REDUCED RANK DETECTION SCHEMES FOR DS-CDMA COMMUNICATION SYSTEMS

Wanshi Chen
Ericsson Wireless Communications
6455 Lusk Blvd.
San Diego, CA 92121
e-mail:
wanshi.chen@ericsson.com

Urbashi Mitra
Dept. of Electrical Engr. – Systems
Communication Sciences Institute
University of Southern California
Los Angeles, CA 90089–2565
e-mail: ubli@usc.edu

Abstract — Several reduced-rank detection schemes for direct-sequence code-division multiple access (DS-CDMA) communication systems are compared. After the simplification of the auxiliary vector filtering (AVF) algorithm [9], it is shown that the AVF algorithm is equivalent to the multistage Wiener filtering (MWF) algorithm of [3]. Furthermore, these schemes can be shown to be equivalent to the multistage linear receiver scheme based on the Cayley Hamilton theorem. The analysis of the reduced rank techniques is extended to multipath fading channels. In particular, a modified reduced rank detection scheme is proposed which outperforms an isolated path combining strategy. In addition, the output signal-to-interference ratio for the noncoherent equal gain combining linear receiver is analyzed to facilitate the study of the tracking behavior of the reduced rank receivers.

I. INTRODUCTION

The linear minimum mean squared error (MMSE) detector for direct-sequence code-division multiple access (DS-CDMA) communications has received considerable attention [5, 7, 11] due to its simplicity of implementation, strong performance, and amenability to adaptive implementation. The adaptive implementation of this receiver can be achieved with prior information comparable to that of the conventional Matched Filtering (MF) receiver *i.e.*, information of the user of interest only and not that of the interfering users.

The study of reduced rank interference suppression for DS-CDMA is motivated by situations where the number of taps to be adaptively tracked by the adaptive MMSE detector is so large that the adaptive receiver responds quite slowly to the time-varying environment. By projecting the received signal onto a subspace of reduced rank, the number of taps in the adaptive filter is reduced thereby improving tracking ability. Reduced rank algorithms based on the exploitation of the Cayley Hamilton theorem are provided in [8]. Therein, approximate MMSE detectors with a multistage linear implementation are presented. In [9], the auxiliary-vector filtering (AVF) method is proposed. In this reduced rank method, an auxiliary vector was derived based on maximizing the cross-correlation between the outputs of the reference vector filter and previously derived auxiliary vector filters. In [3], the multistage Wiener filtering (MWF) method of [2] was applied to DS-CDMA systems. The authors [3] showed that the MWF algorithm reduced rank algorithm required much fewer training samples than the full rank algorithms.

In this paper, we show by theoretical analysis that the MWF, the AVF and the Cayley Hamilton (CH) method of [8] are essentially equivalent. We begin by simplifying the derivation of the auxiliary vectors for the AVF algorithm. Our approach yields a more compact solution for the auxiliary vectors and it greatly reduces the computational complexity as well. By introducing the additional constraint that the blocking matrices in the MWF algorithm are orthonormal in the row space, we prove that the MWF algorithm is equivalent to the AVF algorithm. The proof also naturally leads to the fact that the projection vectors for the MWF algorithm and the Cayley Hamilton approach of [8] share the same subspace. This fact was also shown via an alternative method in [3].

We then extend the reduced rank algorithms to frequency-selective fading channels. The resultant reduced rank detector has performance comparable to the full rank counterpart, but has greatly reduced computational complexity and more importantly, better channel tracking capability for short data records.

The signal to interference-plus-noise ratio (SIR) is an important performance measure for MMSE-based detectors [3, 6]. To facilitate the study of the SIR convergence behavior of the MMSE receivers and the reduced rank algorithms, we analyze the SIR for noncoherent detection in frequency-selective fading channels when fixed short spreading codes are adopted. Simulations show that the theoretical analysis agrees fairly well with the simulation results, especially in the resultant patterns of convergence for different receivers.

II. SYSTEM MODEL

We will consider an asynchronous DS-CDMA system with differential phase shift keying (DPSK) modulation. This approach eliminates the need for estimation of the carrier phases and facilitates the detector design. Without loss of generality, user 1 is taken to be the user of interest. In addition, the time delay for user 1 is assumed to be perfectly known and it is fixed during the transmission. As a result, we can let the time delay for user 1 be 0.

