Wireless Multiple-Access Using SS Time-Hopping and Block Waveform Pulse Position Modulation, Part 1: Signal Design

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ABSTRACT—**In this paper we describe different block waveform pulse-position-modulated (PPM) signal sets constructed using impulse technology. In each case the design method is given, the normalized correlation properties are discussed, the performance in additive white Gaussian noise (AWGN) is analyzed, and receiver simplification for large signal sets is illustrated.**

I. Introduction

COMMUNICATION signals based on impulse technology can be used for multiple-access communication in indoor environments [1] [2] [3] [4]. Due to the ultra-wideband[1] (UWB) nature of these signals, the dense multipath (produced by signals arriving at the receiver with different time delays that can be as small as fractions of nanoseconds [5]) can be resolved, allowing the use of a Rake receiver [6] for signal detection literally at the antenna terminals, making a relatively simple and low-cost, low-power transceiver viable [1].

The use of block-waveform PPM signals is attractive. In one-user environments, it allows to increase the data transmission rate, making efficient use of the signal-to-noise-ratio (SNR) available without increasing the transmission bandwidth. In a multiple-access environment, the use of block waveform signals allows to increase the data transmission rate supported by the system without degrading the multiple access performance for a given number of users.

In this paper we describe different block waveform PPM signal sets constructed using impulse technology. In each case the design method is given, the normalized correlation properties are discussed, the performance in AWGN is analyzed, and receiver simplification for large signal sets is illustrated.

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[1]The range of frequencies occupied by the UWB signals goes from a few hundreds of Kilohertz up to a few Gigahertz

II. Signal description

The signal $w(t)$ is the basic subnanosecond impulse used to convey information. It has duration T_p and energy $E_w = \int_{-\infty}^{\infty} [w(t)]^2\, dt$. The normalized signal correlation function of $w(t)$ is

$$\gamma_w(\tau) \triangleq \frac{1}{E_p}\int_{-\infty}^{\infty} w(t)w(t-\tau)dt > -1\ \forall\tau.$$

The minimum value of $\gamma_w(\tau)$ is denoted $\gamma_{\min}$, and $\tau_{\min}$ denotes the smallest value of $\tau \in (0, T_p]$ such that $\gamma_{\min} = \gamma_w(\tau_{\min})$. The block waveform PPM signals studied in this paper consists of N_s time-shifted impulses

$$S_j(t) = \sum_{k=0}^{N_s-1} w(t - kT_f - \delta_j^k), \quad j = 1, 2, \ldots, M. \quad (1)$$

Given the impulse shape $w(t)$, the j^{th} signal, $1 \le j \le M$, is completely identified by the sequence of time shifts $\{\delta_j^k; k = 0, 1, 2, \ldots, N_s - 1\}$. Each signal $S_j(t)$ has duration N_sT_f and energy $E_S = \int_{-\infty}^{\infty} [S_j(t)]^2\, dt$. In impulse radio, the impulse duration satisfies $T_p << T_f$, where T_f is the time shift value corresponding to the frame period; and the time shift corresponding to the data modulation is $\delta_j^k \in \{\tau_1 < \tau_2 < \ldots < \tau_{\text{N}}\}$, for some N, with $\tau_{\text{N}} + T_w < T_f$. The normalized correlation values are

$$\begin{aligned}\alpha_{ij} &\triangleq \frac{1}{E_S}\int_{-\infty}^{\infty} S_i(t)S_j(t)dt \\ &= \frac{1}{N_s}\sum_{k=0}^{N_s-1} \gamma_w(\delta_i^k - \delta_j^k), \end{aligned} \quad (2)$$

and the matrix of normalized correlation values is

$$, = \begin{bmatrix} 1 & \alpha_{12} & \ldots & \alpha_{1M} \\ \alpha_{21} & 1 & \ldots & \alpha_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{M1} & \alpha_{M2} & \ldots & 1 \end{bmatrix}_{M\times M}. \quad (3)$$

We are interested in block waveform PPM signal sets in which the basic structure of , does not depend on the shape of the impulse waveform.[2] In the next section we describe four signal sets that satisfy this requirement.

