Multiple-Access Performance Limits with Time Hopping and Pulse Position Modulation

Fernando Ramírez-Mireles and Robert A. Scholtz
Communication Sciences Institute, University of Southern California

ABSTRACT— **The use of spread spectrum time hopping in combination with pulse position modulation for multiple access communications is studied. The multiple access performance is described in terms of the number of users supported by the system for a given bit error rate and bit transmission rate. Expressions for the maximum number of users and maximum transmission capacity in bits per second are found. Asymptotic values for these quantities are derived.**

I. INTRODUCTION

Spread Spectrum multiple-access (MA) communication using time hopping (TH) modulation and impulse signal technology was proposed in [1]. This communication technique is called impulse radio (IR) [2]. Impulse radio modulation uses impulse signal technology to generate ultra-wideband (UWB) communication signals[1] that consist of trains of time-shifted subnanosecond impulses. Data is transmitted using pulse-position-modulation (PPM) at a rate of many pulses per symbol, and MA capability is achieved using spread spectrum time hopping (TH). Impulse radio promises to be a viable technique to build relatively simple and low-cost, low-power transceivers that can be used for short range, high speed MA communications over the multipath indoor wireless channel [3].

In [1] the MA performance of IR assuming free space propagation conditions and additive white Gaussian noise (AWGN) was studied. The analysis assumed that binary PPM signals based on binary pulse-position-modulation are shift-coherent detected using a single-channel correlation receiver.[2] In [5] the use of block-waveform encoding PPM signals to increase the number of users supported by the system for a given MA performance and bit transmission rate was investigated. A ten-fold increase in the number of users was shown to be achievable using a receiver of moderate complexity. In this paper we remove the receiver complexity constraint, and proceed to derive expressions for the maximum number of users and maximum transmission capacity in bits per second, calculating their respective asymptotic values.

II. Channel, signals and multiple-access interference models

A. Channel and impulse signal models

The model assumed is a free-space propagation channel impaired with AWGN. The transmitted impulse is $w_{tx}(t) \triangleq \int_{-\infty}^{t} w(\xi)d\xi$ and the received signal is $w(t)+n(t)$.[3] The noise $n(t)$ is AWGN with two-sided power density $\frac{N_o}{2}$. The signal $w(t)$ is the basic subnanosecond impulse used to convey information. It has duration T_w and energy $E_w = \int_{-\infty}^{\infty}[w(t)]^2\,dt$. The normalized signal correlation function of $w(t)$ is $\gamma_w(\tau) \triangleq (1/E_w)\int_{-\infty}^{\infty} w(t)w(t-\tau)dt > -1\ \forall\tau$. We define $\gamma_{\min} = \gamma_w(\tau_{\min})$ as the minimum value of $\gamma_w(\tau)$, $\tau \in (0, T_w]$.

B. PPM signals

The PPM signals consist of N_s time-shifted impulses

$$S_j(t) = \sum_{k=0}^{N_s-1} w(t - kT_f - \delta_j^k), \quad j = 1, 2, \ldots, M.$$

Notice that information is conveyed exclusively in the sequence of time shifts $\{\delta_j^k; k = 0, 1, 2, \ldots, N_s - 1\}$, with $\delta_j^k \in \{0 = \tau_1 < \tau_2 < \ldots < \tau_N\}$ for some $N \geq 2$. In IR, the impulse duration satisfies $T_p << T_f$, where T_f is the time shift value corresponding to the frame period. A single symbol waveform has duration $T_s \triangleq N_s T_f$, with $N_s >> 1$. The M-ary symbol rate is $R_s = T_s^{-1}$. The signals $\{S_j(t)\}$ have correlation values

$$R_{ij} = \int_{-\infty}^{\infty} S_i(\xi)\, S_j(\xi)\, d\xi = E_w \sum_{k=0}^{N_s-1} \gamma_w(\delta_i^k - \delta_j^k),$$

since for $k \neq l$ the pulses are non overlapping. The energy in the j^{th} signal is $E_S = R_{ii} = N_s E_w$, and the normalized correlation value is

$$\alpha_{ij} \triangleq \frac{R_{ij}}{E_S} = \frac{1}{N_s} \sum_{k=0}^{N_s-1} \gamma_w(\delta_i^k - \delta_j^k) \geq \gamma_{\min}\, \forall\tau.$$

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The authors's E-mail are {ramirezm,scholtz}@milly.usc.edu.

