ON DESIGNING THE OPTIMAL TEMPLATE WAVEFORM FOR UWB IMPULSE RADIO IN THE PRESENCE OF MULTIPATH

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ABSTRACT

Using an appropriate template waveform matched to the received signal allows extracting the energy of the received signal efficiently. This efficiency becomes vital for Ultra Wide Bandwidth (UWB) Impulse Radio in the presence of multipath, where each path undergoes a different channel causing distortion in the received pulse shape due to a variety of factors such as different amounts of attenuation for different frequencies [1], [2]. In such a situation, using a clean ideal line of sight path signal as a template may degrade the performance due to the mismatches between the template waveform and the received signal. Furthermore, because of inherent filtering in the RF processing (i.e., antennas, amplifiers, etc.), it is often difficult to determine even such a clean line of sight pulse. In this paper, algorithms for designing optimal template waveforms for UWB Impulse Radio are developed and the improvement over a more traditional template waveform used for this kind of radio is illustrated.

1. INTRODUCTION

Due to sending a sub-nanosecond pulse in each frame period, impulse radio enjoys a very high multipath resolution capability and a very low duty cycle signal with huge spread spectrum processing gain [3], [4], [5]. On the other hand ultra-wide bandwidth suggests that the higher frequencies attenuate more than the lower frequencies [1], [2], causing distortion in the shape of the received pulse. The delay spread of the impulse radio received signal is many many pulse durations even for indoor applications. These phenomena motivate us to design an algorithm which derives an optimal template waveform at the receiver that captures the most amount of energy with the least number of correlations. Since the effects of the channel are somehow embedded in the received signal, we can compute the optimal template waveform based on the received signal online. This makes our algorithm adaptive, since with changes in the channel, the received signal changes, and so does our optimal template based on the received signal. We show the improvement achieved by this optimal template waveform compared to more traditional second order derivative of Gaussian waveform [6], by applying our iterative algorithm to real data obtained from measurement experiments taken in the Wireless Radio Lab of the University of Southern California. Using our template waveform algorithm helps us adapt our template to different environments based on the received signal which embodies all the channel characteristics, including those of the antennas on the waveforms which are sometimes not well-understood. The algorithm developed in this section is not limited to UWB systems only, and can be applied to any kind of communication system.

In Section II, optimal long-tailed template waveform design is presented, which then leads us to design optimal single path template waveform in Section III. Conclusion remarks are made in section IV.

2. LONG-TAILED TEMPLATE

Using the digital sampling oscilloscope, 9 measurements have been taken in the Wireless Radio Lab of the University of Southern California. These measurements are shown in Fig. 1. These are the received signals from a pulser that generates monocycles. It is worth mentioning that each measurement is the average of 256 received profiles at the same location to get a more stable measurement and neglect some transient effects. We sample each measurement at a rate greater than Nyquist rate and normalize them to have unit energy. After these procedures, we represent each measurement by a vector,

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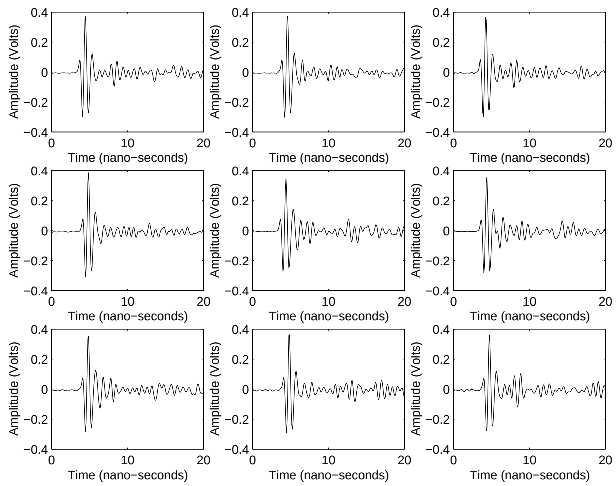

Figure 1: Measurements from the UWB radio received waveform taken at the Wireless Radio Lab at the University of Southern California.