[2]The impulse shape of $w(t)$ is not standardized and depends on the device used to generate the signal.

III. Block waveform PPM signal sets

A. Signal set 1

The first signal set is defined by the time shift pattern

$$\delta_j^k = [(k+j-1) \bmod M]T_1, \qquad (4)$$

where $T_1 > T_w$, and $MT_1 < T_f$ (to avoid impulses overlapping between frames). The signals in (1) with $\{\delta_j^k\}$ in (4) have correlation matrix

$$\Gamma_1 = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}_{M\times M}$$

for any pulse shape $w(t)$.

B. Signal set 2

The second signal set is defined by the time shift pattern $\{\delta_j^k = a_j^k \tau_2\}$, where $a_j^k \in \{0,1\}$ is the j^{th} cyclic shift, $j = 1, 2, \dots, M$, of an m-sequence [7] of length $N_s \geq M$. Based on this definition, it is clear that

$$\delta_j^k = a_j^k \tau_2 \in \{0, \tau_2\}, \qquad (5)$$

where $0 < \tau_2 < T_w$. It can be shown [8] that for the signals in (1) with $\{\delta_i^k\}$ given in (5) the correlation matrix is

$$\Gamma_2 = \begin{bmatrix} 1 & \lambda & \dots & \lambda \\ \lambda & 1 & \dots & \lambda \\ \vdots & \vdots & \ddots & \vdots \\ \lambda & \lambda & \dots & 1 \end{bmatrix}_{M\times M}$$

for any pulse shape $w(t)$, whit the value of λ is given by

$$0 < \lambda_{\min} = \frac{1+\gamma_{\min}}{2} \leq \lambda = \frac{1+\gamma_w(\tau_2)}{2} \leq 1, \qquad (6)$$

for $N_s >> 1$. The actual value of $\gamma_{\min}$ depends on the particular impulse $w(t)$ used in the communications link.

C. Signal set 3

The third signal set is defined by the time shift pattern[3]

$$\delta_j^k = \tau_J + \left[(k + \lfloor \frac{j-1}{N} \rfloor N) \bmod L\right] T_3, \qquad (7)$$

where $J \triangleq j - \lfloor \frac{j-1}{N} \rfloor N$, $1 \leq j \leq M$, $1 \leq J \leq N$, $L \triangleq \frac{M}{N}$, $T_3 \triangleq \tau_N + T_1$ and $LT_3 < T_f$. It can be shown [9] that for the signals in (1) with $\{\delta_j^k\}$ given in (7) the correlation matrix is

$$\Gamma_3 = \begin{bmatrix} \Lambda_{\mathrm{MTSK}} & \mathbf{0} & \dots & \mathbf{O} \\ \mathbf{0} & \Lambda_{\mathrm{MTSK}} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \Lambda_{\mathrm{MTSK}} \end{bmatrix},$$

[3]The notation $\lfloor \xi \rfloor$ indicates the integer part of ξ.

where

$$\Lambda_{\mathrm{MTSK}} = \begin{bmatrix} 1 & \gamma_w(\tau_{21}) & \dots & \gamma_w(\tau_{N1}) \\ \gamma_w(\tau_{21}) & 1 & \dots & \gamma_w(\tau_{N2}) \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_w(\tau_{N1}) & \gamma_w(\tau_{N2}) & \dots & 1 \end{bmatrix},$$

with $\tau_{ij} = \tau_J - \tau_I$. The time-shift values $\{\tau_1 = 0, \tau_{m-1} < \tau_m \leq (m-1)T_w,\ m = 2, 3, \dots, N\}$, are chosen to minimize the maximum correlation value between signals[4], i.e., they are the solution to the problem

$$\underset{\tau=(\tau_1,\tau_2,\dots,\tau_N)\in\mathrm{P}}{minimize} \quad max\Big(\begin{array}{l} \gamma_w(\tau_{12}), \gamma_w(\tau_{13}), \dots, \gamma_w(\tau_{1N}), \\ \gamma_w(\tau_{23}), \gamma_w(\tau_{24}), \dots, \gamma_w(\tau_{2N}), \\ \vdots \\ \gamma_w(\tau_{(N-2)(N-1)}), \gamma_w(\tau_{(N-2)N}),, \\ \gamma_w(\tau_{(N-1)N}) \quad \Big) \end{array} .$$