[1] The range of frequencies occupied by the UWB impulse goes from a few hundreds of Kilohertz up to a few Gigahertz.

[2] This approach is analogous to the one used in [4].

[3] The effect of the antenna system in the UWB transmitted impulse is modeled as a derivation operation.

As in [5], we will work with equally correlated (EC) signals with

$$\alpha_{ij} = \lambda \approx \frac{1+\gamma_w(\tau_2)}{2} > 0 \text{ for } N_s >> 1$$

with $0 < \tau_2 < T_w$.[4]

C. TH PPM signals

Each user's signal is composed of a sequence of fast-hopped frame-shifted versions of one of the M possible PPM symbol waveforms $\{S_j(t)\}$

$$x^{(\nu)}(t) = \sum_{m=0}^{\infty} S_{d_m^{(\nu)}}(t - mN_sT_f - C_m^{(\nu)}(t)),$$

where the superscript (ν), $(1 \le \nu \le N_u)$ indicates user-dependent quantities, N_u is the number of users, m indexes the transmitted symbols, $d_m^{\nu} \in \{1, 2, \ldots, M\}$ is the m^{th} transmitted symbol, and

$$C_m^{(\nu)}(t) \triangleq \sum_{k=mN_s}^{(m+1)N_s-1} T_c\, c_k^{(\nu)} \phi(t - kT_f),$$

$$\phi(t) = \begin{cases} 1, & \text{if } 0 \le t \le T_f \\ 0, & \text{otherwise} \end{cases}.$$

Here T_c is a time-shift value and $\{c_k^{(\nu)}\}$ is the pseudo-random time-hopping sequence assigned to user ν, with $0 \le c_k^{(\nu)} \le N_h$ for some integer N_h. The $\{c_k^{(\nu)}\}$ has periodicity N_p, i.e., $c_{k+lN_p}^{(\nu)} = c_k^{(\nu)}$ for k, l, N_p integers.

D. Multiple-access interference model

In the present analysis, performance computation is based on signal-to-noise ratios (SNR) averaged over random TH sequences $\{c_k^{(\nu)}\}$ and random asynchronous transmission times $\{\tau^{(\nu)}\}$. To this end the following assumptions were made:

(a) The elements $\{c_k^{(\nu)}\}$ for $\nu = 1, 2, \ldots, N_u$ and for all k, are independent, identically and uniformly distributed on the interval $[0, N_h]$. Furthermore, we assume that $N_s \le N_p$.

(b) The transmission time differences $\tau^{(\nu)} - \tau^{(1)} \bmod T_f$, $\nu = 2, \ldots, N_u$, are independent, identically and uniformly distributed on $[0, T_f]$.

(d) We assume that the received monocycle waveform satisfies the relation $\int_{-\infty}^{\infty} w(t)dt = 0$.

III. System performance

A. Multiple-access performance

When N_u transmitters are active and the receiver wishes to determine the data modulating transmitter $\nu = 1$, the received signal $r(t)$ can be viewed as

$$r(t) = A^{(1)} S_{d_m^{(1)}}(t - mN_sT_f - C_m^{(1)}(t) - \tau^{(1)}) + n_{\text{TOT}}(t),$$

