# Wireless Multiple-Access Using SS Time-Hopping and Block Waveform Pulse Position Modulation, Part 2: System Performance

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*ABSTRACT*— **In this paper we analyze the performance of spread spectrum multiple-access (MA) using time hopping (TH) and pulse position modulation (PPM) in combination with impulse technology. For different block waveform (PPM) signal design, the multiple-access performance is analyzed in terms of the number of users supported by the system for a given bit error rate, bit transmission rate, and number of signals in the block waveform set.**

## I. Introduction

SPREAD spectrum MA communications using TH and PPM was proposed in [1]. This communication technique is called impulse radio (IR) modulation, and uses communication waveforms based on impulse signal technology, resulting in signals with ultra-wide bandwidths (UWB)[1]. Impulse radio, together with technological advances in communication circuits and systems, will allow us to build relatively simple and low-cost, low-power transceivers that can be used for license-free, short range, high speed and reliable multiple access communications over multipath wireless channels [1] [2] [3].

In [1] the single-user MA performance of IR assuming free space propagation conditions and additive white Gaussian noise (AWGN) was studied. The analysis assumed that binary PPM signals are coherently detected using a single-channel correlation receiver. The use of block waveform PPM signals allows to improve the MA performance for a given number of users and transmission rate. In [4] the ideas in [1] were generalized to consider the use of equally correlated block-waveform encoding PPM signals based on binary time-shift-keyed modulation. In this paper we extend the work in [4] to include results for other block waveform encoding PPM signals sets presented in [5].

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[1]The range of frequencies occupied by the signals goes from a few hundreds of Kilohertz up to a few Gigahertz

## II. Channel, signals and multiple-access interference models

### *A. Channel and impulse signal models*

The model assumed is a free-space propagation channel impaired with AWGN. The transmitted impulse is $w_{\rm TX}(t) \triangleq \int_{-\infty}^{t} w(\xi)d\xi$ and the received signal is $w(t) + n(t)$.[2] The noise $n(t)$ is AWGN with two-sided power density $\frac{N_o}{2}$. The signal $w(t)$ is the basic subnanosecond impulse used to convey information. It has duration $T_w$ and energy $E_w = \int_{-\infty}^{\infty} [w(t)]^2\, dt$. The normalized signal correlation function of $p(t)$ is $\gamma_w(\tau) \triangleq (1/E_w)\int_{-\infty}^{\infty} w(t)w(t-\tau)dt > -1\ \forall\tau$. We define $\gamma_{\min} = \gamma_w(\tau_{\min})$ as the minimum value of $\gamma_w(\tau)$, $\tau \in (0, T_w]$.

### *B. PPM signals*

The PPM signals consist of $N_s$ time-shifted impulses

$$S_j(t) = \sum_{k=0}^{N_s-1} w(t - kT_f - \delta_j^k), \quad j = 1, 2, \ldots, M.$$

Notice that information is conveyed exclusively in the sequence of time shifts $\{\delta_j^k; k = 0, 1, 2, \ldots, N_s - 1\}$, with $\delta_j^k \in \{\tau_1 < \tau_2 < \ldots < \tau_{\rm N}\}$ for some $N \geq 2$. In IR, the impulse duration satisfies $T_p << T_f$, where $T_f$ is the time shift value corresponding to the frame period. A single symbol waveform has duration $T_s \triangleq N_s T_f$, $N_s >> 1$. The symbol rate is $R_s = T_s^{-1}$. The signals $\{S_j(t)\}$ have correlation values

$$R_{ij} = \int_{-\infty}^{\infty} S_i(\xi)\, S_j(\xi)\, d\xi = E_w \sum_{k=0}^{N_s-1} \gamma_w(\delta_i^k - \delta_j^k),$$

since for $k \neq l$ the pulses are non overlapping. The energy in the $j^{\rm th}$ signal is $E_S = R_{ii} = N_s E_w$, and the normalized correlation value is

$$\alpha_{ij} \triangleq \frac{R_{ij}}{E_S} = \frac{1}{N_s} \sum_{k=0}^{N_s-1} \gamma_w(\delta_i^k - \delta_j^k) \geq \gamma_{\min}.$$

[2]The effect of the antenna system in the UWB transmitted impulse is modeled as a derivation operation.