D. Signal set 4

The fourth signal set is defined by the time shift pattern

$$\delta_j^k = a_j^k \tau_2 + \left[(k + \lfloor \frac{j-1}{N} \rfloor N) \bmod L\right] T_4, \qquad (8)$$

where $0 < \tau_2 < T_w$, $L \triangleq \frac{M}{N}$, $T_4 \triangleq \tau_2 + T_1$, and $LT_4 < T_f$. It can be shown that for the signals in (1) with $\{\delta_j^k\}$ given in (8) the correlation matrix is

$$\Gamma_4 = \begin{bmatrix} \hat{\Gamma}_2 & \mathbf{0} & \dots & \mathbf{O} \\ \mathbf{0} & \hat{\Gamma}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \hat{\Gamma}_2 \end{bmatrix}_{M\times M},$$

where

$$\hat{\Gamma}_2 = \begin{bmatrix} 1 & \lambda & \dots & \lambda \\ \lambda & 1 & \dots & \lambda \\ \vdots & \vdots & \ddots & \vdots \\ \lambda & \lambda & \dots & 1 \end{bmatrix}_{N\times N}$$

for any impulse shape $w(t)$. The value of λ is given in (6).

IV. Performance in AWGN

Signal sets 1 and 2 correspond to orthogonal and equally correlated sets, respectively. Signal sets 3 and 4 both correspond to N-orthogonal[5] sets. Using the union bound, we can calculate the symbol error probability in AWGN for these signals. For signal set 1, the symbol error probability

$$\mathrm{UBP}_e^{(1)} \triangleq (M-1)Q\left(\sqrt{\frac{E_S}{N_o}}\right),$$

[4]This is equivalent to maximize the distance between the signals of the set that are closest together in the signal space.

[5]N-Orthogonal signals are the generalization of bi-orthogonal signals [10] and have the following two properties: (1) The signal set may be divided into L disjoint subsets, each subset containing N signals, and (2) Signals from different subsets are orthogonal.

where $Q(\xi)$ is the Gaussian tail integral. This symbol error probability can be converted to bit error probability as follows [11]

$$\mathrm{UBP}_b^{(1)} \triangleq \frac{M}{2} Q\left(\sqrt{\frac{E_S}{N_o}}\right).$$

For signal set 2, the symbol error probability is

$$\mathrm{UBP}_e^{(2)} \triangleq (M-1) Q\left(\sqrt{\frac{E_S(1-\lambda)}{N_o}}\right).$$

This symbol error probability can be converted to bit error probability as follows

$$\mathrm{UBP}_b^{(2)} \triangleq \frac{M}{2} Q\left(\sqrt{\frac{E_S(1-\lambda)}{N_o}}\right).$$

For signal set 3, the symbol error probability is

$$\mathrm{UBP}_e^{(3)} = \frac{1}{N}\sum_{\substack{i=1 \\ i\neq j}}^{N}\sum_{j=1}^{N} Q\left(\sqrt{\frac{E_S(1-\gamma_w(\tau_{ij}))}{N_o}}\right) + (M-N) Q\left(\sqrt{\frac{E_S}{N_o}})\right).$$

For $N = 2$ the union bound for the bit error probability $\mathrm{UBP}_b^{(3)}$ can be calculated assuming that the L pairs of complementary binary patterns representing data symbols are encoded in the L pairs of signals belonging to disjoint sets. If the decoder decides correctly the orthogonal dimension but errs in the test between the two signals in that subset, every bit of the word is incorrect. If the decoder decides the wrong orthogonal dimension, bit errors are equally distributed. The probability of a given bit being in error is obtained by averaging over the probability of each of these types of error. Hence

$$\mathrm{UBP}_b^{(3)} = Q\left(\sqrt{\frac{E_S(1-\gamma_{\min})}{N_o}}\right) + \frac{(M-2)}{2} Q\left(\sqrt{\frac{E_S}{N_o}}\right).$$