The tapped delay line (TDL) channel model with uniform tap spacing T_c is adopted [10], where T_c is the chip period. For simplicity, throughout the rest of the paper, it is assumed that the maximum value of the summations of the timing delay and maximum path delay for user k, $k = 1, \dots, K$, is less than T_b, where K is the number of users and T_b is the symbol period. Let $N = T_b/T_c$ be the spreading gain. The received baseband signal is first passed through a chip-matched filter before chip-rate sampling. The resulting $N \times 1$ received vector is

0-7803-7225-5/01/$17.00 (c) 2001 IEEE

represented by [1]

$$\mathbf{y}(m) = \sum_{k=1}^{K}\sum_{l=1}^{L} A_{kl}[\gamma_{kl}(m)d_k(m)\mathbf{s}_{kl}^{+} + \gamma_{kl}(m-1) d_k(m-1)\mathbf{s}_{kl}^{-}] + \mathbf{n}(m), \quad (1)$$

where L is the number of multipaths, A_{kl} the amplitude of user k for path l, $d_k(m) \in \{-1,+1\}$ is the m-th differentially encoded data symbol given by $d_k(m) = b_k(m)d_k(m-1)$, where $b_k(m) \in \{-1,+1\}$ is the original data, and $\mathbf{n}(m)$ is the complex additive white Gaussian noise (AWGN) with covariance matrix $\sigma^2\mathbf{I}_N$, where σ^2 is the noise variance and $\mathbf{I}_N$ is the $N \times N$ identity matrix. The normalized fading process $\gamma_{kl}(m)$ is a complex Gaussian random process which satisfies $\mathbf{E}\{\gamma_{k_1l_1}^{*}(m)\gamma_{k_2l_2}(m)\} = \delta(k_1-k_2)\delta(l_1-l_2)$ and $\mathbf{E}\{\gamma_{k_1l_1}^{*}(m)\gamma_{k_2l_2}(m-1)\} = \rho\delta(k_1-k_2)\delta(l_1-l_2)$, where $*$ denotes complex conjugate.

The partial spreading codes $\mathbf{s}_{kl}^{+}$ and $\mathbf{s}_{kl}^{-}$ correspond to the effective spreading codes for the current bit and the previous bit [7], respectively. Both $\mathbf{s}_{kl}^{+}$ and $\mathbf{s}_{kl}^{-}$ are functions of the spreading code, time delay, and chip waveform of user k and they are assumed to be real. The reader is referred to [3, 4, 7] for detailed descriptions of $\mathbf{s}_{kl}^{+}$ and $\mathbf{s}_{kl}^{-}$.

III. REDUCED RANK MMSE FILTERING

Reduced rank techniques reduce the number of taps to be adaptively tracked by projecting the received signal vector onto a lower dimensional subspace. Let D be the resultant lower dimension, where $D < N$, the reduced dimension signal is given by,

$$\tilde{\mathbf{y}}(m) = \mathbf{S}_D^H\mathbf{y}(m), \quad (2)$$

where $\mathbf{S}_D$ is the $N \times D$ projection matrix [2], and the D dimensional signal is denoted by a "tilde" as in [3].

We briefly review the three reduced rank methods to be considered herein: the MWF algorithm [3], the AVF algorithm [9] and the Cayley-Hamilton theorem based algorithm [8]. The MWF algorithm for DS-CDMA was presented in [3]. The equivalent projection matrix is given by

$$\begin{aligned}\mathbf{S}_D^{MW} &= [\ \mathbf{g}_{MW,1}\quad \mathbf{g}_{MW,2}\quad \ldots\quad \mathbf{g}_{MW,D}\ ] \\ &= [\ \mathbf{h}_1\quad \mathbf{B}_1^H\mathbf{h}_2\quad \ldots\quad \prod_{i=1}^{D-1}\mathbf{B}_i^H\mathbf{h}_D\ ],\end{aligned} \quad (3)$$

where $\mathbf{g}_{MW,1} = \mathbf{h}_1 = \mathbf{p}/\|\mathbf{p}\|$, $\mathbf{p}$ is the *steering vector* [3, 4], and $\|.\|$ is the vector norm, the matrix $\mathbf{B}_i$ is an $(N-i)\times(N-i+1)$ blocking matrix, *i.e.*,, $\mathbf{B}_i\mathbf{h}_i = 0$, and the vector $\mathbf{h}_i$ is the normalized correlation vector between the input vector and desired output for the i-th stage.