*C. TH PPM signals*

Each user's signal is composed of a sequence of fast-hopped frame-shifted versions of one of the $M$ possible PPM symbol waveforms $\{S_j(t)\}$

$$x^{(\nu)}(t) = \sum_{m=0}^{\infty} S_{d_m^{(\nu)}}(t - mN_sT_f - C_m^{(\nu)}(t)),$$

where the superscript $(\nu)$, $(1 \leq \nu \leq N_u)$ indicates user-dependent quantities, $N_u$ is the number of users, $m$ indexes the transmitted symbols, $d_m^{(\nu)} \in \{1, 2, \ldots, M\}$ is the $m^{\text{th}}$ transmitted symbol, and $C_m^{(\nu)}(t)$ is a TH function defined by

$$C_m^{(\nu)}(t) \triangleq \sum_{k=mN_s}^{(m+1)N_s-1} T_c\, c_k^{(\nu)} \phi(t - kT_f),$$

where

$$\phi(t) = \begin{cases} 1, & \text{if } 0 \leq t \leq T_f \\ 0, & \text{otherwise} \end{cases}.$$

Here $T_c$ is a time-shift value, and $\{c_k^{(\nu)}\}$ is the pseudo-random time-hopping sequence assigned to user $\nu$, with $0 \leq c_k^{(\nu)} \leq N_h$ for some integer $N_h$, and $c_{k+lN_p}^{(\nu)} = c_k^{(\nu)}$ for some positive integers $N_p$ and $l$.

*D. Multiple-access interference model*

In the present analysis, performance computation is based on signal-to-noise ratios (SNR) averaged over random TH sequences and random asynchronous transmission times. To this end the following assumptions were made:

(a) The elements $\{c_k^{(\nu)}\}$ for $\nu = 1, 2, \ldots, N_u$ and for all $k$, are independent, identically and uniformly distributed on the interval $[0, N_h]$. Furthermore, we assume that $N_s \leq N_p$.

(b) The transmission time differences $\tau_{(\nu)} - \tau_{(1)} \bmod T_f$, $\nu = 2, \ldots, N_u$, are independent, identically and uniformly distributed on $[0, T_f]$.

(d) We assume that the received monocycle waveform satisfies the relation $\int_{-\infty}^{\infty} w(t)dt = 0$.

## III. System performance

*A. Multiple-access performance*

When $N_u$ transmitters are active and the receiver wishes to determine the data modulating transmitter $\nu = 1$, the received signal $r(t)$ can be viewed as

$$r(t) = A^{(1)} S_{d_m^{(1)}}(t - mN_sT_f - C_m^{(1)}(t) - \tau^{(1)}) + n_{\text{TOT}}(t),$$

for $t \in \mathcal{T}_m$, where

$$\mathcal{T}_m \triangleq [mN_sT_f + \tau^{(1)}, ((m+1)N_s - 1)T_f + \tau^{(1)}],$$

and

$$n_{\text{TOT}}(t) \triangleq \sum_{\nu=2}^{N_u} A^{(\nu)} x^{(\nu)}(t) + n(t)$$

includes both MA interference and thermal noise, and is assumed to be a mean-zero Gaussian random process. Standard techniques [6] can then be used to calculate the union bound for the symbol error probability $P_e$ for coherent detection of the TH PPM signals. This union bound is

$$\text{UBP}_e(N_u) \triangleq \frac{1}{M} \sum_{\substack{i=1 \\ i \neq j}}^{M} \sum_{j=1}^{M} Q\left(\sqrt{\mathbf{SNR}_{\text{out}}^{(j,i)}(N_u)}\right), \quad (1)$$

where [7]

$$\mathbf{SNR}_{\text{out}}^{(j,i)}(N_u) = \frac{(A^{(1)})^2 E_s (1 - \alpha_{ji})}{N_o + \sum_{\nu=2}^{N_u} N_{\text{MA}}^{(\nu)}(j,i)}, \quad (2)$$

is the output symbol SNR that the desired user observe in the presence of $(N_u - 1)$ other users, and $N_{\text{MA}}^{(\nu)}$ is the *equivalent* power spectral density level of the multiple-access interference contribution corresponding to the $\nu^{\text{th}}$ user, for $\nu = 2, 3, \ldots, N_u$.

The substitution of the symbol SNR $\mathbf{SNR}_{\text{out}}^{(j,i)}(N_u)$ in (2) into the symbol error probability $\text{UBP}_e(N_u)$ in (1) will provide the desired relation between error probability, number of users and transmission rate.

## IV. Multiple-access symbol SNR

In this section we evaluate $\mathbf{SNR}_{\text{out}}^{(j,i)}(N_u)$ in (2) for the block waveform encoding PPM signal sets discussed in [5]. This requires the calculation of the interference level $N_{MA}^{(\nu)}(j,i)$. The details of this calculation are given in [7]. Here we present the main results.

