Multi-Stage MMSE/MOE Receivers for Frequency Selective Fading Channels in DS-CDMA Systems

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Abstract— **A multi-stage minimum mean square error filter bank (MS-MMSE-FB) and a minimum output energy filter bank (MS-MOE-FB) for the detection of DS-CDMA (Direct-Sequence Code-Division Multi-Access) signals under a dynamic multi-path fading environment are presented. The output signal-to-interference plus noise ratios (SINRs) for both MS-MMSE-FB and MS-MOE-FB are derived. The analytical results reveal how the output SINRs of reduced-rank MS-MMSE-FB and MS-MOE-FB vary with the rank of filters. The findings coincide with results in [1], namely, the rank needed to achieve a target performance does not scale with the system loading in an additive white Gaussian noise (AWGN) channel. The derivation shows that, with the aid of desired user's channel information, the rank of the receivers can be drastically dropped without affecting the steady state SINR, an important factor for interference suppression in a dynamic fading channel with adaptive MS-MMSE-FB/MS-MOE-FB detections.**

I. Introduction

The adaptive reduced-rank interference suppression scheme based on the multi-stage Wiener filter (MSWF) [2] has been developed as a reduced-rank MMSE receiver for interference suppression in DS-CDMA [1], [3] in AWGN and flat fading channels. It is observed that the reduced-rank MSWF can achieve essentially the full-rank performance with a much lower rank than other reduced-rank schemes, *e.g.* the principal-component method. Thus, the number of samples required to estimate the filter parameters of MSWF can be greatly reduced. This feature offers a great advantage for detection in dynamic fading channels.

To obtain the diversity offered by a multi-path fading channel without the knowledge of channel state information, it was suggested in [4], [5] to adopt an adaptive multiple-MOE-filter structure for interference suppression along each resolvable transmission path of the desired user. Then, two adjacent outputs of filters are combined differentially with equal gain to form a decision statistic. This receiver is referred to as the equal-gain combining minimum variance (EGMV) filter [4]. However, according to the simulation results in [6], forming a reduced-rank filter bank which jointly suppresses interference in a multipath fading channel is better than suppressing interference in the same subspace path by path. Based on these arguments and the concept of MSWF [2], we derive a multi-stage (MS) implementation of the MMSE Filter Bank (MMSE-FB) and the MOE Filter Bank (MOE-FB) for interference suppression in multipath fading channels.

For reduced-rank filters, it is important to characterize how the dimension of each subspace, *i.e.* the rank, affects the output SINR. The lower rank implies faster output SINR convergence. For the MSWF filter, an asymptotic analysis of the output SINR for long code CDMA in an AWGN channel was provided in [3]. The spreading code is assumed randomly selected with an infinite dimension in order to reduce the analytical complexity. In this work, we offer an output SINR analysis of MS-MMSE-FB/MS-MOE-FB for more realistic short-code CDMA of dimension N in multi-path Rayleigh fading channels. The theoretically derived steady state output SINR matches well with the simulated output SINR.

The rest of this paper is organized as follows. Section II describes the system model for DS-CDMA in a multi-path fading channel. The MS-MMSE-FB and the MS-MOE-FB receivers in multi-path fading channels are derived in Section III. The performance analysis of the output SINRs for both receivers are given in Section IV. Receivers with channel state information are studied in Section V. Numerical simulation results are presented in Section VI.

II. System Model

A standard model for asynchronous DS-CDMA modulated with binary phase shift keying (BPSK) is considered below. The baseband representation of the transmitted signal of the kth user can be written as

$$x_k = \sum_{m=0}^{M-1} A_k b_k(m) s_k(t - mT_s - \tau_k),$$

where A_k, $b_k(m) \in \{-1, 1\}$ and τ_k are the amplitude, the mth symbol bit and the timing delay of user k, respectively, and T_s is the symbol period. The signature waveform is given by