The projection matrix for the AVF algorithm is given by [9]

$$\mathbf{S}_D^{AV} = [\ \mathbf{g}_{AV,1}\quad \mathbf{g}_{AV,2}\quad \ldots\quad \mathbf{g}_{AV,D}\ ], \quad (4)$$

[1]The more accurate, more realistic three bit scenario can also be treated, but for notational clarity, it is not examined herein.

[2]We note that in all cases, $\mathbf{S}_D$ may not be a true projection, *e.g.* idemptotent, *etc.*. However as the operation above yields a reduced dimension signal, we use this terminology.

where $\mathbf{g}_{AV,1}^H$ is equal to the normalized correlation vector $\mathbf{E}\{d_1(m)\mathbf{y}(m)\} = \mathbf{h}_1$, and $\mathbf{g}_{AV,i}, i = 2,\ldots,D$ are auxiliary vectors, with $\mathbf{g}_{AV,i+1}$ defined by [9]

$$\frac{\mathbf{R}\mathbf{g}_{AV,i}^{Eq} - \mathbf{g}_{AV,1}\left(\mathbf{g}_{AV,1}^H\mathbf{R}\mathbf{g}_{AV,i}^{Eq}\right) - \sum_{j=2}^{i}\mathbf{g}_{AV,j}\left(\mathbf{g}_{AV,j}^H\mathbf{R}\mathbf{g}_{AV,i}^{Eq}\right)}{\|\mathbf{R}\mathbf{g}_{AV,i}^{Eq} - \mathbf{g}_{AV,1}\left(\mathbf{g}_{AV,1}^H\mathbf{R}\mathbf{g}_{AV,i}^{Eq}\right) - \sum_{j=2}^{i}\mathbf{g}_{AV,j}\left(\mathbf{g}_{AV,j}^H\mathbf{R}\mathbf{g}_{AV,i}^{Eq}\right)\|}, \quad (5)$$

where $\mathbf{g}_{AV,i}^{Eq} = \mathbf{g}_{AV,1} - \sum_{j=2}^{i} c_j\mathbf{g}_{AV,j}$, and $c_j, j = 2,\ldots,i$ are the optimized constants [9]. Notice that the auxiliary vectors $\mathbf{g}_{AV,i}, i = 1,\ldots,D$ are restricted to be orthonormal vectors.

Using the Cayley-Hamilton theorem, Moshavi *et al.* [8] proposed the following projection matrix

$$\begin{aligned}\mathbf{S}_D^{CH} &= [\ \mathbf{g}_{CH,1}\quad \mathbf{g}_{CH,2}\quad \ldots\quad \mathbf{g}_{CH,D}\ ] \\ &= [\ \mathbf{h}_1\quad \mathbf{R}\mathbf{h}_1\quad \ldots\quad \mathbf{R}^{D-1}\mathbf{h}_1\ ].\end{aligned} \quad (6)$$

IV. SIMPLIFICATION OF THE AVF ALGORITHM

In the AVF method, the second auxiliary vector $\mathbf{g}_{AV,2}$ is determined by [9]

$$\mathbf{g}_{AV,2} = \arg\max_{\mathbf{g}_{AV,2}} |\mathbf{g}_{AV,2}^H\mathbf{R}\mathbf{g}_{AV,1}|, \quad \text{subject to } \mathbf{g}_{AV,2}^H\mathbf{g}_{AV,1} = 0, \|\mathbf{g}_{AV,2}\| = 1, \quad (7)$$

while $\mathbf{g}_{AV,i+1}, i = 2,\ldots,D-1$ are optimized by

$$\mathbf{g}_{AV,i+1} = \arg\max_{\mathbf{g}_{AV,i+1}} |(\mathbf{g}_{AV,1} - \sum_{j=2}^{i} c_j\mathbf{g}_{AV,j})^H\mathbf{R}\mathbf{g}_{AV,i+1}|, \quad (8)$$

subject to $\mathbf{g}_{AV,i+1}^H\mathbf{g}_{AV,j} = 0, j = 1,\ldots,i$ and $\|\mathbf{g}_{AV,i+1}\| = 1$. We next show how the AVF algorithm can be substantially simplified. This is accomplished by the following proposition.