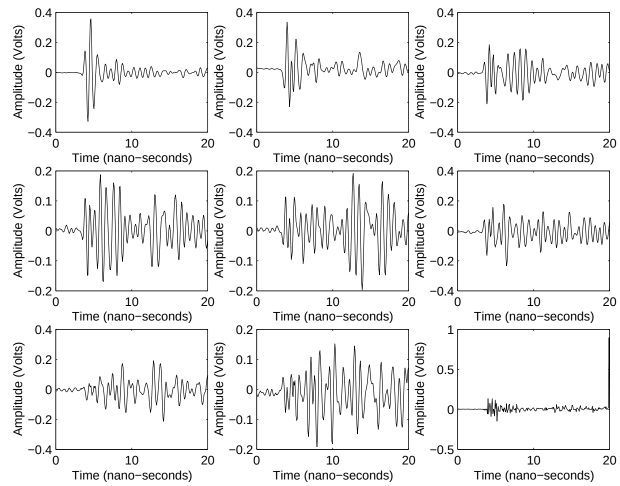

Figure 2: Orthonormal templates as the output of the algorithm.

namely, $\mathbf{r}_i = [r_{i1} r_{i2} ... r_{in}]^t$, for $i = 1, 2, ..., 9$. Now we find the vector $\mathbf{w} = [w_1 w_2 ... w_n]^t$ for which the function,

$$F = \sum_{i=1}^{N} | < \mathbf{r}_i, \mathbf{w} > |^2 \qquad (1)$$

is maximum. In this case, we find the nearest vector to all the measurement vectors in the sense that it captures the most energy out of the measurements if we just want to do a single correlation at the receiver [1]. We set $\mathbf{w}$ to be of unit energy for normalization purposes. This is a constrained optimization problem (e.g., see appendix C in [7]) with $\|\mathbf{w}\| = 1$. Solving this optimization problem, we get $(A + \lambda I)\mathbf{w} = 0$ where $A = M * M^t$ and M is a matrix whose ith column is $\mathbf{r}_i$. Therefore, $\mathbf{w}$ is simply the normalized eigen vector of matrix A corresponding to its largest eigen value since $F = \mathbf{w}^t A \mathbf{w} = \mathbf{w}^t(-\lambda \mathbf{w}) = -\lambda \|\mathbf{w}\|^2 = -\lambda$. This normalized eigen vector is simply the optimal template waveform when we want to do only one correlation against the received signal at the receiver. This problem can be generalized in a straight forward manner to the case when we want to design two or more orthogonal template waveforms that capture the energy of the received signal optimally. The solution is the eigen vectors corresponding to the largest eigen values of matrix A. Since the matrix A is symmetric, these template waveforms can be selected orthogonal. Fig. 2 shows 9 orthonormal template waveforms corresponding to the nine nonzero eigen values of matrix A. The first template waveform at the upper left corner of the figure captures 58.39% of the total energy contained in all the measurements by just one correlation with each measurement. The second template in the upper middle of the figure, captures 23% of the total energy out of all the measurements with just one correlation with each measurement. The rest of the templates capture $8.38\%, 3.61\%, 2.21\%, 1.71\%, 1.20\%, 0.83\%, 0.67\%$ respectively. For nine measurements, we should be able to capture the whole energy with at most nine single long-tailed orthonormal template waveforms. By looking at matrix A, we see that each element somehow computes the average of the correlation between two specified components of each measurement, over all the measurements.

[1]This is equivalent to a LS criterion when $\|\mathbf{w}\|$ is constrained to be constant.

3. SINGLE PATH TEMPLATE

As Fig. 1 shows, there are a lot of paths in the received signal. We want to resolve the paths and find the optimal template waveform based on these individual paths. In that case we desire a short-tailed template waveform (i.e., with support much less than the delay spread) and we may use it to do a selective multiple combining [8] for the most dominant paths. Define the new objective function F as