$$s_k(t) = \sum_{n=0}^{N-1} c_k[n]\psi(t - nT_c),$$

where $\psi(t)$ is the chip pulse shaping function and $T_c$is the chip duration. For simplicity, it is assumed that $\psi(t)$ is a square pulse defined on interval $[0, T_c)$, and the spreading code $\mathbf{c}_k = [c_k[0], c_k[1], \ldots, c_k[N-1]]$ is fixed for each user with a period of N, $c_k(i) \in \{-1, 1\}$. $N = T_s/T_c$ is the spreading gain. Without loss of generality, user 1 is chosen to be the desired user. Its time delay τ_1 is assumed to be fixed and known by the receiver during transmission. Thus, we set τ_1 to 0 in the following discussion.

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The received baseband signal is passed through a chip-matched filter and sampled at the chip rate. Assume that the maximum path delay for each user is less than T_s. The discrete time signal vector $\mathbf{y}$ is obtained by collecting N consecutive samples of the received signal and is given by [4]

$$\begin{aligned}\mathbf{y}(m) &= \sum_{k=1}^{K}\sum_{l=1}^{L_k} A_{kl}\gamma_{kl}(m)\cdot \\ &\quad [\mathbf{s}_{kl}^{+}b_k(m) + \mathbf{s}_{kl}^{-}b_k(m-1)] + \mathbf{n}(m), \quad (1)\end{aligned}$$

where K is the number of users, L_k is the number of paths of user k, A_{kl} is the amplitude of the signal on path l of user k, $\mathbf{s}_{kl}^{+}$ and $\mathbf{s}_{kl}^{-}$ are the partial spreading codes corresponding to the current bit $b(m)$ and the previous bit $b(m-1)$ over the sampling interval, respectively. The filtered noise vector $\mathbf{n}(m)$ is complex Gaussian distributed with covariance $2\mathbf{N_0}\mathbf{I}$. Finally, the fading process, γ_{kl}, is a complex zero-mean Gaussian process that satisfies

$$E\{\gamma_{k_1 l_1}^{*}(m)\gamma_{k_2 l_2}(m)\} = \delta(k_1-k_2)\delta(l_1-l_2).$$

III. Multi-Stage MMSE/MOE Filter Bank

The MMSE receiver in a frequency selective fading channel is simply equal to [7]

$$\hat{\omega} = \arg\min_{\omega} E\|b_1 - \omega^{\mathbf{H}}\mathbf{y}\|^2 = \mathbf{R}^{-1}\mathbf{S}_1^{+}\mathbf{A}_1 E(\Gamma_1),$$

where $\mathbf{R} = E(\mathbf{y}\mathbf{y}^H)$, $\mathbf{A}_1 = diag([A_{11},\ldots,A_{1L}])$, $\mathbf{S}_1^{+} = [\mathbf{s}_{11}^{+},\ldots,\mathbf{s}_{1L}^{+}]$ and $\Gamma_1 = [\gamma_{11},\ldots,\gamma_{1L}]^T$. For simplicity, we use L to denote L_1, which is the number of paths of the desired user. Note that $E(\Gamma_1)$ involved in the MMSE filter will degenerate to zero in a fast fading channel during the observation window. A modified MMSE filter for exploiting the multi-path diversity without desired user's channel information can be formulated as

$$\hat{\omega} = \arg\min_{\omega} E\|b_1\Gamma_1 - \omega^{\mathbf{H}}\mathbf{y}\|^2 = \mathbf{R}^{-1}\mathbf{S}_1^{+}\mathbf{A}_1\Sigma_1, \quad (2)$$

where Σ_1 is the covariance matrix of the fading vector Γ_1 and is equal to $\mathbf{I}$. The decision statistic $\mathbf{z}$ and the decision rule for differential encoded BPSK (DBPSK) are

$$\mathbf{z}(m) = \hat{\omega}^H(m)\mathbf{y}(m), \quad (3)$$

$$\hat{b_1}(m) = sgn\{Re(\mathbf{z}^H(m)\mathbf{z}(m-1))\}. \quad (4)$$

The receiver is referred to as the differential Equal Gain Combining of the MMSE Filter (EGMSE) [4].

