System Performance Analysis of Impulse Radio Modulation

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ABSTRACT— **In this paper we present a system performance analysis of impulse radio modulation using block waveform encoding signal sets. We analyze both the multiple-access performance in a channel with additive white Gaussian noise, and the single-link performance in a channel with dense multipath.**

I. Introduction

Impulse radio (IR) is a spread-spectrum multiple-access (MA) technique that uses impulse signal technology to generate ultra-wideband (UWB) communication signals[1] that consist of trains of time-shifted subnanosecond impulses [1]. Data is transmitted using pulse-position-modulation (PPM) at a rate of many pulses per symbol, and multiple-access (MA) capability is achieved using spread spectrum time hopping (TH). The UWB nature of IR makes it convenient for MA communications in radio channels impaired with dense multipath (such as the wireless indoor channel [2]). With UWB TH PPM signals the dense multipath can be resolved, allowing the use of a Rake receiver [3] for signal demodulation. The radio links can be operated with reduced fade margins and the UWB TH PPM waveforms can be received by correlation detection literally at the antenna terminals, making a relatively simple and low-cost, low-power transceiver viable [1]

Although an IR system and a CDMA system operating with the same bandwidth can be shown to be quite comparable when used in a MA environment, the current impulse technology gives an advantage to IR on the basis of achievable effective processing gains for the two systems. In IR high processing gains are automatically achievable with the use of subnanosecond pulses, allowing a large number of users to be accommodated in the system.

When compared with other technologies capable of supporting G-Hz bandwidths, IR has an advantage over infrared technology since radio communication can easily penetrate the structure of buildings facilitating wireless communications. Impulse radio potentially is cheaper than millimeter wave communications for the same short-range communications environment.

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[1]The range of frequencies occupied by the UWB impulse goes from a few hundreds of Kilohertz up to a few Gigahertz.

In this work we present a system performance analysis of IR using block waveform encoding signal sets. We analyze the multiple-access performance in a channel with additive white Gaussian noise (AWGN), and the single-link performance in a channel with dense multipath.

II. Multiple-access performance in additive noise

In this section the multiple-access performance is analyzed in terms of bit error rate for a given number of users and bit transmission rate. All the users transmit asynchronously and the desired user signal is detected using a single-channel correlation receiver for coherent detection of the symbol waveform.[2]

A. Channels and signals description

A.1 IR-AWGN channel model

Wireless IR channel with free space propagation conditions and AWGN. In this model the transmitted impulse is $p_{\text{TX}}(t) \triangleq \int_{-\infty}^{t} p(\xi)d\xi$ and the received signal is $p(t) + n(t)$.[3] The noise $n(t)$ is AWGN with two-sided power density $\frac{N_o}{2}$. The signal $p(t)$ is the basic subnanosecond impulse used to convey information. It has duration T_p and energy $E_p = \int_{-\infty}^{\infty} [p(t)]^2\, dt$. The normalized signal correlation function of $p(t)$ is $\gamma_p(\tau) \triangleq (1/E_p)\int_{-\infty}^{\infty} p(t)p(t-\tau)dt > -1\ \forall\tau$. We define $\gamma_{\min} = \gamma_p(\tau_{\min})$ as the minimum value of $\gamma_p(\tau)$, $\tau \in (0, T_p]$.

B. PPM signals in IR-AWGN

The PPM signals received over the IR-AWGN channel consist of N_s time-shifted impulses

$$\Psi_j(t) = \sum_{k=0}^{N_s-1} p(t - kT_f - \rho_j^k), \quad j = 1, 2, \ldots, M. \quad (1)$$

Here $\Psi_j(t)$ represents the j^{th} signal in an ensemble of signals, each signal completely identified by the sequence of time shifts $\{\rho_j^k; k = 0, 1, 2, \ldots, N_s - 1\}$. Each signal $\Psi_j(t)$ has duration $T_s \triangleq N_s T_f$ and energy $E_\Psi = N_s E_p$. In IR, the impulse duration satisfies $T_p << T_f$, where T_f is the time shift value corresponding to the frame period; and the time shift corresponding to the data modulation is $\rho_j^k \in \{\tau_1 < \tau_2 < \ldots < \tau_{\text{N}}\}$

[2]This approach is analogous to the one used in [4].