Proposition 1: For any given normalized vector $\mathbf{v}_0$ and $\mathbf{R}$, let vector $\mathbf{v}_1$ be determined by

$$\mathbf{v}_1 = \arg\max_{\mathbf{v}_1} |\mathbf{v}_1^H\mathbf{R}\mathbf{v}_0|, \quad \text{subject to } \mathbf{v}_1^H\mathbf{v}_1 = 1, \mathbf{v}_1^H\mathbf{v}_0 = 0, \mathbf{v}_1^H\mathbf{R}\mathbf{v}_0 \text{ real},$$

Now let $\mathbf{v}_2$ be another vector such that $\mathbf{v}_2^H\mathbf{v}_2 = 1, \mathbf{v}_2^H\mathbf{v}_0 = 0$, and $\mathbf{v}_2^H\mathbf{v}_1 = 0$. Then we have

$$\mathbf{v}_2^H\mathbf{R}\mathbf{v}_0 = 0. \quad (9)$$

The proof can be found in [1].

Using Proposition 1, we can obtain an equivalent, but more efficient way to derive the auxiliary vectors for the AVF algorithm

$$\mathbf{g}_{AV,i+1} = \arg\max_{\mathbf{g}_{AV,i+1}} |\mathbf{g}_{AV,i}^H\mathbf{R}\mathbf{g}_{AV,i+1}|, \quad (10)$$

subject to $\mathbf{g}_{AV,i+1}^H\mathbf{g}_{AV,i+1} = 1$ and $\mathbf{g}_{AV,i+1}^H\mathbf{g}_{AV,j} = 0, j = 1,\ldots,i$. Notice that with our implementation, we do not need to determine the constants $c_i, i = 2,\ldots,D$ as in [9].

In addition, we have

$$\mathbf{g}_{AV,i}^{H}\mathbf{R}\mathbf{g}_{AV,j} = 0, \text{if } |i-j| > 1. \quad (11)$$

It can be shown that the simplified solution for $\mathbf{g}_{AV,i+1}$ is [1]

$$\frac{\mathbf{R}\mathbf{g}_{AV,i} - \mathbf{g}_{AV,i}(\mathbf{g}_{AV,i}^{H}\mathbf{R}\mathbf{g}_{AV,i}) - \mathbf{g}_{AV,i-1}(\mathbf{g}_{AV,i-1}^{H}\mathbf{R}\mathbf{g}_{AV,i})}{\|\mathbf{R}\mathbf{g}_{AV,i} - \mathbf{g}_{AV,i}(\mathbf{g}_{AV,i}^{H}\mathbf{R}\mathbf{g}_{AV,i}) - \mathbf{g}_{AV,i-1}(\mathbf{g}_{AV,i-1}^{H}\mathbf{R}\mathbf{g}_{AV,i})\|}, \quad (12)$$

where Equation (11) has been used. That is, in deriving $\mathbf{g}_{AV,i+1}$, we need to focus on vectors $\mathbf{g}_{AV,i}$ and $\mathbf{g}_{AV,i-1}$ only. The auxiliary vectors $\mathbf{g}_{AV,j}, j = 1, \ldots, i-2$ will not have an effect on optimizing $\mathbf{g}_{AV,i+1}$. Therefore, the derivation of the auxiliary vectors for the AVF algorithm is greatly simplified.

V. EQUIVALENCE OF THE REDUCED RANK MMSE FILTERING METHODS

In addition to the constraint that $\mathbf{B}_i\mathbf{h}_i = 0$ as in [3], we further let

$$\mathbf{B}_i\mathbf{B}_i^H = \mathbf{I}_{N-i}, i = 1, \ldots, D-1, \quad (13)$$

i.e., $\mathbf{B}_i$ is restricted to be orthonormal in the row space [3]. Given this additional constraint, it can be proven that the performance of the MWF algorithm is not a function of the choice of the set $\mathbf{B}_i, i = 1, \ldots, D-1$, which satisfy the desired constraints. More interestingly, this additional constraint makes the necessary connection between the AVF algorithm and the MWF algorithm and leads us to the proof of the equivalence of the two algorithms.

Proposition 2: The projection matrix $\mathbf{S}_D^{MW}$ for the D stage MWF algorithm is independent of $\mathbf{B}_i, i = 1, \ldots, D-1$. Consequently, the performance of the MWF algorithm is independent of $\mathbf{B}_i, i = 1, \ldots, D-1$.

Corollary 2.1:

$$\mathbf{S}_D^{MW} = \mathbf{S}_D^{AV}. \quad (14)$$

That is, the MWF method is exactly equivalent to the AVF method. We also have

Corollary 2.2:

$$span\{\mathbf{g}_{MW,1}, \mathbf{g}_{MW,2}, \ldots, \mathbf{g}_{MW,D}\} = span\{\mathbf{h}_1, \mathbf{R}\mathbf{h}_1, \ldots \mathbf{R}^{D-1}\mathbf{h}_1\}. \quad (15)$$

The derivations of Proposition 2 and its Corollaries can be found in [1].