For the case of the constrained MOE problem

$$\hat{\omega} = \arg\min_{\omega} E\|\omega^H\mathbf{y}\|^2, \quad (5)$$

subject to

$$\omega^{\mathbf{H}}\mathbf{S}_1^{+} = (\mathbf{S}_1^{+})^{\mathbf{H}}\mathbf{S}_1^{+}.$$

The solution follows

$$\hat{\omega} = \mathbf{R}^{-1}\mathbf{S}_1^{+}((\mathbf{S}_1^{+})^{\mathbf{H}}\mathbf{R}^{-1}\mathbf{S}_1^{+})^{-1}(\mathbf{S}_1^{+})^{\mathbf{H}}\mathbf{S}_1^{+}. \quad (6)$$

The corresponding decision rule (4) is referred to as the differential Equal Gain Combining of the Minimum Variance Filter(EGMV) [4].

In the following, the multistage representation of the corresponding EGMSE and EGMV filters will be derived, which are denoted by MS-MMSE-FB and MS-MOE-FB, respectively.

A. MS-MMSE Filter Bank

Based on a similar procedure in deriving the Multi-Stage Wiener Filter (MSWF) as given in [2], the MS-MMSE-FB filter for (2) can be obtained. This procedure is outlined as follows. Assume matrix $\mathbf{S}_1^{+}$ is of the full column rank, then it can be factorized by the Gram-Schmidt process as

$$\mathbf{S}_1^{+} = \mathbf{H}_1\mathbf{U}_1, \quad (7)$$

where $\mathbf{S}_1^{+}$ is the matrix of steering vectors of dimension $N \times L$. Let us choose an unitary matrix $\mathbf{T}$ such that

$$\mathbf{T} \equiv \left[\frac{\mathbf{H}_1^{\mathbf{H}}}{\mathbf{B}_1}\right], \quad (8)$$

where $\mathbf{B}_1 \perp \mathbf{H}_1$ is the blocking matrix of dimension $(N-L)\times N$, which can be obtained by the QR factorization of $\mathbf{S}_1^{+}$. By invoking the invariance of MSE due to a unitary transform of $\mathbf{y}$, *i.e.*

$$\min_{\omega} E\|b_1\Gamma_1 - \omega^H\mathbf{y}\|^2 = \min_{\omega} E\|b_1\Gamma_1 - \omega^H\mathbf{T}\mathbf{y}\|^2. \quad (9)$$

The solution to (9) is

$$\begin{aligned}\hat{\omega} &= \arg\min_{\omega} E\|b_1\Gamma_1 - \omega^H\mathbf{T}\mathbf{y}\|^2 \\ &= (\mathbf{T}\mathbf{R}\mathbf{T}^{\mathbf{H}})^{-1}\mathbf{T}\mathbf{S}_1^{+}\mathbf{A}_1 \quad (10)\end{aligned}$$

The objective of choosing such a form of $\mathbf{T}$ is to map the received signal $\mathbf{y}$ onto the subspace $\mathbf{H}_1$ spanned by the steering vectors $\mathbf{S}_1^{+}$ and its orthogonal complement. We then use the projection of $\mathbf{y}$ on $\mathbf{B}_1$ to suppress the residual interference in the projection of $\mathbf{y}$ on $\mathbf{H}_1$. Substituting (8) into (10) and applying the Matrix Inversion Lemma to $(\mathbf{T}\mathbf{R}\mathbf{T}^H)^{-1}$, the MMSE filter bank $\hat{\omega}$ becomes