[3]The effect of the antenna system in the UWB transmitted impulse is modeled as a derivation operation.

for some $N \geq 2$. The signals $\{\Psi_j(t)\}$ have normalized correlation values

$$\beta_{ij} \triangleq \frac{\int_{-\infty}^{\infty} \Psi_i(t)\, \Psi_j(t)\, dt}{\int_{-\infty}^{\infty} \Psi_i^2(t)\, dt}$$

C. TH PPM signals

In this multiple access system each user's signal is composed of a sequence of fast-hopped frame-shifted versions of one of the M possible PPM symbol waveforms $\{\Psi_j(t)\}$

$$x^{(\nu)}(t) = \sum_{m=0}^{\infty} \Psi_{d_m^{(\nu)}}(t - mN_sT_f - C_m^{(\nu)}(t)),$$

where the superscript (ν), $(1 \leq \nu \leq N_u)$ indicates user-dependent quantities, N_u is the number of users, m indexes the transmitted symbols, $d_m^{\nu} \in \{1, 2, \ldots, M\}$ is the m^{th} transmitted symbol, and

$$C_m^{(\nu)}(t) \triangleq \sum_{k=mN_s}^{(m+1)N_s-1} T_c\, c_k^{(\nu)} \phi(t - kT_f),$$

$$\phi(t) = \begin{cases} 1, & \text{if } 0 \leq t \leq T_f \\ 0, & \text{otherwise} \end{cases}.$$

here T_c is a time-shift value, and $\{c_k^{(\nu)}\}$ is the pseudo-random time-hopping sequence assigned to user ν, with $0 \leq c_k^{\nu} \leq N_h$ for some integer N_h. To simplify the analysis, we further assume that $N_hT_c + 2(\tau_{\text{N}} + T_p) < T_f/2$. For a fixed T_f, the M-ary symbol rate $R_s = T_s^{-1}$ determines the number $N_s >> 1$ of impulses that are modulated by a given symbol.

D. Multiple-access interference model

The following assumptions are made to facilitate our analytical treatment.

(a) To estimate performance without choosing a hopping code, we assume that the elements $\{c_k^{(\nu)}\}$ for $\nu = 1, 2, \ldots, N_u$ and for all k, are independent, identically distributed (iid) random variables with uniform distribution on $[0, N_h]$.

(b) The transmission time differences $\tau_k - \tau_1$, $k = 2, \ldots, N_u$, are iid random variables, with $\tau_k - \tau_1 \bmod T_f$ being uniformly distributed on $[0, T_f]$.

(d) We assume that the received monocycle waveform satisfies the relation $\int_{-\infty}^{\infty} p(t)dt = 0$.

E. Multiple-access system performance

When N_u links are active in this MA system, the received signal $r(t)$ can be modeled as

$$r(t) = \sum_{\nu=1}^{N_u} A^{(\nu)} x^{(\nu)}(t - \tau^{(\nu)}) + n(t) \qquad (2)$$

where $A^{(\nu)}$ is the attenuation of user ν's signal over the IR channel, $\tau^{(\nu)}$ represents clock asynchronisms between user ν transmitter and the receiver, and the signal $n(t)$ represents non MA interference modeled as AWGN. Let's assume that the receiver wishes to determine the data modulating transmitter $\nu = 1$. The received signal $r(t)$ in (2) can be viewed as

$$r(t) = A^{(1)} \Psi_{d_m^{(\nu)}}(t - \tau^{(1)}) - mN_sT_f - C_m^{(\nu)}(t)) + n_{\text{tot}}(t),$$

for $t \in \mathcal{T}_m$, where

$$n_{\text{tot}}(t) \triangleq \sum_{\nu=2}^{N_u} A^{(\nu)} x^{(\nu)}(t) + n(t) \qquad (3)$$

includes both MA interference and thermal noise, and is assumed to be a mean-zero Gaussian random process. Standard techniques [5] can then be used to calculate the union bound on the bit error probability UBPb for coherent detection of *equally correlated* TH PPM signals. This bound is given by

$$\text{UBPb}(N_u) = \frac{M}{2} \int_{\sqrt{\mathbf{SNRb}_{\text{MA}(N_u)}}}^{\infty} \frac{\exp(-\xi^2/2)}{\sqrt{2\pi}}\, d\xi,$$

where [1] [6]

$$\mathbf{SNRb}_{\text{MA}}(N_u) = \left[[\mathbf{SNRb}_{\text{ONE}}(1)]^{-1} + \left[\frac{1}{R_b} \frac{(2E_p[1-\beta])^2}{2T_f \sigma_a^2 \sum_{\nu=2}^{N_u} \left(\frac{A^{(\nu)}}{A^{(1)}}\right)^2} \right]^{-1} \right]^{-1},$$

$$\sigma_a^2 = \frac{1}{T_f} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} p(t-s)[p(t) - p(t - \tau_{\min})]dt \right]^2 ds,$$

and

$$\mathbf{SNRb}_{\text{ONE}}(1) = \frac{1}{\log_2(M)} \frac{(A^{(1)})^2 E_s[1-\beta]}{N_o}.$$

