Multiple-Access with Time Hopping and Block Waveform PPM Modulation

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Abstract — The use of time-hopped block waveform encoding PPM signals sets for multiple access communications is studied. The multiple access performance is analyzed in terms of the number of users supported by the system for a given bit error rate and bit transmission rate. The analysis shows that this technique is potentially able to provide multiple-access communications with a combined transmission capacity of over 500 Megabits per second at bit error rates in the range $10^{-4}$ to $10^{-8}$ using receivers of moderate complexity.

I. INTRODUCTION

Multiple-access (MA) communication using a time-hopping modulation employing impulse signal technology was proposed in [1]. This communication technique is called impulse radio (IR) [2]. Impulse radio modulation uses subnanosecond impulse technology to build ultra-wideband (UWB) communication waveforms that consist of trains of time-shifted subnanosecond pulses using time hopping (TH) for spread-spectrum sequence modulation, and pulse position modulation (PPM) for the data modulation. Impulse radio promises to be a viable technique to build relatively simple and low-cost, low-power transceivers that can be used for short range, high speed MA communications over the multipath indoor wireless channel.

In [1] the single-user MA performance of IR assuming free space propagation conditions and additive white Gaussian noise (AWGN) was studied. The analysis assumed that binary PPM signals based on binary time-shift-keyed modulation are detected using a correlation receiver. The analysis in [1] is quite similar to that for code-division MA made in [3] and is based on the fact that both designs use single-channel correlation receivers for phase-coherent detection of the bit waveform. In this paper we generalize the ideas in [1] to investigate the use of block-waveform encoding PPM signals to increase the number of users supported by the system for a given MA performance and bit transmission rate, without increasing each user's transmitted power. We also illustrate some of the tradeoffs between MA performance and receiver complexity.

II. CHANNEL, SIGNALS AND MULTIPLE-ACCESS INTERFERENCE MODELS

A. Channel and Impulse Signal Models

The model assumed is a channel with ideal propagation condition. The transmitted pulse is $w(t) \triangleq \int_{-\infty}^{\infty} w(\xi) d\xi$ and the received pulse is $Aw(t-\tau) + n(t)$. The constants $A$ and $\tau$ represent the attenuation and propagation delay, respectively, that the signal experiences over the link path between the transmitter and receiver. The noise $n(t)$ is AWGN with two-sided power density $\frac{N_0}{2}$. The signal $w(t)$ is the basic subnanosecond impulse used to convey information. It has duration $T_w$, two-sided bandwidth $W$, and energy $E_w = \int_{-\infty}^{\infty} [w(t)]^2 dt$. The normalized signal correlation function of $w(t)$ is

$$
\gamma_w(\tau) \triangleq \frac{1}{E_w} \int_{-\infty}^{\infty} w(t) w(t-\tau) dt > -1 \forall \tau.
$$

The minimum value of $\gamma_w(\tau)$ will be denoted $\gamma_{\min}$, and $\tau_{\min}$ will denote the smallest value of $\tau$ in $[0, T_w]$ such that $\gamma_{\min} = \gamma_w(\tau_{\min})$. The correlation value between $w(t-\tau_i)$ and $w(t-\tau_j), i \neq j$, is given by $\gamma_w(\tau_i - \tau_j)$. Note that the signals $w(t-\tau_i)$ and $w(t-\tau_j)$ are linearly independent, hence they can never be antipodal.

B. TH PPM Signals

The TH PPM signal conveying information exclusively in the time shifts is

$$
x^{(\nu)}(t) = \sum_{k=0}^{\infty} w(t - kT_f - c_k^{(\nu)}T_i - \delta_k^{(\nu)}T_c) 1_{[k/N_u]}(t).
$$

The superscript $(\nu)$, $(1 \leq \nu \leq N_u)$ indicates user-dependent quantities. The index $k$ is the number of time hops that the signal $x^{(\nu)}(t)$ has experienced, and

The effect of the antenna system in the UWB transmitted pulse is modeled as a derivation operation.

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also the number of impulses that has been transmitted. The impulse duration satisfies $T_w < T_f$, where $T_f$ is the frame (impulse repetition) time and equals the average time between pulse transmissions. The $\{c_k^{(v)}\}$ is the pseudo-random time-hopping sequence assigned to user $v$. It is periodic with period $N_p$ (i.e., $c_{k+N_p}^{(v)} = c_k^{(v)}$ for all integers) and each sequence element is an integer in the range $0 \leq c_k^{(v)} \leq N_h$. The time hopping code provides an additional time shift to each impulse, each time shift being a discrete time value between $0 \leq c_k^{(v)} T_c < N_h T_c$ seconds.