$$F = |\mathbf{r} - \sum_{j=1}^{L} c_j \mathbf{w}(n - n_j)|^2 \qquad (2)$$

where $\mathbf{r}$ is a typical received waveform, n_j is the delay associated to its jth path and c_j is the corresponding amplitude. As the above formula suggests, we assume only the first L dominant paths in our model. In order to minimize F with respect to $\mathbf{w}$, c_j's, and n_j's, we first minimize F conditioned on a given waveform $\mathbf{w}$ as an initial estimation. A good initial estimation can be the truncated version of the long-tailed template waveform obtained in the last section. After finding the optimized values of c_j's and n_j's for $j = 1, 2, ..., L$ based on this initial waveform, we use these values of the coefficients c_j's and delays n_j's to find the optimized waveform $\mathbf{w}$ of length m. Now we use this new optimized waveform $\mathbf{w}$ to compute the new values of the coefficients and delays and we repeat this procedure again and again until convergence occurs for the waveform $\mathbf{w}$. The length of $\mathbf{w}$, m, is a design parameter. If we assign a very small length for the template waveform, then it will not be effective, since it requires more correlations against the received signal to capture the same amount of energy. Therefore, we can choose an initial value for m, and then obtain the template waveform and compute the number of correlations to capture a specified amount of energy out of the received signal. Then we increase m, and repeat the same procedure again. If the reduction in the number of correlations to capture energy is significant, we increase m again up to the point where the reduction is not worth choosing a longer template waveform or we get negligible values for the template waveform after some point. For the case when we know the width of one received pulse, we can simply choose m such that it meets the width of the pulse. So $F = |(\mathbf{r} - \sum_{j=1}^{L} c_j \mathbf{w}_I(n - n_j))|^2$ where subscript I means the initial estimation for $\mathbf{w}$. Assume the received signal before sampling as $r(t) = s(t) + \tilde{n}(t)$ where $\tilde{n}(t)$ is the additive white Gaussian noise with power spectral level of $\frac{N_0}{2}$. The received signal $r(t)$ consists of several paths at specific delays $n_i = i\Delta n, i = 1, 2, ..., L$, and amplitudes c_i's, for $i = 1, 2, ..., L$. Assuming selective combining for the first L dominant paths, we ignore the rest of the paths:

$$r(t) = \sum_{i=1}^{L} c_i \mathbf{w}(t - n_i) + \tilde{n}(t) \tag{3}$$

Finding the maximum likelihood (ML) estimator is equivalent to finding the Minimum Mean Squared Estimates (MMSE) of $\hat{c}_i$'s and $\hat{n}_i$'s, because $\tilde{n}(t)$ is AWGN. Defining $\mathbf{c} = [c_1 c_2 ... c_L]^t$ and $\mathbf{n} = [n_1 n_2 ... n_L]^t$ and ignoring the irrelevant term of the above integral in calculating the MMSE of $\mathbf{c}$ and $\mathbf{n}$ we get the following estimations [6]:

$$\hat{\mathbf{n}} = \arg\max(\mathbf{X}^{+}(\mathbf{n})\mathbf{R}^{-1}\mathbf{X}(\mathbf{n})) \tag{4}$$

and

$$\hat{\mathbf{c}} = \mathbf{R}^{-1}\mathbf{X}(\hat{\mathbf{n}}) \tag{5}$$

where

$$\mathbf{X}(\mathbf{n}) = \int_0^T r(t) \begin{pmatrix} w(t-n_1) \\ w(t-n_2) \\ \vdots \\ w(t-n_L) \end{pmatrix} dt \tag{6}$$

and the correlation matrix R is

$$\mathbf{R} = \begin{pmatrix} R(n_1-n_1) & R(n_1-n_2) & \cdots & R(n_1-n_L) \\ R(n_2-n_1) & R(n_2-n_2) & \cdots & R(n_2-n_L) \\ \vdots & & & \\ R(n_L-n_1) & R(n_L-n_2) & \cdots & R(n_L-n_L) \end{pmatrix} \tag{7}$$

where $R(n_i - n_j) = \int_0^T w(t-n_i)w(t-n_j)\,dt$.

Using the values obtained for delays and amplitudes in (4) and (5) to compute the optimal $\mathbf{w}$, we repeat the whole procedure with our new $\mathbf{w}$ instead of $\mathbf{w}_I$ until the optimal template waveform converges to its final format. In order to compute the new $\mathbf{w}$, we need to minimize $F = |\mathbf{r} - \sum_{j=1}^{L} \hat{c_j}\mathbf{w}(n - \hat{n_j})|^2$. In this equation, $\mathbf{r}$ is an $n \times 1$ vector, and $\mathbf{w}$ is an $m \times 1$ vector, and in order to write the above equation correctly, we need to add zeros to each $\mathbf{w}(n - \hat{n_j})$ such that it becomes a vector of order $n \times 1$ too, i.e., $\mathbf{w}(n - \hat{n_j}) = [00...0w_1w_2w_3...w_m000...0]^t$ where we have added $\hat{n_j}$ zeros at the beginning of the vector, and then we have the unknown coefficients, w_j's, which are to be determined, and finally we add $n - m - \hat{n_j}$ zeros to complete the dimension as an $n \times 1$ vector.