The $\mathbf{SNRb}_{\text{MA}}(N_u)$ and $\mathbf{SNRb}_{\text{ONE}}(1)$ are equivalent to the output bit signal-to-noise-ratio (SNR) that one might observe in multiple-users and single-link communications, respectively. The $\beta \triangleq \frac{1+\gamma_{\min}}{2}$ is the normalized correlation value of the PPM equally correlated signals[4]. Clearly, the σ_a depend on $p(t)$ and $\tau_{\min}$, and therefore is dependent on the signal design. Note that the bit transmission rate $R_b = \frac{log_2(M)}{N_sT_f}$.

III. EXAMPLE

In this section we illustrate the potential MA performance of this system for a specific design. In IR modulation, the UWB received impulse $p(t)$ can be modeled by

$$p(t) = \left[1 - 4\pi\left[\tfrac{t}{t_n}\right]^2\right] \exp\left(-2\pi\left[\tfrac{t}{t_n}\right]^2\right)$$

where the value $t_n = 0.4472$ ns was used to fit the model $p(t)$ to a measured waveform $p_n(t)$ from a particular experimental IR link. The normalized signal correlation function corresponding to $p(t)$ is

$$\gamma_p(t) = \left[1 - 4\pi\left[\tfrac{t}{t_n}\right]^2 + \tfrac{4\pi^2}{3}\left[\tfrac{t}{t_n}\right]^4\right] \exp\left(-\pi\left[\frac{t}{t_n}\right]^2\right).$$

[4]For construction of PPM equally correlated signals see [7].

In this case $\tau_{\min} = 0.2419$ ns and $\gamma_{\min} = -0.6183$.

Figure (1) shows the MA performance curves. The curves represent the bit error probability with different values of M for the case in which the one-user bit SNR value $\mathbf{SNRb}_{\mathrm{ONE}}(1) = 10.5$ dB, so that without MA noise UBPb(1) $\simeq 4 \times 10^{-4}$ with M=2. The curves were calculated using $T_f = 100$ ns and $R_b = 9.6$ Kbps for each user. Perfect power control (i.e. $A^{(\nu)} = A^{(1)}$ for $\nu = 1, 2, \ldots, N_u$) was assumed.

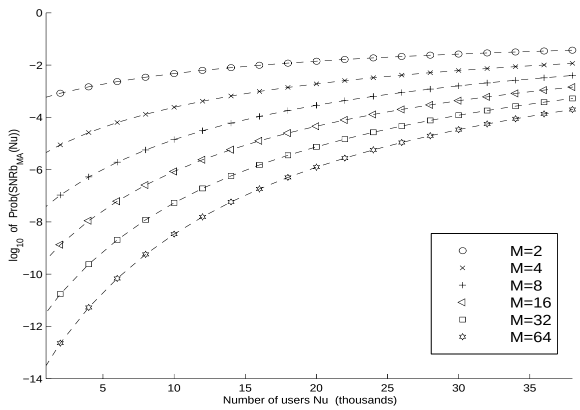

Fig. 1. The base 10 logarithm of the probability of bit error, as a function of number of simultaneous users N_u for different values of M under perfect power control conditions.

Similar curves can be calculated using high data transmission rates, in which the system is able to support hundreds of users transmitting over a Megabit per second with bit error probability in the range 10^{-5} to 10^{-8}.

IV. Single-link performance in dense multipath

In this section we make an assessment of the performance of IR modulation in an indoor multipath environment with detection using a Rake receiver. In the present analysis we assume one user and perfect synchronization. Under these circumstances, the spread spectrum time-hopping sequence modulation has no effect on the correlation properties of the communication signals, and will be omitted in the expressions defining the signals and their correlation values.