The time shift corresponding to the data modulation is $c_k^{(v)} = \tau_d$ $\in \{\tau_1 = 0 < \tau_2 < \ldots < \tau_{N_d}\}$, with $\tau_{N_d}$ small relative to $T_f$. To simplify the analysis, we further assume that $N_h T_c + 2(\tau_{N_d} + T_w) < T_f/2$. The data sequence $\{d_m^{(v)}\}$ of user $v$ is an M-ary $(1 \leq d_m^{(v)} \leq M)$ symbol stream that conveys information in some form. Impulse radio is a fast hopping system, meaning that there are $N_s$ impulses transmitted per symbol. The data symbol changes only every $N_s$ hops, and assuming that a new data symbol begins with pulse index $k = 0$, the index of the data modulating pulse $k$ is $[k/N_s]$ (Here the notation $[q]$ denotes the integer part of $q$). If we define

$$C_m^{(v)}(t) = \sum_{k=mN_s}^{(m+1)N_s-1} T_s c_k^{(v)} p(t-kT_f),$$

$$p(t) = \begin{cases} 1, & \text{if } 0 \leq t \leq T_f \\ 0, & \text{otherwise} \end{cases}$$

and

$$S_i(t) = \sum_{k=0}^{N_s-1} w(t-kT_f - \delta_i^k)$$

for $i = 1, 2, \ldots, M$, then (1) can be rewritten

$$x^{(v)}(t) = \sum_{m=0}^{\infty} S_m^{(v)}(t-mN_s T_f - C_m^{(v)}(t))$$

$$= \sum_{m=0}^{\infty} X_m^{(v)}(t),$$

where $m$ indexes the transmitted symbols. Hence the user’s signal $x^{(v)}(t)$ is composed of a sequence of signals $X_m^{(v)}(t)$, where each frame-shifted $X_m^{(v)}(t)$ is a fast-hopped version of one of the $M$ possible PPM symbol waveforms $\{S_i(t)\}$. A single symbol waveform has duration $T_s = N_s T_f$. For a fixed $T_f$, the M-ary symbol rate $R_s = T_f^{-1}$ determines the number $N_s$ of impulses that are modulated by a given symbol. Note that when the hopping pattern in (2) is known, the signals $\{S_i(t)\}$ and $\{X_m^{(v)}(t)\}$ have the same correlation properties

$$R_{ij} \triangleq \int_{-\infty}^{\infty} X_{m_i}^{(v)}(\xi) X_{m_j}^{(v)}(\xi) d\xi$$

$$= \int_{-\infty}^{\infty} S_i(\xi) S_j(\xi) d\xi$$

$$= \delta_{ij} E_w \sum_{k=0}^{N_s-1} \gamma_w (\delta_i^k - \delta_j^k),$$

since for $k \neq l$ the pulses are non overlapping. The energy in the $i^\text{th}$ signal is $E_S = R_{ii} = N_s E_w$, and the normalized correlation value is

$$\alpha_{ij} = \frac{R_{ij}}{E_S} = \frac{1}{N_s} \sum_{k=0}^{N_s-1} \gamma_w (\delta_i^k - \delta_j^k) \geq \gamma_{\min} \forall i, j.$$

### C. PPM signals

The PPM signal $S_i(t)$ in (3) represents the $i^\text{th}$ signal in an ensemble of $M$ information signals, each signal completely identified by the sequence of time shifts $\{\delta_i^k; k = 0, 1, 2, \ldots, N_s - 1\}$.

One way to construct equally correlated (EC) signals $\{S_i(t)\}$ for $M \leq N_s$ consists of using the $(0,1)$ pattern of an m-sequence [4] of length $N_s$. Let $a_k^i$, $k = 0, 1, 2, \ldots, N_s - 1$ be the $i^\text{th}$ cyclic shift of the m-sequence, for $i = 1, 2, \ldots, M$. This pattern of 1’s and 0’s can be used to define the time shift pattern $\{\delta_i^k = a_k^i \tau_2; k = 0, 1, 2, \ldots, N_s - 1\}$ representing the $i^\text{th}$ signal. Hence, the EC PPM signals can be written

$$S_i(t) = \sum_{k=0}^{N_s-1} w(t-kT_f - a_k^i \tau_2)$$

for $i = 1, 2, \ldots, M$. It can be shown that the signals in (4) have normalized correlation values $\alpha_{ij} = 1$ and $\alpha_{ij} = \lambda_i, i \neq j$. By using $\tau_2 = \tau_{\min}$, the minimum value of $\lambda$ is

$$\lambda_{\min} = \frac{N_s-1}{N_s} \frac{\gamma_{\min}}{2} \approx 1 + \frac{\lambda_{\min}}{2} > 0 \text{ for } N_s \gg 1.$$

### D. Multiple-access interference model

The following assumptions are made to facilitate our analytical treatment.