In order to minimize F with respect to w_j's for $j = 1, 2, ..., m$, we need to take the derivatives of F with respect to each w_j and equate them to zero.

$$\frac{\partial F}{\partial w_p} = 2\sum_{k=1}^{L} c_k^2 w_p + 2\sum_{k=1}^{L}\sum_{l<k} c_k c_l (w_{p-\delta_{lk}} + w_{p+\delta_{lk}}) - 2\sum_{i=1}^{L} r_{n_i+p} c_i = 0, \text{p=1,2,...,m.} \tag{8}$$

where $\delta_{lk} = |n_k - n_l|$. In (8) set $w_j = 0$ for $j > m$, or $j < 0$. The above linear system of equations can be solved for the given values of c_i's and n_j's using any standard algorithm for solving a linear system of equations available in any numerical computation book.

The last step is to normalize the template waveform obtained from all the above steps in order to make it of unit energy as the constraint of our optimization, so that we can compare its performance with any other unit energy template in terms of the amount of the captured energy after correlation.

$$\mathbf{w}_{opt} = \frac{\mathbf{w}}{||\mathbf{w}||} \qquad (9)$$

where $\mathbf{w}_{opt}$ is the optimal template waveform as the output of our algorithm.

In order to determine whether the algorithm has converged to $\mathbf{w}_{opt}$ or not, we can use the following criterion: If $||\mathbf{w}_{opt}^{(k+1)} - \mathbf{w}_{opt}^{(k)}|| < \alpha$ for some positive α, then stop running the algorithm, otherwise continue from step two. The output of the algorithm after the kth iteration has been denoted by $\mathbf{w}_{opt}^{(k)}$. Here, α depends on the accuracy needed. The smaller the α, the better the approximation. Fig. 3 demonstrates the output of the algorithm, $\mathbf{w}_{opt}$, along with the second-order derivative of Gaussian waveform. Fig. 4 shows the flow chart of the algorithm.

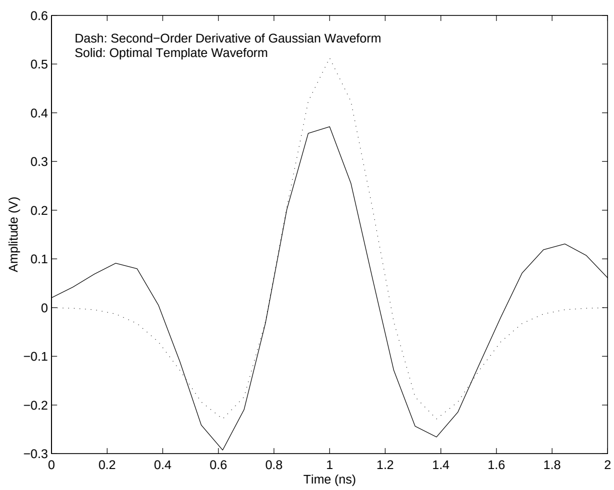

Figure 3: Optimal template along with the second-order derivative of Gaussian waveform

As (4) suggests, there is a nonlinear complexity associated to the exhaustive search for finding the optimal values of the dominant paths's arrival times. However, we can simply use a suboptimal linear search when we assume a negligible overlap between adjacent paths. This can be explained by looking at (7). In this case, we can see that matrix R becomes strongly diagonal, so does its inverse in (4). Therefore, (4) suggests that we need to search for those values of $\mathbf{n}$ where $||\mathbf{X}||^2$ becomes maximum. Since we can search for each dominant path independently in this case, this simply means to find those values of