A. IR-MP channel description

Wireless indoor IR multipath channel. In this model the transmitted signal is $p_{\mathrm{TX}}(t)$ and the received signal is $\sqrt{\overline{E}_p}\tilde{p}(u,t) + n(t)$. The pulse $\sqrt{\overline{E}_p}\tilde{p}(u,t)$ is a time spreaded version of $p(t)$, with duration $T_m >> T_p$ and average energy $\overline{E}_p$. The u indexes an event taking place in the sample space of a certain random experiment. Hence, $\tilde{p}(u,t)$ is a random process.[5] For performance analysis purpose, it will be assumed that the IR-MP channel can be characterized by the ensemble of pulses responses

$$\{\tilde{p}(u_o,t)\}\,,\; u_o = 1, 2, \ldots, u_*.$$

The normalized signal correlation function of $\tilde{p}(u_o,t)$ is

$$\gamma_{\mathrm{MP}}(u_o,\tau) \triangleq \frac{\int_{-\infty}^{\infty} \tilde{p}(u_o,t)\tilde{p}(u_o,t-\tau)dt}{\int_{-\infty}^{\infty} [\tilde{p}(u_o,t)]^2 dt}$$

B. PPM signals in IR-MP

The PPM signals received over the IR-MP channel consist of N_s time-shifted impulses

$$\tilde{\Psi}_j(u_o,t) = \sum_{k=0}^{N_s-1} \tilde{p}(u_o, t - kT_f - \rho_j^k), \quad j = 1, 2, \ldots, M.$$

Each signal $\tilde{\Psi}_j(u_o,t)$ has duration $\tilde{T}_s \triangleq N_s T_m$, energy $\tilde{E}_\Psi(u) = \int_{-\infty}^{\infty}[\tilde{\Psi}(u_o,\xi)]^2 d\xi$, and average energy $\overline{E}_\Psi = N_s \overline{E}_p$. For the multipath analysis we further assume the channel varies slowly with respect to $\tilde{T}_s$, and that $\tau_{\mathrm{N}} + T_m < T_f$. The signals $\{\tilde{\Psi}(u_o,t)\}$ have normalized correlation values

$$\tilde{\beta}_{ij}(u_o) \triangleq \frac{\int_{-\infty}^{\infty} \tilde{\Psi}_i(u_o,\xi)\, \tilde{\Psi}_j(u_o,\xi)\, d\xi}{\int_{-\infty}^{\infty} [\tilde{\Psi}_i(u_o,\xi)]^2\, d\xi}.$$

C. Single-link multipath performance

When the signal $\int_{-\infty}^{t} \Psi_j(\xi)d\xi$ is transmitted over the IR-MP channel, the received signal in a symbol interval $\tilde{T}s = N_s T_f$ can be written

$$r(u_o,t) = \tilde{\Psi}_j(u_0,t) + n(t),\; 0 \le t \le \tilde{T}_s$$

Conditioned on the measurement location u_o, the received signal consists of the communication signal $\tilde{\Psi}_j(u_0,t)$ plus AWGN (of course, for different u_o, the sets of received signals are different). For every u_o, the *optimum* receiver is a bank of filters matched to the M signals $\tilde{\Psi}_j(u_0,t)$, $j = 1, 2, \ldots, M$. The problem is that this receiver must be able to match the random variations in the received signal for every possible value u_o. With this motivation, we introduce the concept of *perfect Rake* (PRake) receiver, a super Rake receiver that has an unlimited number of correlation resources and is able to construct a reference signal $\tilde{\Psi}_j(u_0, \tilde{T}_s - t)$ that is perfectly matched to the signal received $\tilde{\Psi}_j(u_0,t)$ over the multipath channel. For every u_o, performance analysis using the PRake receiver can be calculated, and the average performance can be obtained by averaging over all values of u_o. The PRake receiver provides a theoretical bound for the best performance attainable.

Conditioned on $u = u_o$, the union bound on the bit error probability Pe for *equally correlated* UWB PPM signals detected with the PRake receiver can be written

$$\mathrm{U\tilde{B}Pb}(u_o) = \frac{M}{2} \int_{\sqrt{\mathbf{SNRb}_{\mathrm{MP}(u_o)}}}^{\infty} \frac{\exp(-\xi^2/2)}{\sqrt{2\pi}}\, d\xi,$$

[5] For the IR-MP channel the random experiment is a measurement experiment performed in an office building where $\tilde{p}(u_o,t)$ denotes the IR-MP channel pulse response to $p_{\mathrm{TX}}(t)$, measured at position u_o in the absence of noise.

where [8]

$$\mathbf{SNRb}_{\mathrm{MP}}(u_o) = \frac{\tilde{E}_\Psi(u_o)}{N_o}\log_2(M)(1-\tilde{\beta}(u_o)).$$

The $\tilde{E}_\Psi(u_o)$ value accounts for variations in the received signal energy due to fading caused by multipath. The $\tilde{\beta}(u_o) \triangleq \frac{1+\gamma_{\mathrm{MP}}(u_o,\tilde{\tau}_{\min})}{2}$ is the normalized correlation value of the UWB PPM equally correlated signals. The $\tilde{\beta}(u_o)$ value accounts for distortions in the signal correlation function caused by multipath [9].