For signal set 1, defined by time shift pattern

$$\delta_j^k = [(k + j - 1) \bmod M]T_1,$$

we have found that

$$\mathbf{SNR}_{\text{out}}^{(1)}(N_u) = \frac{(A^{(1)})^2 E_s}{N_o + \sum_{\nu=2}^{N_u} N_1^{(\nu)}},$$

where

$$N_1^{(\nu)} \triangleq \frac{(A^{(\nu)})^2 E_w}{T_f} \int_{-(T_w+T_1)}^{(T_w+T_1)} \frac{[\gamma_w(\varsigma) - \gamma_w(\varsigma - T_1)]^2}{[\gamma_w(0) - \gamma_w(T_1)]} d\varsigma$$

for $\nu = 2, 3, \ldots, N_u$ and $\forall i \neq j$.

For signal set 2, defined by time shift pattern

$$\delta_j^k = a_j^k \tau_2 \in \{0, 0 < \tau_2 < T_w\},$$

with $\{a_j^k\} \in \{0, 1\}$ the $j^{\text{th}}$ cyclic shift of an m-sequence, we have found that

$$\mathbf{SNR}_{\text{out}}^{(1)}(N_u) = \frac{(A^{(1)})^2 E_s (1 - \lambda)}{N_o + \sum_{\nu=2}^{N_u} N_2^{(\nu)}},$$

where

$$N_2^{(\nu)} \triangleq \frac{(A^{(\nu)})^2 E_w}{T_f} \int_{-(T_w+\tau_2)}^{(T_w+\tau_2)} \frac{[\gamma_w(\varsigma) - \gamma_w(\varsigma - \tau_2)]^2}{[\gamma_w(0) - \gamma_w(\tau_2)]} d\varsigma$$

for $\nu = 2, 3, \ldots, N_u$ and $\forall i \neq j$, with $0 < \lambda < 1$.

For signal set 3, defined by time shift pattern

$$\delta_j^k = \tau_J + [(k+m) \bmod L] T_3,$$

where $J = j - \lfloor \frac{j-1}{N} \rfloor N$ and $m = \lfloor \frac{j-1}{N} \rfloor N$, we have found that

$$\mathbf{SNR}_{\text{out}}^{(3),(j,i)}(N_u) = \begin{cases} \mathbf{SNR}_{\text{out}}^{(1)}(N_u), & \text{for } \lfloor \frac{j-1}{N} \rfloor \neq \lfloor \frac{i-1}{N} \rfloor \\ \mathbf{SNR}_{\text{out}}^{(3'),(j,i)}(N_u), & \text{for } \lfloor \frac{j-1}{N} \rfloor = \lfloor \frac{i-1}{N} \rfloor \end{cases},$$

where $\mathbf{SNR}_{\text{out}}^{(1)}(N_u)$ was calculated for signal set 1, and

$$\mathbf{SNR}_{\text{out}}^{(3')(j,i)}(N_u) = \frac{(A^{(1)})^2 E_s (1 - \gamma_w(\tau_J - \tau_I))}{N_o + \sum_{\nu=2}^{N_u} N_{3'}^{(\nu)}(j,i)},$$

where

$$N_{3'}^{(\nu)}(j,i) \triangleq \frac{(A^{(\nu)})^2 E_w}{T_f} \int_{-2(T_w+\tau_N)}^{2(T_w+\tau_N)} \frac{[\gamma_w(\varsigma - \tau_J) - \gamma_w(\varsigma - \tau_I)]^2}{[\gamma_w(0) - \gamma_w(\tau_J - \tau_I)]} d\varsigma,$$

for $\nu = 2, 3, \ldots, N_u$, $J = j - \lfloor \frac{j-1}{N} \rfloor N$, $I = i - \lfloor \frac{i-1}{N} \rfloor N$ and $\forall i \neq j$,

