_o}) = \begin{bmatrix} 1 & \alpha_{21}^{opt}(\frac{E_S}{N_o}) & \ldots & \alpha_{N1}^{opt}(\frac{E_S}{N_o}) \\ \alpha_{21}^{opt}(\frac{E_S}{N_o}) & 1 & \ldots & \alpha_{2N}^{opt}(\frac{E_S}{N_o}) \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{N1}^{opt}(\frac{E_S}{N_o}) & \alpha_{N2}^{opt}(\frac{E_S}{N_o}) & \ldots & 1 \end{bmatrix} \quad (17)$$

where

$$\alpha_{ij}^{opt}(\tfrac{E_S}{N_o})=\gamma_w(\tau_i^{opt}(\tfrac{E_S}{N_o})-\tau_j^{opt}(\tfrac{E_S}{N_o})) \quad (18)$$

Note that the time shift values are interrelated. For example, let $\tau_{i,j} \overset{\triangle}{=} \tau_i - \tau_j$, then $\tau_{1,4} = \tau_{1,2} + \tau_{2,3} + \tau_{3,4}$

For any given $w(t)$, this optimization problem is very difficult to solve both by analytic methods and by computer search. Besides, when the solution exists, the correspondence between the optimum cross correlation matrix Λ_{M-TSK}^{opt} and the optimum time shifts $\tau_1^{opt}, \tau_2^{opt}, \ldots, \tau_1^{opt}$ is not linear and might not be one to one.

A reasonable approximate solution for this problem is to substitute $P_e(\frac{E_S}{N_o},\Lambda_{M-TSK})$ by the union bound on the probability of error [Simon, 1996]

$$UBP_e(\tfrac{E_S}{N_o},\Lambda_{M-TSK})=\sum_{\substack{k=1\\k\neq l}}^{N_s}\sum_{l=1}^{N_s} Q\left(\sqrt{\tfrac{E_S}{N_o}(1-\alpha_{ij})}\right)$$

and find by computer search the time shift values $\hat{\tau}_1^{opt}, \hat{\tau}_2^{opt}, \ldots, \hat{\tau}_1^{opt}$ that minimize

$$UBP_e(\tfrac{E_S}{N_o},\Lambda_{M-TSK}(\tau_1,\tau_2,\ldots,\tau_N))$$

The corresponding optimization problem is

$$\underset{(\tau_1,\tau_2,\ldots,\tau_N)\in \mathbf{P}}{minimize} \quad \sum_{\substack{k=1\\k\neq l}}^{N_s}\sum_{l=1}^{N_s} Q\left(\sqrt{\tfrac{E_S}{N_o}(1-\gamma_w(\tau_{ij}))}\right) \quad (19)$$

where $\mathbf{P}$ is the region defined by

$$[\tau_1 = 0] \times [\tau_1 < \tau_2 \leq T_w] \times \ldots \times [\tau_{N-1} < \tau_N \leq (N-1)T_w]$$

To find the solution for high $(\frac{E_S}{N_o})$ values, [4] the calculation can be approximated by

$$\begin{aligned} \underset{(\tau_1,\tau_2,\ldots,\tau_N)\in \mathbf{P}}{minimize} \quad max(\quad & \gamma_w(\tau_{12}),\gamma_w(\tau_{13}),\ldots,\gamma_w(\tau_{1N}), \\ & \gamma_w(\tau_{23}),\gamma_w(\tau_{24}),\ldots,\gamma_w(\tau_{2N}), \\ & \vdots \\ & \gamma_w(\tau_{(N-2)(N-1)}),\gamma_w(\tau_{(N-2)N}),, \\ & \gamma_w(\tau_{(N-1)N}) \quad) \end{aligned} \quad (20)$$

[4] At High SNR values the performance is dominated by the two signals with highest cross correlation value.

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