The value $\tilde{\mathrm{UBPb}}(u_o)$ is conditioned on the event $u = u_o$, and depends on $\tilde{\Psi}_j(u_o,t)$ (i.e, $\tilde{p}(u_o,t)$), the channel waveform measured in the absence of noise at position u_o. Taking the expected value $\mathbf{E}_u\{\cdot\}$ with respect to u over all positions

$$\overline{\mathrm{UBPb}}\left(\frac{\overline{E}_\Psi}{N_o}\right) = \mathbf{E}_u\{\tilde{\mathrm{UBPb}}\,(u)\}$$

where

$$\left(\frac{\overline{E}_\Psi}{N_o}\right) \triangleq \mathbf{E}_u\left\{\mathbf{SNRb}_{\mathrm{MP}}(u)\right\}$$

is the average received symbol SNR.

D. EXAMPLE

The expected value $\overline{\mathrm{UBPb}}\left(\frac{\overline{E}_\Psi}{N_o}\right)$ can be approximated by the sample mean value

$$\overline{\mathrm{UBPb}}\left(\frac{\overline{E}_\Psi}{N_o}\right) \approx \frac{1}{u_*}\sum_{u_o=1}^{u_*}\tilde{\mathrm{UBPb}}\,(u_o)$$

Figure (2) shows the curves for $\overline{\mathrm{UBPb}}\left(\frac{\overline{E}_\psi}{N_o}\right)$, $M = 2,4,8,16,32,64$, using the same impulse shape $p(t)$ used in the MA example. In this case the value $t_n = 0.7531$ ns was used to fit the model $p(t)$ to the measured waveform $p_m(t)$. The UWB impulse $p_m(t)$ is a unitary-energy template with duration $\tilde{T}_p = 1.5$ ns that was taken from a multipath-free and noise-free measurement. The resultant correlation function of $p(t)$ has a minimum $\gamma_{\min} = -0.6183$ as before, but the minimum is attained at the time shift value $\tilde{\tau}_{\min} = 0.7531$ ns. The channel pulse responses $\tilde{p}(u_o,t), u_o = 1,2,\ldots,u_* = 392$ come from signal propagation data recorded in an UWB measurements experiment [10].

V. Conclusions

Results from this analysis show that for applications requiring high data rate (1024 Kbps) combined with low probability of bit error (10^{-6}), IR modulation is potentially able to support hundreds of users. Similarly , for applications requiring low data rate (9.6 Kbps) and moderate probability of error (10^{-4}), IR is potentially able to support dozens of thousands of users. In either case, the combined transmission rates give a transmission capacity near to 500 Megabits per second using receivers of moderate complexity.

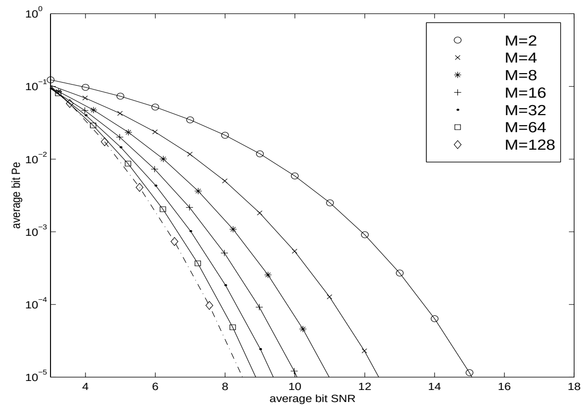

Fig. 2. The $\overline{\mathrm{UBPb}}\left(\frac{\overline{E}_\Psi}{N_o}\right)$ Curves for $M = 2,4,8,16,32,64$ signals.

The real payoff of IR will be in wireless transmission in multipath channels. This analysis show that for a single-link case and symbol error probability of 10^{-3}, the performance in the presence of multipath using an idealized Rake receiver is, on average, a few dB worse than performance in the absence of multipath.

These results support the conclusion that IR will have good multiple access performance even in dense multipath channels.

VI. Acknowledgments

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