For signal set 4, defined by the time shift pattern

$$\delta_j^k = a_j^k \tau_2 + \left[ (k + \lfloor \frac{j-1}{N} \rfloor N) \bmod L \right] T_4,$$

we have found that

$$\mathbf{SNR}_{\text{out}}^{(4),(j,i)}(N_u) = \begin{cases} \mathbf{SNR}_{\text{out}}^{(1)}(N_u), & \text{for } \lfloor \frac{j-1}{N} \rfloor \neq \lfloor \frac{i-1}{N} \rfloor \\ \mathbf{SNR}_{\text{out}}^{(4')}(N_u), & \text{for } \lfloor \frac{j-1}{N} \rfloor = \lfloor \frac{i-1}{N} \rfloor \end{cases},$$

where

$$\mathbf{SNR}_{\text{out}}^{(4')(j,i)}(N_u) = \frac{(A^{(1)})^2 E_s (1 - \lambda)}{N_o + \sum_{\nu=2}^{N_u} N_{4'}^{(\nu)}},$$

where

$$N_{4'}^{(\nu)} \triangleq \frac{(A^{(\nu)})^2 E_w}{T_f} \int_{-2(T_w+\tau_2)}^{2(T_w+\tau_2)} \frac{[\gamma_w(\varsigma) - \gamma_w(\varsigma - \tau_2)]^2}{[\gamma_w(0) - \gamma_w(\tau_2)]} d\varsigma$$

for $\nu = 2, 3, \ldots, N_u$ and $\forall i \neq j$, with $0 < \lambda < 1$.

## V. Numerical example

In this section we illustrate the potential MA performance of this system under perfect power control (i.e. $A^{(\nu)} = A^{(1)}$ for $\nu = 1, 2, \ldots, N_u$) for a specific IR link. In IR modulation, the UWB received impulse $w(t)$ can be modeled by

$$w(t) = \left[ 1 - 4\pi \left[ \frac{t}{t_n} \right]^2 \right] \exp \left( -2\pi \left[ \frac{t}{t_n} \right]^2 \right)$$

where the value $t_n = 0.4472$ ns was used to fit the model $w(t)$ to a measured waveform $w_m(t)$ from a particular experimental IR link. The duration of the impulse is $T_w = 1.2$ ns. The normalized signal correlation function corresponding to this impulse is

$$\gamma_w(t) = \left[ 1 - 4\pi \left[ \frac{t}{t_n} \right]^2 + \frac{4\pi^2}{3} \left[ \frac{t}{t_n} \right]^4 \right] \exp\left( -\pi [\tfrac{t}{t_n}]^2 \right)$$

In this case $\tau_{\min} = 0.2419$ ns and $\gamma_{\min} = -0.6183$.

Given the impulse $w(t)$, we can evaluate $N_m^{(\nu)}$, $m = 1, 2, 3, 4$, using the following time shift values. For $N_1^{(\nu)}$, we have $T_1 = T_w$, for $N_2^{(\nu)}$ we use $\tau_2 = \tau_{\min}$, for $N_3^{(\nu)}$ we use $N = 2$, $\tau_1 = 0$, $\tau_2 = \tau_{\min}$, and $T_3 = \tau_{\min} + T_w$, and for $N_4^{(\nu)}$ we use $N = 2$, $\tau_2 = \tau_{\min}$ and $T_4 = \tau_{\min} + T_w$.

Figures 1, 2, 3 and 4, shows the MA performance curves. The curves were calculated using $T_f = 100$ ns and bit transmission rate $R_b = 9.6$ Kilobits per second.

Similar curves can be calculated using high data transmission rates (e.g. 1024 Kilobits per second), in which the system is able to support hundreds of users each one transmitting over a Megabit per second with bit error probability in the range $10^{-5}$ to $10^{-8}$.

## VI. Conclusion

From figures 1, 2, 3 and 4, the benefits of using block waveform modulation are evident. By using higher values of $M$ other than 2, it is possible either to increase the number of users for a fixed probability of error, or to improve the probability of detection for a fixed number of users $N_u$, without increasing each user's signal power. It can be seen that the benefit in going from one value of $M$ to the next value actually decreases as $M$ increases. It is also clear that signal set 3 ranks first, set 1 ranks second, set 4 ranks third and set 2 ranks fourth in terms of multiple-access performance. This is expected, since the signal sets can be ranked, in that order, in terms of "good" correlation properties. This analysis, together with the results in [5], shows that impulse radio is potentially able to provide multiple-access communications with a combined transmission capacity of over 500 Megabits per second at bit error rates in the range $10^{-4}$ to $10^{-8}$ using receivers of moderate complexity.