$t \in \mathcal{T}_m \triangleq [mN_sT_f + \tau^{(1)}, ((m+1)N_s - 1)T_f + \tau^{(1)}]$, where

$$n_{\text{TOT}}(t) \triangleq \sum_{\nu=2}^{N_u} A^{(\nu)} x^{(\nu)}(t) + n(t)$$

includes both MA interference and thermal noise, and is assumed to be a mean-zero Gaussian random process. Standard techniques [7] can then be used to calculate the M-ary bit error probability P_e for coherent detection of the EC TH PPM signals. This P_e values is

$$P_e(N_u) = \frac{M}{M-1}\left[1 - \int_{-\infty}^{\infty}\left(1 - Q\left[\xi + \sqrt{2\log_2(M)\,\mathbf{SNRb}_{\text{out}}(N_u)}\right]\right)^{M-1}\right] \quad (1)$$

where $Q[\xi]$ is the Gaussian-tail integral, and [5]

$$\mathbf{SNRb}_{\text{out}}(N_u) = \left[[\mathbf{SNRb}_{\text{out}}(1)]^{-1} + \left[\frac{1}{R_b}\frac{\mu/T_f}{\sum_{\nu=2}^{N_u}\left(\frac{A^{(\nu)}}{A^{(1)}}\right)^2}\right]^{-1}\right]^{-1} \quad (2)$$

is the output bit SNR that the desired user observe in the presence of $N_u - 1$ other users,

$$\mu = \frac{(E_w[1 - \gamma_w(\tau_2)])^2}{2\sigma_a^2}$$

is a normalized SNR parameter which is defined in terms of the pulse shape $w(t)$ and the time shift τ_2,

$$\sigma_a^2 = \frac{1}{T_f}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} w(t-s)[w(t) - w(t-\tau_2)]dt\right]^2 ds,$$

and

$$\mathbf{SNRb}_{\text{out}}(1) = \frac{1}{\log_2(M)}\frac{(A^{(1)})^2 E_s[1-\lambda]}{N_o}$$

is equivalent to the output bit SNR that the desired user might observe in single-link communications. The substitution of the bit SNR $\mathbf{SNRb}_{\text{out}}(N_u)$ in (2) into the bit error probability $P_e(N_u)$ in (1) will provide the desired relation between bit error probability P_e, number of users N_u and bit transmission rate R_b.

B. Multiple-access degradation factor

Let's define $\mathbf{SNRb}_{\text{spec}}$ to be the specified operating bit SNR to achieve the desired probability of error. Recall that $\mathbf{SNRb}_{\text{out}}(1)$ is the bit SNR value when only user one is active, and that $\mathbf{SNRb}_{\text{out}}(N_u) < \mathbf{SNRb}_{\text{out}}(1)$ is the actual bit SNR when N_u users are active in the system.

[4] For construction of PPM equally correlated signals see [6].

The $\mathbf{SNRb}_{\text{rec}}(N_u) > \mathbf{SNRb}_{\text{spec}}$ is the required value of $\mathbf{SNRb}_{\text{out}}(1)$ that makes $\mathbf{SNRb}_{\text{out}}(N_u) = \mathbf{SNRb}_{\text{spec}}$, so user one can still meet the specified value of bit error probability even when N_u users are active. The value of $\mathbf{SNRb}_{\text{rec}}(N_u)$ can be calculated solving

$$\mathbf{SNRb}_{\text{spec}} = \frac{\mathbf{SNRb}_{\text{rec}}(N_u)}{1+\mathbf{SNRb}_{\text{rec}}(N_u)\left[\frac{1}{R_b}\frac{\mu/T_f}{\sum_{\nu=2}^{N_u}\left(\frac{A^{(\nu)}}{A^{(1)}}\right)^2}\right]^{-1}} \quad (3)$$

to get

$$\mathbf{SNRb}_{\text{rec}}(N_u) = \frac{\mathbf{SNRb}_{\text{spec}}}{1-\mathbf{SNRb}_{\text{spec}}\left[\frac{1}{R_b}\frac{\mu/T_f}{\sum_{\nu=2}^{N_u}\left(\frac{A^{(\nu)}}{A^{(1)}}\right)^2}\right]^{-1}}$$