We note that the same result is found in [3]. However, our proof of Proposition 2 leads to a straightforward alternative proof of this statement.

Therefore, the MWF method, the AVF method and the Cayley Hamilton method of [8] are equivalent to each other.

VI. REDUCED RANK ALGORITHMS FOR MULTIPATH FADING CHANNELS

In multipath fading channels, two different cases are often considered in designing MMSE type receivers, corresponding to two different scenarios where the channel coefficient for each path of the user of interest is (1) known *a priori*, and (2) unknown *a priori* [4].

[3]Interestingly, in computer simulations, we observed that the choice of blocking matrices (without the row space constraint) did not affect performance.

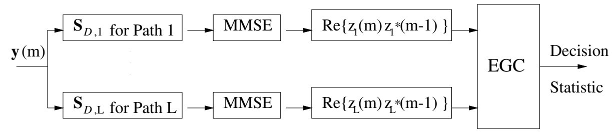

Figure 1: System diagram for the direct extension of the reduced rank techniques for multipath fading channels.

A. KNOWN CHANNEL

The steering vector [4] can now be explicitly constructed as

$$\mathbf{p}_1(m) = \sum_{l=1}^{L} A_{1l}\gamma_{1l}(m)\mathbf{s}_{1l}^{+} = \check{\mathbf{s}}_{1,1:L}^{+} diag(\Gamma_1(m)), \quad (16)$$

where $\check{\mathbf{s}}_{1,1:L}^{+} = \mathbf{s}_{1,1:L}^{+}\mathbf{A}_1$, $\mathbf{s}_{1,1:L}^{+} = [\mathbf{s}_{11}^{+}, \mathbf{s}_{12}^{+}, \ldots, \mathbf{s}_{1L}^{+}]$, $\Gamma_1(m) = [\gamma_{11}(m), \ldots, \gamma_{1L}(m)]^T$, $\mathbf{A}_1 = diag([A_{11}, \ldots, A_{1L}])$, and $diag(.)$ means to diagonalize. The projection matrix can thus be constructed from $\mathbf{p}_1(m)$ and the reduced rank detector follows naturally.

B. UNKNOWN CHANNEL

In this case, the channel coefficients $\gamma_{1l}(m), l = 1, \ldots, L$ are unknown *a priori*. However, it is assumed that the number of multiple paths, L, and the path delays, $\tau_{1l}, l = 1, \ldots, L$, are known. Given that noncoherent MMSE receivers offer improved performance for fast fading multipath environments, we consider such receivers herein. The noncoherent equal gain combiner of minimum variance detector (EGMV) was proposed in [7]. The $N \times L$ dimensional coefficients $\mathbf{C}_{EGMV}(m)$ are given by [7]

$$\mathbf{C}_{EGMV}(m) = \hat{\mathbf{R}}^{-1}(m)\mathbf{s}_{1,1:L}^{+}\left((\mathbf{s}_{1,1:L}^{+})^H\hat{\mathbf{R}}^{-1}(m)\mathbf{s}_{1,1:L}^{+}\right)^{-1} (\mathbf{s}_{1,1:L}^{+})^H\mathbf{s}_{1,1:L}^{+}. \quad (17)$$

Notice that $\mathbf{C}_{EGMV}(m)$ is an $N \times L$ matrix, with each column corresponding to the linear mapping for each resolvable path. The outputs of the L paths are then differentially detected before equally combined.

Now let us extend the reduced rank techniques for multipath fading channels. One immediate solution is to design the reduced rank algorithm for each path since the spreading code for the path is known. The reduced-dimensional received signal after the projection matrices for each path is then individually optimized based on the MMSE criterion before noncoherently equally combined, as shown in Figure 1, where $\mathbf{S}_{D,l}, l = 1, 2, \ldots, L$ is the projection matrix of the reduced rank scheme for the l-th path. By collecting the L reduced rank detectors for L paths to a $D \times L$ matrix, we have

$$\mathbf{C}_{MRR}(m) = [(\mathbf{S}_{D,1}^H\mathbf{R}\mathbf{S}_{D,1})^{-1}\mathbf{e}_1, (\mathbf{S}_{D,2}^H\mathbf{R}\mathbf{S}_{D,2})^{-1}\mathbf{e}_1, \ldots, (\mathbf{S}_{D,L}^H\mathbf{R}\mathbf{S}_{D,L})^{-1}\mathbf{e}_1], \quad (18)$$

where $\mathbf{e}_1 = [1, 0, \ldots, 0]^T$ of dimension $D \times 1$. However, it turns out that the performance of this detector is significantly worse than the EGMV solution.