$$\begin{aligned}\hat{\omega} &= \left(\left[\frac{\mathbf{H}_1^{\mathbf{H}}}{\mathbf{B}_1}\right]E(\mathbf{y}\mathbf{y}^H)\left[\,\mathbf{H}_1 \mid \mathbf{B}_1^{\mathbf{H}}\,\right]\right)^{-1}\left[\frac{\mathbf{H}_1^{\mathbf{H}}}{\mathbf{B}_1}\right]\mathbf{S}_1^{+}\mathbf{A}_1 \\ &= \left(\left[\begin{array}{c|c}\mathbf{K}_{\mathbf{d}_1} & \mathbf{r}_{\mathbf{y}_1\mathbf{d}_1}^{\mathbf{H}} \\ \hline \mathbf{r}_{\mathbf{y}_1\mathbf{d}_1} & \mathbf{K}_{\mathbf{y}_1}\end{array}\right]\right)^{-1}\left[\frac{\mathbf{U}_1}{\mathbf{0}}\right]\mathbf{A}_1 \\ &= \left[\frac{\mathbf{I}}{-\mathbf{K}_{\mathbf{y}_1}^{-1}\mathbf{r}_{\mathbf{y}_1\mathbf{d}_1}}\right]\mathbf{E}_1^{-1}\mathbf{U}_1\mathbf{A}_1, \quad (11)\end{aligned}$$

where $\mathbf{d}_1 \equiv \mathbf{H}_1^{\mathbf{H}}\mathbf{y}$, $\mathbf{y}_1 \equiv \mathbf{B}_1\mathbf{y}$, $\mathbf{K}_{\mathbf{d}_1}$ and $\mathbf{K}_{\mathbf{y}_1}$ are the covariance matrices of $\mathbf{d}_1$ and $\mathbf{y}_1$, respectively, $\mathbf{r}_{\mathbf{y}_1\mathbf{d}_1} \equiv \mathbf{E}(\mathbf{y}_1\mathbf{d}_1^{\mathbf{H}})$ is the cross-covariance matrix of $\mathbf{y}_1$ and $\mathbf{d}_1$, and

$$\mathbf{E}_1 \equiv (\mathbf{K}_{\mathbf{d}_1} - \mathbf{r}_{\mathbf{y}_1\mathbf{d}_1}^{\mathbf{H}}\mathbf{K}_{\mathbf{y}_1}^{-1}\mathbf{r}_{\mathbf{y}_1\mathbf{d}_1}).$$

The decision statistic can be obtained according to

$$\mathbf{z} = \hat{\omega}^H\mathbf{T}\mathbf{y} = \mathbf{A}_1\mathbf{U}_1^{\mathbf{H}}\mathbf{E}_1^{-1}[\mathbf{d}_1 - \omega_1^{\mathbf{H}}\mathbf{y}_1], \quad (12)$$

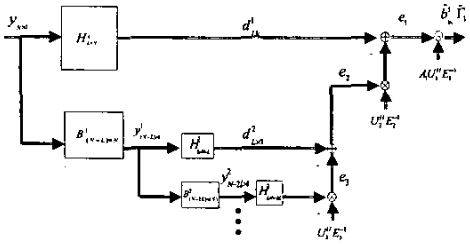

Fig. 1. The structure of MS-MMSE-FB/MS-MOE-FB in multi-path fading channels. For MS-MOE-FB, the term $\mathbf{E}_1^{-1}\mathbf{U}_1\mathbf{A}_1$ is replaced by $\mathbf{U}_1$.

where

$$\omega_1 \equiv \mathbf{K}_{\mathbf{y}_1}^{-1}\mathbf{r}_{\mathbf{y}_1\mathbf{d}_1} \tag{13}$$

is the MMSE filter bank that minimizes $E\|\mathbf{d}_1 - \omega_1^{\mathbf{H}}\mathbf{y}_1\|^2$. Thus, by choosing another unitray matrix $\mathbf{T}$ having $\mathbf{r}_{\mathbf{y}_1\mathbf{d}_1} \equiv \mathbf{H}_2\mathbf{U}_2$ and $\mathbf{B}_2 \perp \mathbf{H}_2$, the same procedure from (9) to (12) for calculating $\hat{\omega}$ can also be applied to ω_1. By recursively employing the above steps, an MS-MMSE-FB filter for decision statistics (3) can be obtained, which is shown in Fig. 1. By concatenating two consecutive soft outputs of MS-MMSE-FB, we get the statistics for decision rule (4). Note that MS-MMSE-FB reduces to MSWF when the rank of $\mathbf{H}_1$ is reduced to $L = 1$. That is the case for flat fading channels.