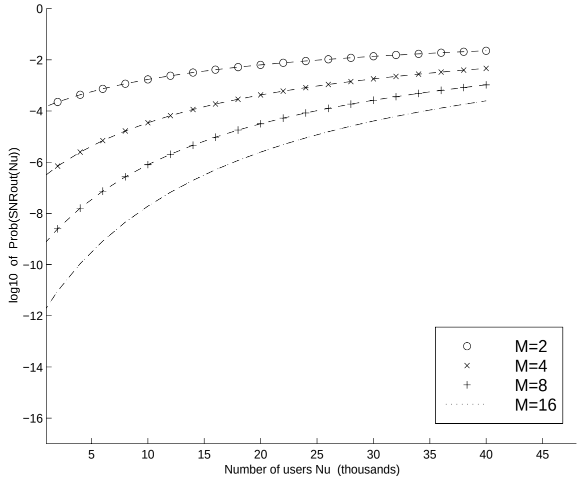


Fig. 1. The base 10 logarithm of the probability of bit error for signal set 1, as a function of $N_u$ for different values of $M$, using $R_b = 9.6$ Kbps, and $\mathbf{SNR}_{\text{out}}^{(1)}(1) = 11.40$ dB.

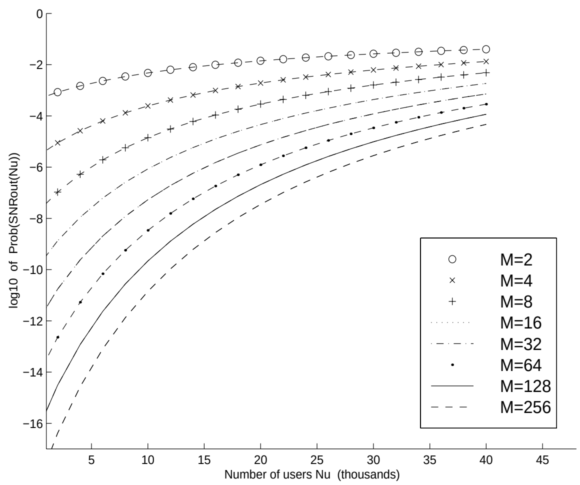


Fig. 2. The base 10 logarithm of the probability of bit error for signal set 2, as a function of $N_u$ for different values of $M$, using $R_b = 9.6$ Kbps, and $\mathbf{SNR}_{\text{out}}^{(2)}(1) = 10.48$ dB.

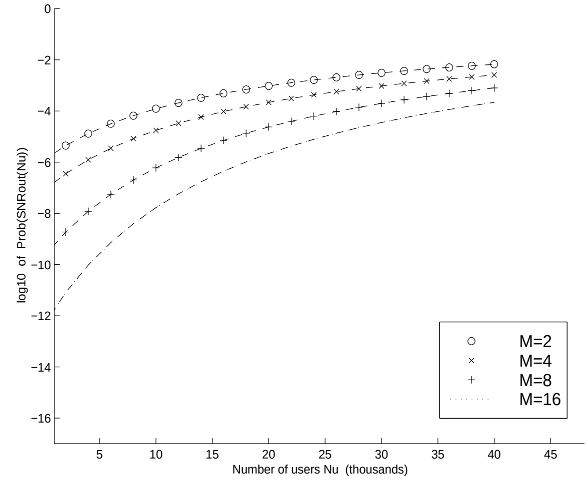


Fig. 3. The base 10 logarithm of the probability of bit error for signal set 3, as a function of $N_u$ for different values of $M$, using $R_b = 9.6$ Kbps, and $\mathbf{SNR}_{\text{out}}^{(3')}(1) = 13.49$ dB.

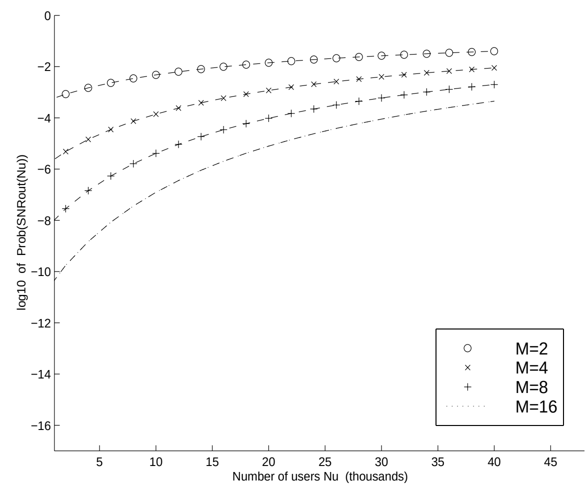


Fig. 4. The base 10 logarithm of the probability of bit error for signal set 4, as a function of $N_u$ for different values of $M$, using $R_b = 9.6$ Kbps, and $\mathbf{SNR}_{\text{out}}^{(4')}(1) = 13.49$ dB.

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