For signal set 4, the symbol error probability is

$$\mathrm{UBP}_e^{(4)} \triangleq (N-1) Q\left(\sqrt{\frac{E_S(1-\lambda)}{N_o}}\right) + (M-N)\, Q\left(\sqrt{\frac{E_S}{N_o}}\right),$$

For $N = 2$ the union bound for the bit error probability is

$$\mathrm{UBP}_b^{(4)} = Q\left(\sqrt{\frac{E_S(1-\lambda)}{N_o}}\right) + \frac{(M-2)}{2} Q\left(\sqrt{\frac{E_S}{N_o}}\right).$$

A. Example

In this example we calculate the error probability in AWGN for the four types of signals just discussed. In IR modulation, the UWB received pulse $w(t)$ can be modeled by

$$w(t) = \left[1-4\pi\left[\tfrac{t}{t_n}\right]^2\right] \exp\left(-2\pi\left[\tfrac{t}{t_n}\right]^2\right), \quad (9)$$

where the value $t_n = 0.4472$ ns was used to fit the model $w(t)$ to an impulse $w_m(t)$ measured in a particular radio link. The impulse duration is $T_w = 1.2$ ns. The signal correlation function corresponding to $w(t)$ is

$$\gamma_w(\tau) = \left[1-4\pi\left[\tfrac{\tau}{t_n}\right]^2+\tfrac{4\pi^2}{3}\left[\tfrac{\tau}{t_n}\right]^4\right] \exp\left(-\pi\left[\tfrac{\tau}{t_n}\right]^2\right).$$

In this case $\tau_{\min} = 0.2419$ ns and $\gamma_{\min} = -0.6183$.

Figure 1 shows $\mathrm{UBP}_b^{(1)}$, $\mathrm{UBP}_b^{(2)}$, $\mathrm{UBP}_b^{(3)}$ and $\mathrm{UBP}_b^{(4)}$for $M = 8$, $N = 4$ and $L = 2$, calculated using the impulse in (9). For signal set 1 we used $T_1 = T_w$, for signal set 2 we used $\tau_2 = \tau_{\min}$, for signal set 3 we used $N = 2$, $L = 4$, $\tau_1 = 0$, $\tau_2 = \tau_{\min}$ and $T_3 = \tau_{\min} + T_w$, for signal set 4 we used $N = 2$, $L = 4$, $\tau_2 = \tau_{\min}$ and $T_4 = \tau_{\min} + T_w$.

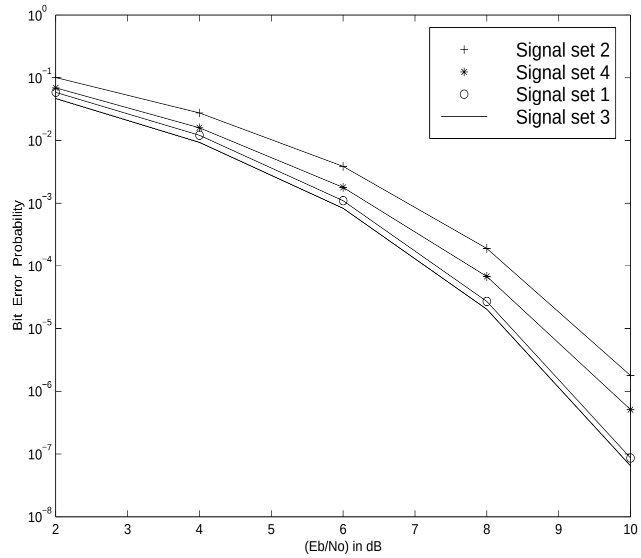

Fig. 1. The $\mathrm{UBP}_b^{(1)}$, $\mathrm{UBP}_b^{(2)}$, $\mathrm{UBP}_b^{(3)}$ and $\mathrm{UBP}_b^{(4)}$for $M = 8$, $N = 2$ and $L = 4$.