B. MS-MOE Filter Bank

By applying transformation $\mathbf{T}$ to the discrete time received signal $\mathbf{y}$, equation (5) becomes

$$\hat{\omega} = \arg\min_{\omega} E\|\omega^H \mathbf{T}\mathbf{y}\|^2,$$

subject to

$$\omega^H \mathbf{T}\mathbf{S}_1^+ = (\mathbf{S}_1^+)^{\mathbf{H}}\mathbf{T}^{\mathbf{H}}\mathbf{T}\mathbf{S}_1^+.$$

Let

$$\begin{aligned} \mathbf{R}_\mathbf{T} &= \mathbf{T}\mathbf{R}\mathbf{T}^{\mathbf{H}}, \\ \mathbf{C}_\mathbf{T} &= \mathbf{T}\mathbf{S}_1^+. \end{aligned}$$

The optimal filter bank is given by

$$\hat{\omega} = \mathbf{R}_\mathbf{T}^{-1}\mathbf{C}_\mathbf{T}(\mathbf{C}_\mathbf{T}^{\mathbf{H}}\mathbf{R}_\mathbf{T}^{-1}\mathbf{C}_\mathbf{T})^{-1}\mathbf{C}_\mathbf{T}^{\mathbf{H}}\mathbf{C}_\mathbf{T}. \tag{14}$$

By using the same transformation $\mathbf{T}$ as in (8), and following a similar procedure from (11) to (12), it is straightforward to show that MS-MOE-FB and the corresponding decision statistic are

$$\begin{aligned} \hat{\omega} &= \left[\frac{\mathbf{I}}{-\mathbf{K}_{\mathbf{y}_1}^{-1}\mathbf{r}_{\mathbf{y}_1\mathbf{d}_1}}\right]\mathbf{U}_1 && (15) \\ \mathbf{z} &= \mathbf{U}_1^{\mathbf{H}}[\mathbf{d}_1 - \omega_1^{\mathbf{H}}\mathbf{y}_1] && (16) \end{aligned}$$

It is worthwhile to point out that the MS-MMSE-FB (11) and the MS-MOE-FB (15) differ by only the constant matrices $\mathbf{E}_1^{-1}$ and $\mathbf{A}_1$. Thus, both of the filter banks share exactly the same structure except for the constant matrices $\mathbf{E}_1^{-1}$ and $\mathbf{A}_1$

C. Reduced-Rank MS-MMSE/MOE Filter Banks

For improved tracking and estimation error variance [8], over a full rank algorithm, we consider a reduced-rank algorithm. Many articles have proposed reduced-rank algorithms based on the signal and noise subspace separation method. A reduced-rank MMSE filter for DS-CDMA in flat fading channels based on the MSWF developed in [2] was proposed in [1]. The subspace spanned by the reduced-rank MSWF was given in [3]. For MS-MMSE-FB or MS-MOE-FB, the reduced rank implementation also follows the same rule by keeping the first D stages of the multi-stage filter bank. The subspace spanned by the reduced-rank MS-MMSE/MS-MOE filter bank is given by,

Proposition 1:

A. If the blocking matrix $\mathbf{B}_i$ at each stage i as shown in Fig. 1 is taken to be $\mathbf{B}_i \equiv \mathbf{I} - \mathbf{H}_i\mathbf{H}_i^{\mathbf{H}}$, the matrix $\mathbf{H}_i$ $(N \times L)$ of matched filters at each stage is of the form:

$$\begin{aligned} \mathbf{H}_i\mathbf{U}_i &= E(\mathbf{y}_{i-1}\mathbf{d}_{i-1}^{\mathbf{H}}) = \mathbf{B}_{i-1}\mathbf{E}(\mathbf{y}_{i-2}\mathbf{y}_{i-2}^{\mathbf{H}})\mathbf{H}_{i-1} \\ &= \prod_{j=i-1}^{1}(\mathbf{I} - \mathbf{H}_j\mathbf{H}_j^{\mathbf{H}})\mathbf{R}\mathbf{H}_{i-1} && (17) \\ & \quad i = 2\ldots D,\ \mathbf{y}_0 \equiv \mathbf{y}. \\ & \quad \mathbf{H}_i \perp \mathbf{H}_j,\ \mathbf{i} \neq \mathbf{j} && (18) \end{aligned}$$