The ratio

$$\begin{aligned}\mathrm{DF}(N_u) &= \frac{\mathbf{SNRb}_{\text{rec}}(N_u)}{\mathbf{SNRb}_{\text{spec}}(1)} \\ &= \frac{1}{1-\mathbf{SNRb}_{\text{spec}}\left[\frac{1}{R_b}\frac{\mu/T_f}{\sum_{\nu=2}^{N_u}\left(\frac{A^{(\nu)}}{A^{(1)}}\right)^2}\right]^{-1}} \quad (4)\end{aligned}$$

is a degradation factor that measures the additional amount of SNR required by user one to overcome the negative effect of the multiple-access interference caused by the N_u users. Under ideal power control conditions (i.e., when $A^{(\nu)} = A^{(1)}$ for $\nu = 2, 3, \ldots, N_u$), $\mathrm{DF}(N_u)$ in (4) can be written

$$\mathrm{DF}(N_u) = \frac{1}{1-\mathbf{SNRb}_{\text{spec}}\left[\frac{1}{R_b}\frac{\mu/T_f}{(N_u-1)}\right]^{-1}}. \quad (5)$$

The expression in (5) gives $\mathrm{DF}(N_u)$ as a function of N_u. It is also possible to get an expression for $N_u(\mathrm{DF})$ as a function of DF as follows

$$N_u(\mathrm{DF}) = \frac{1}{\mathbf{SNRb}_{\text{spec}}}\frac{1}{R_b}\frac{\mu}{T_f}(1-\frac{1}{\mathrm{DF}})+1. \quad (6)$$

The maximum number of users is

$$N_{\max} \triangleq \lim_{\mathrm{DF}\to\infty} N_u(\mathrm{DF}) = \frac{1}{\mathbf{SNRb}_{\text{spec}}}\frac{1}{R_b}\frac{\mu}{T_f}+1. \quad (7)$$

Similarly, it is also possible to get an expression for $R_b(\mathrm{DF})$ as a function of DF as follows

$$R_b(\mathrm{DF}) = \frac{1}{\mathbf{SNRb}_{\text{spec}}}\frac{1}{N_u-1}\frac{\mu}{T_f}(1-\frac{1}{\mathrm{DF}}).$$

The maximum bit transmission rate is

$$R_{\max} \triangleq \lim_{\mathrm{DF}\to\infty} N_u(\mathrm{DF}) = \frac{1}{\mathbf{SNRb}_{\text{spec}}}\frac{1}{N_u-1}\frac{\mu}{T_f}. \quad (8)$$

The values $N_{\max}$ and $R_{\max}$ are the largest values that N_u and R_b can attain, respectively, when the performance is determined by the amount of multiple-access interference produced by N_u active users.

Notice that there is a limit on how large $\mathbf{SNRb}_{\text{spec}}$ can be for a given number of users N_u. This maximum value can be found if we let $\mathbf{SNRb}_{\text{rec}}(N_u)$ take on large values in (3) to get

$$\begin{aligned}\mathbf{SNRb}_{\text{lim}}(N_u) &\triangleq \lim_{\mathbf{SNRb}_{\text{rec}}(N_u)\to\infty} \mathbf{SNRb}_{\text{spec}} \\ &= \frac{1}{R_b}\frac{\mu/T_f}{(N_u-1)}.\end{aligned}$$

Notice that by decreasing $\mathbf{SNRb}_{\text{spec}}$ we can increase $R_{\max}$ or $N_{\max}$. The limit on how small $\mathbf{SNRb}_{\text{spec}}$ can be for a given number of users N_u is investigated in the next subsection.

C. Multiple-access transmission capacity

It is well known from communication theory that P_e in (1) has the following limiting behavior[5] [7]

$$\lim_{\mathrm{M}\to\infty} P_e = \begin{cases} 1, & \text{if } \mathbf{SNRb}_{\text{spec}} < \log_e(2) \\ 0, & \text{if } \mathbf{SNRb}_{\text{spec}} > \log_e(2) \end{cases} \quad (9)$$

We can use the condition in (9) together with (7) to write

$$N_{\max} < N_{\text{IR}} \triangleq \frac{1}{\log_e(2)}\frac{1}{R_b}\frac{\mu}{T_f}+1.$$