The proposed reduced rank technique is shown in Figure 2. Instead of working on the D-dimensional received signal after the projection operation for each path, we are now working on the $DL \times 1$ dimensional signal, which is the group of all the

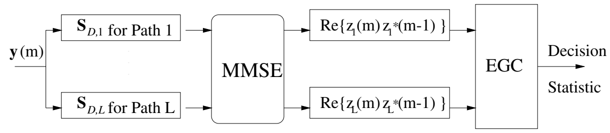

Figure 2: System diagram for the proposed reduced rank techniques for multipath fading channels.

L branches' outputs. Again the MMSE criterion is applied to to obtain the optimum coefficients of dimension $DL \times L$ for noncoherent demodulation, which can be derived as [1]

$$\mathbf{C}_{new}(m) = \mathbf{f}(m)\left((\mathbf{S}_{D,1:L}^H \mathbf{s}_{1,1:L}^+)^H \mathbf{f}'(m)\right)^{-1} \left(\mathbf{S}_{D,1:L}^H \mathbf{s}_{1,1:L}^+\right)^H \mathbf{S}_{D,1:L}^H \mathbf{s}_{1,1:L}^+. \quad (19)$$

where $\mathbf{S}_{D,1:L}$ is the $N \times DL$ matrix given by $\mathbf{S}_{D,1:L} = [\ \mathbf{S}_{D,1} \quad \mathbf{S}_{D,2} \quad \cdots \quad \mathbf{S}_{D,L}\ ]$, and

$$\mathbf{f}(m) = (\mathbf{S}_{D,1:L}^H \mathbf{R} \mathbf{S}_{D,1:L})^{-1} \mathbf{S}_{D,1:L}^H \mathbf{s}_{1,1:L}^+. \quad (20)$$

VII. SIR ANALYSIS FOR NON-COHERENT MMSE RECEIVERS

For the EGMV detector and the corresponding reduced rank techniques (as shown Figure 1 and Figure 2), there are L multiple filters for the L multipaths. For both receivers, the decision statistic for noncoherent combining can be given by,

$$z(m) = \sum_{l=1}^{L} Re\{\mathbf{c}_l(m)^H \mathbf{y}(m)\mathbf{y}(m-1)^H \mathbf{c}_l(m-1)\}. \quad (21)$$

The linear mapping, $\mathbf{C}(m) = [\mathbf{c}_1(m), \mathbf{c}_2(m), \ldots, \mathbf{c}_L(m)]$ above refers to either (19) or (20) above.

Proposition 3: The mean and variance of the decision statistic for the EGMV/RR detectors with noncoherent equal gain combining can be approximated by [1]

$$\mathbf{E}\{z(m)|b_1(m)\} \approx b_1(m)\rho \sum_{l=1}^{L} \mathbf{c}_l(m)^H \check{\mathbf{s}}_{1,1:L}^+ (\check{\mathbf{s}}_{1,1:L}^+)^H \mathbf{c}_l(m-1), \quad (22)$$

and

$$\mathbf{Var}\{z(m)|b_1(m)\} \approx \sum_{l_1=1}^{L}\sum_{l_2=1}^{L} \left\{ |\mathbf{c}_{l_1}(m)^H \mathbf{R} \mathbf{c}_{l_2}(m)|^2 - |\mathbf{c}_{l_1}(m)^H \mathbf{R}_u \mathbf{c}_{l_2}(m)|^2 \right\}, \quad (23)$$

where

$$\mathbf{R} = \sum_{k=1}^{K} \left\{ (\check{\mathbf{s}}_{k,1:L}^+)^H \check{\mathbf{s}}_{k,1:L}^+ + (\check{\mathbf{s}}_{k,1:L}^-)^H \check{\mathbf{s}}_{k,1:L}^- \right\} + \sigma^2 \mathbf{I}_N, \quad (24)$$

and

$$\mathbf{R}_u \triangleq \mathbf{E}\{\mathbf{y}_1(m)\mathbf{y}_1(m)\} = (\check{\mathbf{s}}_{1,1:L}^+)^H \check{\mathbf{s}}_{1,1:L}^+. \quad (25)$$

where $\mathbf{y}_1(m)$ denotes the signal part corresponding to user 1 in $\mathbf{y}(m)$, $\check{\mathbf{s}}_{k,1:L}^+ = \mathbf{s}_{k,1:L}^+ \mathbf{A}_k$, $\mathbf{A}_k = diag([A_{k1}, A_{k2}, \ldots, A_{kL}])$ and $\mathbf{c}_l(m) \approx \mathbf{c}_l(m-1)$ has been used. Notice that $\mathbf{R}$ in Equation (24) corresponds to the case when the channel variations are so fast that the receiver can only track the mean power of the active users [4].