B. The subspace $\mathbf{T}_{LD}^H(N \times DL)$ spanned by the reduced-rank MS-MMSE-FB (11) of stage D is equivalent to the subspace spanned by the reduced-rank MS-MOE-FB (15) of the same stage, which is

$$\begin{aligned} \mathbf{T}_{LD}^H &\equiv \left[\mathbf{H}_1 |\ \mathbf{B}_1\mathbf{H}_2|\ \ldots\ |\prod_{\mathbf{i}=1}^{\mathbf{D}-1}\mathbf{B}_i\mathbf{H}_\mathbf{D}\right], \\ &= [\mathbf{H}_1 |\ \mathbf{H}_2|\ \ldots\ |\ \mathbf{H}_\mathbf{D}]. && (19) \end{aligned}$$

The reduced-rank MS-MMSE-FB/MS-MOE-FB are equivalent to replacing the transformation matrix $\mathbf{T}$ in Eq.(10) and Eq.(14) by $\mathbf{T}_{LD}$

IV. Performance Analysis of Output SINR

For reduced-rank filters, it is important to characterize how the dimension of the subspace affects the output SINR. For the MSWF in DS-CDMA, an asymptotic analysis of the output SINR was conducted in [3] via a random sequence analysis. Here, we present the SINR analysis of the MS-MMSE/MS-MOE filters for the short code DS-CDMA of dimension N in a multi-path Rayleigh fading channel. First, we obtain the analytic auto-correlation matrix $\mathbf{R}$ of the received signal $\mathbf{y}$ for an asynchronous DS-CDMA system with multi-path fading, which is given by

$$\mathbf{R} = \sum_{k=1}^{K}\mathbf{S}_k^+\mathbf{A}_k^2(\mathbf{S}_k^+)^H + \mathbf{S}_k^-\mathbf{A}_k^2(\mathbf{S}_k^-)^H + 2N_0\mathbf{I} \tag{20}$$

where $\mathbf{A}_k = diag([A_{k1},\ldots,A_{kL}])$, $\mathbf{S}_k^+ = [\mathbf{s}_{k1}^+,\ldots,\mathbf{s}_{kL}^+]$, $\mathbf{S}_k^- = [\mathbf{s}_{k1}^-,\ldots,\mathbf{s}_{kL}^-]$ and $\mathbf{I}$ stands for the identity matrix of $N \times N$. Then, we insert $\mathbf{R}$ into Eq.(19) to obtain $\mathbf{T}_{LD}$, and reformulate this matrix as

$$
\begin{aligned}
\mathbf{T}_{LD}^{H} &\equiv [\mathbf{H_1} |\ \mathbf{H_1^{\perp}}] \\
\mathbf{H_1^{\perp}} &= [\mathbf{H_2} |\ \ldots\ |\ \mathbf{H_D}].
\end{aligned}
$$

Let z be the decision statistic as defined in (3), Γ_1 corresponds to the desired user's complex Gaussian fading process and A_1 is the desired user's signal amplitude. Ignoring the effect of ISI, the output SINRs for both of the MS-MMSE and the MS-MOE filter bank are given in the following proposition.