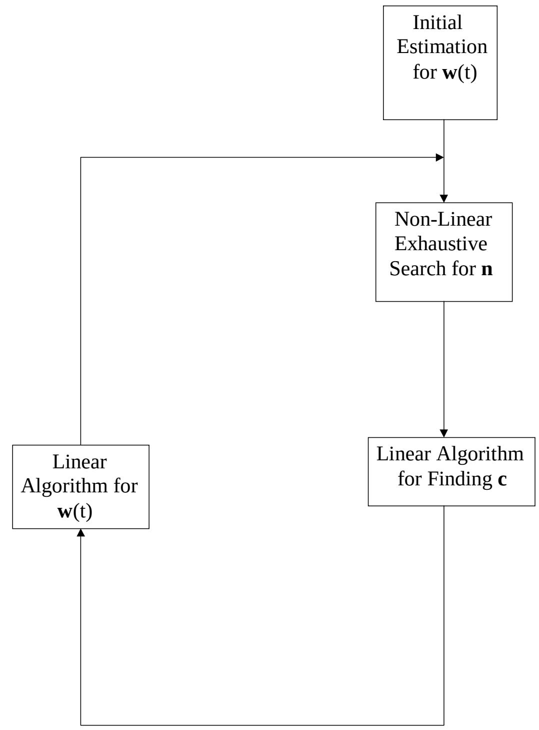

Figure 4: Flow chart of the algorithm

$\mathbf{n}$ for which the magnitude of each component of $\mathbf{X}$ is maximum. This is a linear complex search in terms of the number of components of $\mathbf{X}$. It is worth mentioning that this suboptimal algorithm becomes optimal for the case when there is no overlap between the adjacent paths at all. Because of the excellent multipath resolution capability of impulse radio due to its ultra wide bandwidth, we can employ the suboptimal algorithm with some confidence. Since the results obtained by the suboptimal algorithm match those of optimal algorithm with a high precision, the results presented here reflect those obtained by running the fast linear suboptimal algorithm. Running the suboptimal algorithm on a various generated data using computer simulation has shown that the algorithm resolves the paths successfully under different multipath scenarios where two different paths can even overlap with each other.

For any given $w(t)$ at each iteration of the algorithm, c_i's and n_i's are the maximum likelihood estimates of the amplitudes and delays of different paths. Specifically, n_i's are obtained through an exhaustive search to minimize the magnitude of the difference between $\sum_{i=1}^{L} c_i \mathbf{w}(t - n_i)$ and the received waveform $r(t)$. Also at the same time, due to the AWGN nature of the problem, c_i's are mean squared estimations which minimize the mean squared error. By these explanations, we see that after estimating the amplitudes and delays of different paths, the mean-squared error becomes smaller during any iteration. For the second part of each iteration, given the estimates of amplitudes and delays, we compute the shape of the template waveform by taking derivatives to minimize the mean-squared error; therefore, we get a smaller mean-squared error after the second half of each iteration. Since the sequence of mean-squared errors is a decreasing sequence bounded from below by zero, we conclude that this sequence is convergent.

Running the algorithm when $L = 3$ for the measurement shown in the upper left of Fig. 1 and with the initial estimation as the input to the algorithm to be the second-order derivative of Gaussian waveform, demonstrates about 0.93 dB improvement in terms of the captured energy out of the three most dominant paths with respect to that of second order derivative of Gaussian. Similar results are obtained by running the algorithm on the rest of the measurements. Also, the algorithm converges very fast, and in fact after the second iteration, there is no more improvement. This verifies the optimality of our template waveform shown in Fig. 3 over the second order derivative of Gaussian.

To demonstrate the robustness of the algorithm, we consider a very bad initial estimation of the template waveform, which is just a unit energy rectangular pulse (Flat Template Waveform) over the interval $0 \leq t \leq 2$ nanoseconds. The captured energy using this template waveform is only 7 percent of the total energy. Fig. 3 shows the template waveform after only two iterations of the algorithm. It captures 97 percent of the energy and it is identical to the template waveform obtained using the second-order derivative of Gaussian as the initial estimation. The 0.9 dB improvement in extracting the energy out of the received signal using the optimal template waveform compared to the second order derivative of Gaussian, can even further mitigate the already low fading margin in UWB impulse radio.

4. CONCLUSION

Two algorithms to design optimal template waveforms were introduced: One for optimal one-time correlation long-tailed templates and the other for multi-correlation short-tailed templates. We showed how we can design optimal templates in the sense of capturing the most amount of energy with the least number of correlations against the received signal in the presence of multipath. Simulation results verifies the robustness and accuracy of our iterative algorithm. Applying our algorithm to real data obtained from measurements, we notice the capability of the algorithm in resolving the dominant paths of the multipath along with estimating the optimal template waveform shape which showed around 0.9 dB improvement over more traditionally used templates. This is a general algorithm that can be used for any communication system and not just the impulse radio; however, due to the very fine multipath resolvablity of the impulse radio, we can use suboptimal algorithm in which an exhaustive search can be replaced by a linear complexity block in the algorithm. Also since this algorithm uses the received paths to estimate the template waveform shape, channel characteristics will be implicitly embedded in the design of the optimal template, which could not be accounted for had they not been modelled mathematically.

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