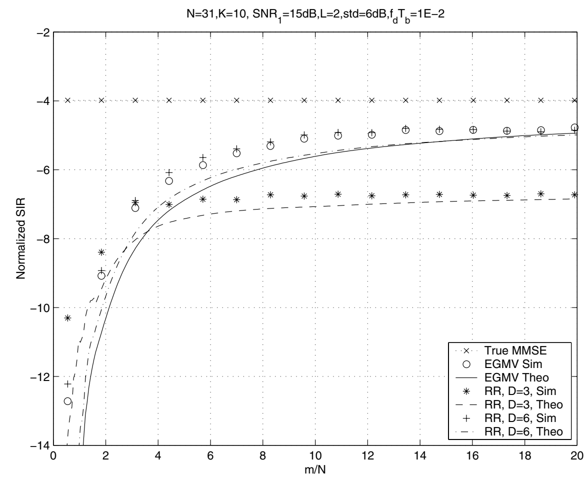

Figure 3: Normalized SIR tracking comparison for the simulations and theoretical analysis, N=31,K=10, $SNR_1 = 15dB$,L=2,std=6dB, $f_d T_b = 10^{-2}$.

Thus, the SIR for the EGMV/RR detectors is given by $\mathbf{E}^2\{z(m)|b_1\}/\mathbf{Var}\{z(m)|b_1\}$. independent

Figure 3 shows the SIR tracking versus the number of observations for the theoretical analysis and computer simulations, where $N = 31$,$K = 10$,$L = 2$ and the Doppler frequency is set to 100Hz. Three detectors are shown for comparison: the EGMV detector and the RR detectors with $D = 3$ and $D = 6$. It can be seen that the approximate analysis agrees fairly well with the simulated results, especially when the number of observations increases. Furthermore, the SIR analysis can be employed to predict the relative behaviour of several MMSE-based algorithms.

VIII. SIMULATION RESULTS

In the following simulations, if not specified, $D = 8$, the spreading codes are Gold codes [10] of length $N = 31$, the number of multipaths L is 2 with Rayleigh fading for each path, the forgetting factor $\lambda = 0.995$, the product of the Doppler frequency and the symbol period $f_d T_b = 10^{-2}$, and the powers of the active users are randomly chosen from the log-normal distribution with a deviation of 6 dB.

In multipath fading channels, when the channel parameters of the user of interest are known, it is straightforward to construct the single blind adaptive MMSE filter (denoted as `S. Blind MMSE known') and subsequently, the single reduced rank algorithm filter (denoted as `S. RR known'). In the case of unknown channels, the EGMV detector and the two reduced rank techniques discussed in Section VI will be examined. For convenience, the direct extension scheme is denoted as `M. RR unknown', while the modified reduced rank detection scheme is denoted as `New RR unknown'. It can be seen from Figure 4 that the reduced rank detection for the known channel case produces comparably performance to the full-rank blind adaptive MMSE detector. For the unknown channel case, the reduced rank detection which operates independently on each path (M. RR unknown) has much worse performance than the

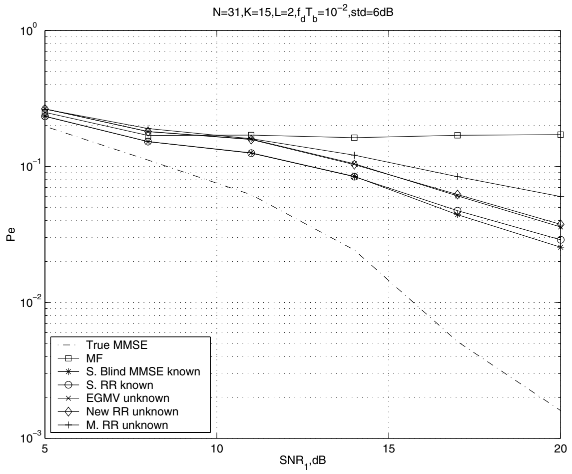

Figure 4: Performance comparison of the MMSE and the reduced rank detectors in multipath fading channels, N=31,K=15,L=2,D=8,λ=0.995, and $f_d T_b = 10^{-2}$.