(a) To estimate performance without choosing a hopping code, we assume that the elements $\{c_k^{(v)}\}$ for $v = 1, 2, \ldots, N_u$ and for all $k$, are independent,

The signal $S_i(t)$ is the received signal when $\int_{-\infty}^{T_f} S_i(\xi) d\xi$ is transmitted over the channel in the absence of noise and interference.
identically distributed random variables. Each $c_{k}^{(r)}$ is uniformly distributed on the interval $[0, N_k]$, and performance computation is based on signal-to-noise ratios averaged over the TH sequence variables.

(b) To ensure that no hopping code random variables occur more than once in a symbol time, we assume that $N_s \leq N_p$.

(c) Asynchronous transmission dictates that the transmission time differences $\tau_k - \tau_1$, $k = 2, \ldots, N_u$, are independent, identically distributed random variables, with $\tau_k - \tau_1 \pmod{T_f}$ being uniformly distributed on $[0, T_f]$.

(d) We assume that the received monocycle waveform satisfies the relation $\int_{-\infty}^{\infty} w(t) dt = 0$.

### III. System performance

#### A. Receiver signal processing

When $N_u$ links are active in this MA system, then the received signal $r(t)$ can be modeled as

$$r(t) = \sum_{\nu=1}^{N_u} A^{(\nu)} x^{(\nu)}(t - \tau^{(\nu)}) + n(t)$$

where $A^{(\nu)}$ is the attenuation of user $\nu$'s signal over the IR channel, $\tau^{(\nu)}$ represents time asynchronism between the clocks of user transmitter $\nu$ and the receiver, and the signal $n(t)$ represents non MA interference modeled as AWGN.

Let's assume that the receiver wants to demodulate the signal of user $\nu = 1$ corresponding to the $m$-th data symbol $d_m^{(1)} = j$. If only that user is present, then

$$r(t) = A^{(1)} X_{m,d_m^{(1)}}^{(1)} (t - \tau^{(1)}) + n(t)$$

When the receiver is perfectly synchronized to the first user signal, e.g., having learned the value of $\tau^{(1)}$, or at least $\tau^{(1)} \pmod{N_p T_f}$, the receiver can determine the sequence $\{T_m\}$ of time intervals, with interval $T_m$ containing the waveform representing data symbol $d_m^{(1)}$ (or $d_{\text{mon}}^{(1)}$). In this case the detection problem consists of deciding between $M$ equal energy signals in AWGN. Therefore, the optimum receiver consists of $M$ filters matched to the signals $\{X_{m,j}^{(1)}(t - \tau^{(1)})\}$, $t \in T_m = m N_p T_f + \tau^{(1)}, ((m+1) N_s - 1) T_f + \tau^{(1)}$, $j = 1, 2, \ldots, M$, followed by samplers and a decision circuit that selects the signal corresponding to the largest output.

When more than one link is active in the MA system, the optimal receiver is a complicated structure that takes advantage of all of the receiver's knowledge regarding the characteristics of the MA interference [5] [6]. For simplicity, we will use the M-ary correlation receiver even when there are many transmitters active, and the detection problem will be the coherent detection of $M$ equal energy signals in the presence of mean-zero $M$ equal energy signals in addition to AWGN.