Hence, N_{IR} is attainable, in principle, using block waveforms signals with $M \to \infty$. Similarly, we can use the condition in (9) together with (8) to write

$$R_{\max} < C_{\text{IR}}(N_u) \triangleq \frac{1}{\log_e(2)}\frac{\mu/T_f}{(N_u-1)} \quad (10)$$

Hence, the term $C_{\text{IR}}(N_u)$ plays the role of *multiple-access channel capacity per user* of IR in bits per second. Another way to see this is by using Shannon's formula for channel capacity [7]

$$C(B) = B\,\log_2(1 + \frac{1}{B}P_{\text{lim}}(N_u))$$

with bandwidth $B \triangleq \frac{1}{T_w}$ in the order of Gigahertz, and

$$P_{\text{lim}}(N_u) \triangleq R_b\,\mathbf{SNRb}_{\text{lim}}(N_u)$$

playing the role of *effective signal power to power noise density* ratio. Hence

$$\begin{aligned}C(B) &= B\,\log_2(1+\frac{1}{B}\frac{\mu/T_f}{N_u-1}) \\ &= \frac{B}{\log(2)}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\left(\frac{1}{B}\frac{\mu/T_f}{N_u-1}\right)^k. \quad (11)\end{aligned}$$

[5] In (9) we have assumed that $\mathbf{SNRb}_{\text{out}}(1) = \mathbf{SNRb}_{\text{rec}}(N_u)$ so that the condition $\mathbf{SNRb}_{\text{out}}(N_u) = \mathbf{SNRb}_{\text{spec}}$ can be met.

With B in the order of Gigahertz, μ in the order of hundreds, T_f in the order of hundreds of nanoseconds, and $N_u >> 1$, it is clear that

$$\left(\frac{1}{B}\frac{\mu/T_f}{N_u - 1}\right) < 0.01$$

and (11) can be approximated

$$\begin{aligned} C(B) &\simeq \frac{1}{\log(2)}\frac{\mu/T_f}{N_u - 1} \\ &= C_{\mathrm{IR}}(N_u) \end{aligned}$$

If C_{IR} is the multiple-access capacity per user of IR in bits per second, then

$$\begin{aligned} C_{\mathrm{TOT}} &\triangleq N_u C_{IR} \\ &\simeq \frac{1}{\log(2)}\frac{\mu}{T_f},\ N_u >> 1, \end{aligned}$$

plays the role of *total multiple-access capacity* of IR in bits per second and gives an upper bound on the total combined bit transmission rate that can can be attained when the performance is determined by the amount of multiple-access interference with N_u users active.

IV. Numerical example

In this section we evaluate $N_u(\mathrm{DF})$ in (6) and $C_{\mathrm{IR}}(N_u)$ in (10) for a specific design. In IR modulation, the UWB received impulse $w(t)$ can be modeled by

$$w(t) = \left[1 - 4\pi\left[\frac{t}{t_n}\right]^2\right] \exp\left(-2\pi\left[\frac{t}{t_n}\right]^2\right)$$

where the value t_n (ns) is used to fit the model $w(t)$ to a measured waveform from a particular experimental IR link. The normalized signal correlation function corresponding to this impulse is

$$\gamma_w(t) = \left[1 - 4\pi\left[\frac{t}{t_n}\right]^2 + \frac{4\pi^2}{3}\left[\frac{t}{t_n}\right]^4\right] \exp\left(-\pi\left[\frac{t}{t_n}\right]^2\right)$$

In this case $\tau_{\min}$ (ns) depends on t_n, and $\gamma_{\min} = -0.6183$ for any t_n.

The value of μ was calculated for three different pulse widths t_n, using $\tau_2 = \tau_{\min}$ and $T_f = 100$ ns. These values are shown in table I.

Figure 1 shows $N_u(\mathrm{DF})$ for the EC TH PPM signal set, using $R_b = 1048$ Kilobits per second per user and $2 \leq M \leq 16$ with $Pe(1) \simeq 10^{-7}$. The curves were calculated using $\tau_2 = \tau_{\min} = 0.2419$ ns and $T_f = 100$ ns. Perfect power control was assumed.