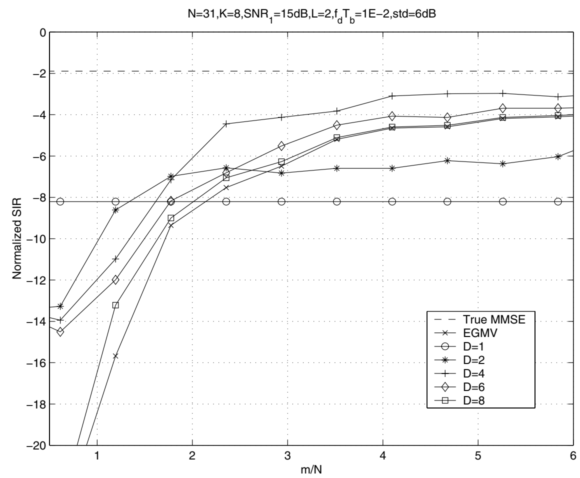

Figure 5: Normalized SIR tracking vs. the number of observations. N=31,K=8,L=2,$SNR_1 = 15dB$,$f_d T_b = 10^{-2}$, std=6dB.

modified reduced rank scheme (New RR unknown). The proposed reduced rank detection also achieves nearly the same performance as the full-rank EGMV detector.

Figure 5 shows the normalized SIR as a function of the number of observations (normalized by N), using the approximate analysis presented in Section VII. The systems parameters are detailed in the figure. It can be seen that for $m < N$, $D = 1$ performs better than others. The RR detector with $D = 2$ will then be the superior detector until m approaches $3N$ where $D = 4$ takes over. For a short data record, the full rank EGMV detector is inferior to the reduced rank detectors. However, it can be expected that when the number of observations is large enough, the full rank EGMV will eventually outperform the reduced rank detectors.

IX. CONCLUSIONS

This paper simplifies the AVF algorithm [9] and proves the equivalence of three reduced rank algorithms. It then extends the reduced rank algorithms to multipath fading channels. The output SIR for the noncoherent equal gain combining linear receiver is also derived.

References

[1] W. Chen. *Adaptive MMSE Receivers For DS/CDMA Communications In Fast Fading Environments*. Master Thesis, the Ohio State Univ., 2001.

[2] J. S. Goldstein, I. S. Reed, and L. L Scharf. A multistage representation of the Wiener filter based on orthogonal projections. *IEEE Trans. on Info. Theory*, 44(7):2943–2959, November 1998.

[3] M. L. Honig and J. S. Goldstein. Adaptive reduced-rank interference suppression based on the multi-stage Wiener filter. *to appear on IEEE Trans. Comm.*

[4] M. L. Honig, S. L. Miller, M. J. Shensa, and L. B. Milstein. Performance of adaptive linear interference suppression in the presence of dynamic fading. *IEEE Trans. Comm.*, 49(4):635–645, April 2001.

[5] U. Madhow and M. L. Honig. MMSE interference suppression for DS/SS CDMA. *IEEE Trans. Comm.*, 42:3178–3188, December 1994.

[6] U. Madhow, L. J. Zhu, L. Galup, and M. B. Matthews. Differential MMSE: new adaptive algorithms for equalization, interference suppression, and beamforming. *Thirty-Second Asilomar Conference on Signals, Systems and Computers*, 1:640–644, 1998.

[7] S. L. Miller, M. L. Honig, and L. B. Milstein. Performance analysis of MMSE receivers for DS-CDMA in frequency-selective fading channels. *IEEE Trans. Comm.*, 48:1919–1929, November 2000.

[8] S. Moshavi, E. G. Kanterakis, and D. L. Schilling. Multistage linear receivers for DS-CDMA systems. *International Journal of Wireless Information Networks*, 3(1):1–17, 1996.

[9] D. A. Pados and S. N. Batalama. Joint space-time auxiliary-vector filtering for DS/CDMA systems with antenna arrays. *IEEE Trans. Comm.*, 47(9):1406–1415, September 1999.

[10] J. G. Proakis. *Digital Communications*. NY: McGraw-Hill, 3rd ed., 1995.

[11] Z. Xie, R. T. Short, and C. K. Rushforth. A family of suboptimum detectors for coherent multi-user communications. *IEEE Journal on Selected Areas on Communications*, 8:683–690, May 1990.