#### B. Multiple-access system performance

When $N_u$ transmitters are active and the receiver wishes to determine the data modulating transmitter $\nu = 1$, the received signal $r(t)$ in (5) can be viewed as

$$r(t) = A^{(1)} X_{m,d_m^{(1)}}^{(1)} (t - \tau^{(1)}) + n_{\text{tot}}(t), \quad t \in T_m$$

where

$$n_{\text{tot}}(t) \triangleq \sum_{\nu=2}^{N_u} A^{(\nu)} x^{(\nu)}(t) + n(t)$$

includes both multiple-access interference and thermal noise, and is assumed to be a mean-zero Gaussian random process. Standard techniques [7] can then be used to calculate the union bound on the bit error probability $\text{UBP}_b$ for coherent detection of the EC TH PPM signals $X_{m,j}^{(1)}(t - \tau^{(1)})$, $j \in T_m$, $j = 1, 2, \ldots, M$. This bound is given by

$$\text{UBP}_b(N_u) = \frac{M}{2} \int_{-\infty}^{\infty} \frac{\text{exp}(-\xi^2/2)}{\sqrt{2\pi}} d\xi,$$

where

$$\text{SNR}_{\text{tot}}(N_u) = \frac{1}{\log_2(M)} \frac{m^2}{\sigma_{\text{tot}}^2(N_u)}$$

and

$$m = \int_{t \in T_m} A^{(1)} X_{m,j}^{(1)} (t - \tau^{(1)}) Y_{m,j;i}^{(1)} (t - \tau^{(1)}) dt$$

$$= A^{(1)} [E_s - R_{ij}] = A^{(1)} E_s [1 - \lambda] \quad \forall i \neq j,$$

where

$$Y_{m,j;i}^{(1)} (t - \tau^{(v)}) \triangleq [X_{m,j}^{(v)} (t - \tau^{(v)}) - X_{m,i}^{(v)} (t - \tau^{(v)})]$$

and

$$\sigma_{\text{tot}}^2(N_u) = \mathbb{E} \left\{ \left[ \int_{t \in T_m} n_{\text{tot}}(t) Y_{m,j;i}^{(1)} (t - \tau^{(1)}) dt \right]^2 \right\}$$

where $\mathbb{E}\{\cdot\}$ is the expected value operator. If only the desired transmitter is active, then

$$\sigma_{\text{tot}}^2(1) = \mathbb{E} \left\{ \left[ \int_{t \in T_m} n(t) Y_{m,j;i}^{(1)} (t - \tau^{(1)}) dt \right]^2 \right\}$$

$$= N_s E_s [1 - \lambda] \quad \forall i \neq j = \sigma_{\text{re}}^2$$
\[ \text{SNR}_{\text{b, out}}(1) = \frac{1}{\log_2(M)} \frac{(A^{(1)})^2 E_s [1 - \lambda_{\min}]}{N_0} \]

Thus, \( \text{SNR}_{\text{b, out}}(1) \) is equivalent to the output bit signal-to-noise-ratio (SNR) that one might observe in single-link communications. To complete the calculation of \( \text{SNR}_{\text{b, out}}(N_u) \), we must quantify the total noise variance \( \sigma_{\text{tot}}^2(N_u) \) and confirm that the mean of the total noise \( n_{\text{tot}}(t) \) is zero. This is done making use of the assumptions made in subsection II-D.

When the waveform \( w(t) \) is averaged over uniformly distributed random time shifts as in (c), then (d) in most situations gives that the mean value of the average is zero. Hence this condition is sufficient for \( E\{n_{\text{tot}}(t)\} = 0 \).

Assumptions (a) and (b) insure that the interference created by different monocoly transmitters is independent, and therefore

\[ \sigma_{\text{tot}}^2(N_u) = \sigma_{\text{re}}^2 + \sum_{\nu=2}^{N_u} (A(x)^2) \mathbb{E}\{n(x)^2\} \]

where

\[ n(x) = \int_{t \in T_m} x(x) (t - \tau(x)) y_{N_j,i}^1(t - \tau(x)) dt \]

For the particular waveforms and parameters that we have investigated we have found that

\[ \mathbb{E}\{n(x)^2\} \approx \frac{N_u}{2} \sigma_{\text{re}}^2 \quad \forall i \neq j \]

where

\[ \sigma_{\text{re}}^2 = \frac{1}{T_f} \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} w(t - s) [w(t) - w(t - \tau_2)] \right]^2 ds \]

Substituting (10) and (11) into (9), and using (13) and (14), we get

\[ \text{SNR}_{\text{b, out}}(N_u) = \left[ \text{SNR}_{\text{b, out}}(1) \right]^{-1} + \left[ \frac{1}{R_b} \frac{(E_s [1 - \gamma_{\min}])}{2T_f \sigma_{\text{re}}^2 \sum_{\nu=2}^{N_u} (A(x)^2)} \right]^{-1} \]

for use in the bit error probability bound in (8). In (15) we used the fact that the bit transmission rate \( R_b = \frac{\log_2(M)}{N_u T_f} \).