Figure 2 shows $N_u(\mathrm{DF})$ for the EC TH PPM signal set, this time using $R_b = 9.6$ Kilobits per second per user and $2 \leq M \leq 1024$ with $Pe(1) \simeq 10^{-3}$. Also shown is the

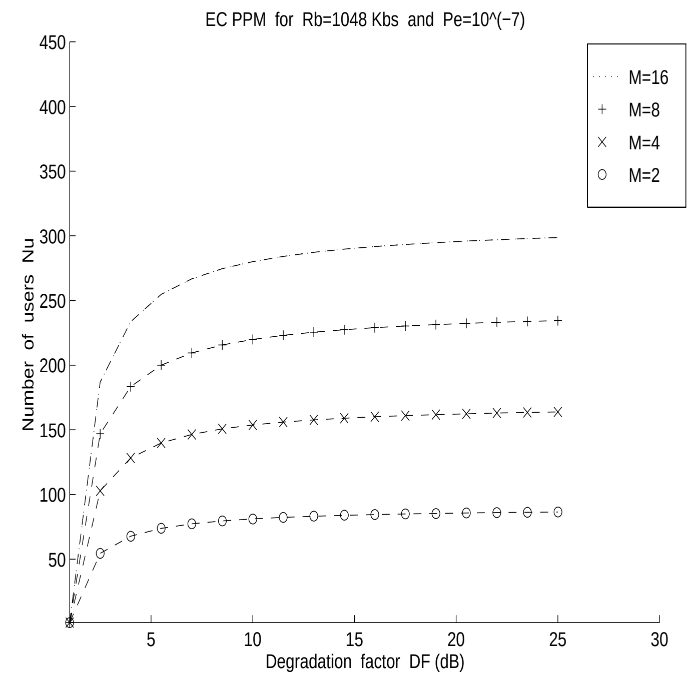

Fig. 1. The number of users $N_u(\mathrm{DF})$ for EC PPM signals, calculated using $2 \leq M \leq 16$ with $P_e(1) \simeq 10^{-7}$. The curves were calculated using $R_b = 1048$ Kilobits per second and set 2 of parameters in table I.

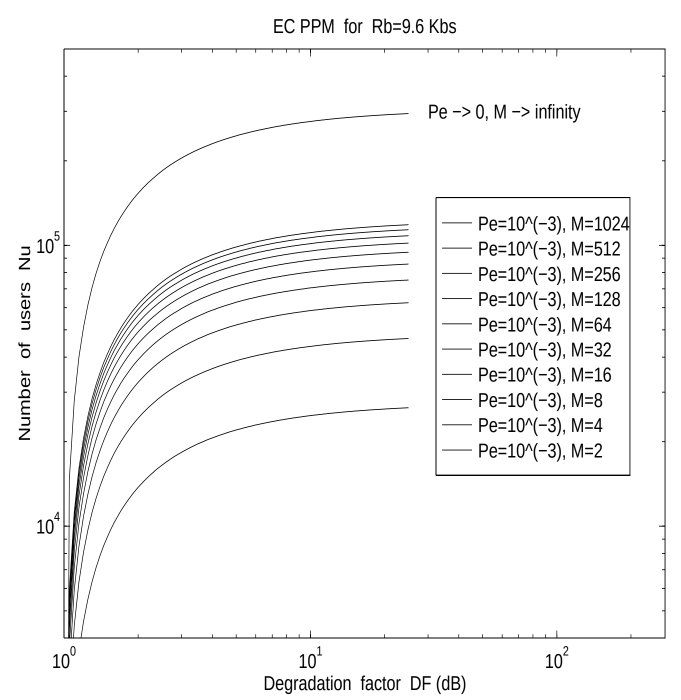

Fig. 2. The number of users $N_u(\mathrm{DF})$ for EC PPM signals, calculated using $2 \leq M \leq 1024$ with $P_e(1) \simeq 10^{-3}$. Also shown is the value of $N_u(\mathrm{DF}) \to N_{\mathrm{IR}}$ for large values of both DF and M. The curves were calculated using $R_b = 9.6$ Kilobits per second and set 2 of parameters in table I.