V. Receiver simplification

To detect the received signal we need to correlate this signal with M reference signals. For large M this can result in a receiver of considerable complexity. We can take advantage of the structure of the PPM signals to simplify the construction of the receiver.

Let $x(t) = S_i(t)+n(t)$, where $S_i(t)$ is one of the PPM signals in (1) and $n(t)$ is AWGN. In the receiver, each of the M channel correlation output can be written

$$y_j = \int_0^{N_s T_f} x(t)\, S_j(t)\, dt$$

$$= \sum_{k=0}^{N_s-1} \int_{kT_f}^{(k+1)T_f} x(t)\, w(t - kT_f - \delta_j^k)\, dt \qquad (10)$$

For signal set 1, y_j in (10) can be written

$$y_j = \sum_{k=0}^{N_s-1} \sum_{q=0}^{M-1} \delta_{q,[(k+j-1) \bmod L]M}\, z(k,q),$$

where

$$z(k,q) \triangleq \int_{kT_f+qT_1}^{kT_f+(q+1)T_1} x(t)\, w(t - kT_f - qT_1)\, dt,$$

and $\delta_{q,q'}$ is the Kronecker delta. From the expression for y_j, $j = 1, 2, \ldots, M$, it is clear that the receiver needs only 1 correlator and M store and sum circuits.

For signal set 2, y_j in (10) can be written

$$y_j = \sum_{k=0}^{N_s-1} \sum_{m=1}^{2} \delta_{(m-1),a_j^k}\, z_m(k),$$

where

$$z_m(k) \triangleq \int_{kT_f}^{kT_f+T_w+\tau_2} x(t)\, w(t - kT_f - \tau_m)\, dt,$$

for $m = 1, 2$. From the expression for y_j, $j = 1, 2, \ldots, M$, it is clear that the receiver needs only 2 correlators and M store and sum circuits.

For signal set 3, y_j in (10) can be written

$$y_j = \sum_{k=0}^{N_s-1} \sum_{q=0}^{L-1} \delta_{q,[(k+\lfloor \frac{j-1}{N} \rfloor N) \bmod L]}\, z_J(k,q),$$

where

$$z_J(k,q) \triangleq \int_{kT_f+qT_3}^{kT_f+(q+1)T_3} x(t)\, w(t - kT_f - \tau_J - qT_3)\, dt$$

for $1 \leq J \leq N$. From the expression for y_j, it is clear that the receiver needs only N correlators and M store and sum circuits.

For signal set 4, y_j in (10) can be written

$$y_j = \sum_{k=0}^{N_s-1} \sum_{q=0}^{L-1} \sum_{m=1}^{2} \delta_{(m-1),a_j^k}\, z_m(k,q),$$

where

$$z_m(k,q) \triangleq \int_{kT_f+qT_3}^{kT_f+(q+1)T_3} x(t)\, w(t - kT_f - \tau_m - qT_3)\, dt,$$

for $m = 1, 2$. From the expression for y_j, it is clear that the receiver needs only 2 correlators and M store and sum circuits.

In the four cases the decision variables $\{y_j\}$ can be calculated while $x(t)$ is received and no symbol delay occur.

VI. Conclusion

This paper described the construction of four different sets of ultra-wideband block waveform PPM signals using impulse technology. This signal sets can be used in ultra-wideband spread spectrum multiple-access communications [2]. From figure 1, performance of signal sets 3 and 2 are the best and the worse, respectively. The performance of sets 1 and 4 (in that order of performance) falls within sets 3 and 2. From the construction methods (with fixed N_s and T_f), the number of signals in sets 2 and 1 are the largest and the shortest, respectively. The number of signals in sets 4 and 3 (in that order of size) falls within sets 2 and 1. Also, in order of degree of dependence on impulse shape, signal sest 1 and 3 are the less and the more dependent, respectively, with signal sets 2 and 3 with about same degree of dependence. With respect to receiver complexity, signal sets 3 and 1 have the highest and the lowest complexity , respectively. Signal sets 2 and 4 have the same complexity.

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