IV. RECEIVER IMPLEMENTATION

To detect the \( M \) signals we will need to correlate the input signal with \( M \) reference signals. For large \( M \) these can result in a receiver of great complexity. We can take advantage of the structure of the EC PPM signals to simplify the construction of the receiver. We illustrate this idea using the representation in (4). Let the received signal be \( r(t) \) in (6). In the receiver, each of the \( M \) channel correlation outputs can be written

\[ y_i = \int_{t \in T_m} r(t) X_{N_i,i}^1(t - \tau(1)) dt \]

where

\[ z_m(k) \text{ def } = \int_{kT_f + \tau(1)}^{(k+1)T_f + \tau(1)} r(t) w(t - kT_f - \tau(1) - \epsilon(1)) dt \]

for \( m = 1, 2 \). From the expression for \( y_i \), \( i = 1, 2, \ldots, M \), it is clear that the receiver needs only 2 correlators and \( M \) store and sum circuits. The \( y_i \) can be calculated while \( r(t) \) is received and no symbol delay occurs. This is illustrated in figure 1.
V. NUMERICAL EXAMPLE

In this section we illustrate the potential MA performance of this system for a specific design. In IR modulation, the UWB received impulse \( w(t) \) can be modeled by

\[
w(t) = \left[ 1 - 4\pi \left( \frac{t}{t_n} \right)^2 \right] \exp \left( -2\pi \left( \frac{t}{t_n} \right)^2 \right)
\]

where the value \( t_n = 0.4472 \) ns was used to fit the model \( w(t) \) to a measured waveform from a particular experimental IR link. The normalized signal correlation function corresponding to this pulse is

\[
\gamma_w(t) = \left[ 1 - 4\pi \left( \frac{t}{t_n} \right)^2 + \frac{4\pi^2}{3} \left( \frac{t}{t_n} \right)^4 \right] \exp \left( -\pi \left( \frac{t}{t_n} \right)^2 \right)
\]

In this case \( \tau_{\text{min}} = 0.2419 \) ns and \( \gamma_{\text{min}} = -0.6183 \).

Given the pulse \( w(t) \), the proper signal design depends on the good choice of \( \tau_2 \) in (4). If \( (\text{SNR}_{\text{out}}(1))^{-1} \) dominates in the denominator in (15) (e.g., when \( N_u = 1 \) or the AWGN dominates), then the value \( \tau_2 = \tau_{\text{min}} \) (i.e., \( \lambda = \lambda_{\text{min}} \)) can be shown to minimize the probability bound in (8). On the other hand, when MA noise dominates in \( \text{SNR}_{\text{out}}(N_u) \), then \( \sigma_n^2/[1 - \lambda]^2 \) is the quantity that should be minimized by the proper choice of \( \tau_2 \).

Figure (2) shows the MA performance curves. The curves represent the bit error probability for the case in which the one-user bit SNR value \( \text{SNR}_{\text{out}}(1) = 10.5 \) dB, so that without MA noise \( \text{UBP}(1) \approx 4 \times 10^{-4} \) with \( M=2 \). The curves were calculated using \( \tau_2 = \tau_{\text{min}} = 0.2419 \) ns and \( T_f = 100 \) ns. Perfect power control (i.e., \( A^{(\nu)} = A^{(1)} \) for \( \nu = 1, 2, \ldots, N_u \)) was assumed. Similar curves can be calculated using high data transmission rates, in which the system is able to support hundreds of users transmitting over a Megabit per second with bit error probability in the range \( 10^{-5} \) to \( 10^{-8} \).

VI. CONCLUSIONS

Using EC PPM sets with large values of \( M \), it is possible to increase the number of users supported by the system for a given multiple-access performance and bit transmission rate, making efficient use of the signal-to-noise-ratio available. Furthermore, we can take advantage of the structure of the EC PPM signals to reduce the complexity of the receiver to 2 correlators and \( M \) store and sum circuits without introducing a symbol delay. This analysis shows that impulse radio is potentially able to provide multiple-access communications with a combined transmission capacity of over 500 Megabits per second at bit error rates in the range \( 10^{-4} \) to \( 10^{-8} \) using receivers of moderate complexity. For communications in the presence of multipath, the greatest potential for impulse radio comes from the fine time resolution produced by the subnanosecond pulses [8]. Propagation paths with differential delays in the order of this impulse width or more can be resolved and coherently combined using a Rake receiver, hence combating the normally degrading effects of multipath.

REFERENCES