set 1 of parameters	set 2 of parameters	set 3 of parameters
$t_n = 0.2877$ ns $T_w = 0.75$ ns $\tau_{\min} = 0.1556$ ns	$t_n = 0.4472$ ns $T_w = 1.2$ ns $\tau_{\min} = 0.2419$ ns	$t_n = 0.7531$ ns $T_w = 2.0$ ns $\tau_{\min} = 0.4073$ ns
$\mu = 253.42$	$\mu = 162.28$	$\mu = 96.37$

TABLE I

Value of μ calculated using $\tau_2 = \tau_{\min}$ and $T_f = 100$ ns.

value of $N_u(\mathrm{DF}) \to N_{\mathrm{IR}}$ for large values of both DF and M. The curves were calculated using $\tau_2 = \tau_{\min} = 0.2419$ ns and $T_f = 100$ ns. Again, perfect power control was assumed.

Figure 3 show the multiple access capacity of IR $C_{IR}(N_u)$ in bits per second corresponding to sets 1, 2, 3 of parameters in table I.

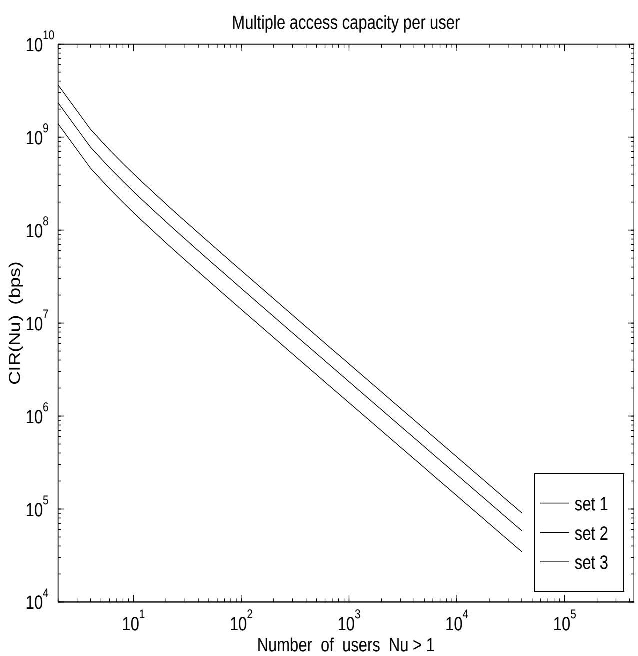

Fig. 3. The multiple access capacity per user $C_{IR}(N_u)$ in bits per second as a function of N_u, calculated using sets 1, 2, 3 of parameters in table I.

V. Discussion of results

Figures 1 and 2 shows how by using higher values of M it is possible to increase the maximum number of users $N_{\max}$, for fixed values of bit transmission rate and probability of bit error. In figure 2 the value $N_{\max}$ is shown to reach a maximum N_{IR} no matter how large M becomes.

From figure 3 it is clear that the multiple-access capacity per user $C_{\mathrm{IR}}(N_u)$ using set 1 is larger than $C_{\mathrm{IR}}(N_u)$ using set 2, which in turn is larger than $C_{\mathrm{IR}}(N_u)$ using the set 3 of parameters in table I. Similarly, the total multiple-access capacity $C_{\mathrm{TOT}} = 3.3964$ Gigabits using set 1 is larger than $C_{\mathrm{TOT}} = 2.3412$ Gigabits using set 2, which in turn is larger than $C_{\mathrm{TOT}} = 1.3903$ Gigabits using the set 3 of parameters. This is to be expected since the set 1 corresponds to "more impulsive" signals, which means that the TH-PPM signals corresponding to different users are less likely to suffer collisions among them. From the frequency point of view, a narrower impulse implies more spreading gain in the frequency domain.

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