Proposition 2:
Define

$$\mathbf{S_D} = \mathbf{H_1^H R H_1} - \mathbf{H_1^H R H_1^{\perp}}((\mathbf{H_1^{\perp}})^{\mathbf{H}}\mathbf{R H_1^{\perp}})^{-1}(\mathbf{H_1^{\perp}})^{\mathbf{H}}\mathbf{R H_1}$$

A. The output SINR for MS-MMSE-FB of rank DL follows

$$
\begin{aligned}
SINR_{mmse} &\equiv E(\Gamma_1^H z\ |b_1 = 1)^2/Var(\Gamma_1^H z\ |b_1 = 1) \\
&= \mathbf{tr}(\mathbf{A_1 U_1^H S_D^{-1} U_1 A_1}) \qquad (21)
\end{aligned}
$$

B. The output SINR for MS-MOE-FB of rank DL follows

$$
\begin{aligned}
SINR_{moe} &\equiv E(\Gamma_1^H \mathbf{A}_1 z\ |b_1 = 1)^2/Var(\Gamma_1^H \mathbf{A}_1 z\ |b_1 = 1) \\
&= [\mathbf{tr}(\mathbf{U_1 A_1^2 U_1^H})]^2/\mathbf{tr}(\mathbf{A_1 U_1^H S_D U_1 A_1}) \qquad (22)
\end{aligned}
$$

V. Pre-Combining Multi-Stage Filters

If the desired user's fading channel process can be estimated and the phase information is recovered accurately enough, the soft output of the MS-MMSE-FB (11) and the MS-MOE-FB (15) can be combined with the desired user's estimated channel coefficients. We assume that it is equal to $\mathbf{\Gamma_1}$. In this case, the decision statistics of the MS-MOE-FB (16) can be reformulated to include channel coefficients Γ_1 as

$$
\begin{aligned}
\mathbf{z} &= \Gamma_1^H \mathbf{A_1 U_1^H}[\ \mathbf{d_1} - \mathbf{r_{y_1 d_1}^H K_{y_1}^{-1} y_1}] \\
&= [d_1 - E(d_1 \mathbf{y_1^H})\mathbf{K_{y_1}^{-1} y_1}] \\
&= [d_1 - \underline{\omega}_1^H \mathbf{y_1}], \qquad (23)
\end{aligned}
$$

where

$$
\begin{aligned}
d_1 &\equiv \Gamma_1^H \mathbf{A_1}(\mathbf{S_1^{+}})^{\mathbf{H}}\mathbf{y}, \\
\underline{\omega}_1 &\equiv \mathbf{y_1^H K_{y_1}^{-1}} E(\mathbf{y_1} d_1^*).
\end{aligned}
$$

Note that the matrix of steering vectors degenerates to $\mathbf{S_1^{+} A_1 \Gamma_1}$ of dimension $N \times 1$ and that $\underline{\omega}_1$ is now a vector instead of a matrix. The MS-MOE-FB is equivalent to the pre-combining MOE filter as shown in Fig. 2. The subspace spanned by the MOE filter of stage D, defined as $\mathbf{T}_{L+D-1}^{H}$, is equal to

$$\mathbf{T}_{L+D-1}^{H} = [\mathbf{H_1} |\ \mathbf{h_2} |\ \ldots\ |\ \mathbf{h_D}].$$

The rank for each vector $\mathbf{h_i}$, $\mathbf{i = 2 \ldots D}$ is one. The total rank of the MS-MOE filter becomes $L + D - 1$ instead of LD. For the case of the MS-MMSE-FB, it degenerates to the MSWF proposed in [2]. The subspace spanned by the MSWF of stage D was given in [3], *i.e.*

$$\mathbf{T}_D^H = [\mathbf{S_1^{+} A_1 \Gamma_1} |\ \mathbf{h_2} |\ \ldots\ |\ \mathbf{h_D}].$$

The total rank of the reduced-rank MSWF filter is D. As a result, with the knowledge of desired user's fading coefficients Γ_1, the rank of the MS-MMSE and the MS-MOE can drop from LD to D and $L + D - 1$, respectively, without affecting the receiver's steady state performance. However, the convergence of their output SINRs are expected to be faster than that of the MS-MMSE-FB and the MS-MOE-FB as shown in the simulation result. This is an important factor to enhance the receiver's performance in a non-stationary fading channel.

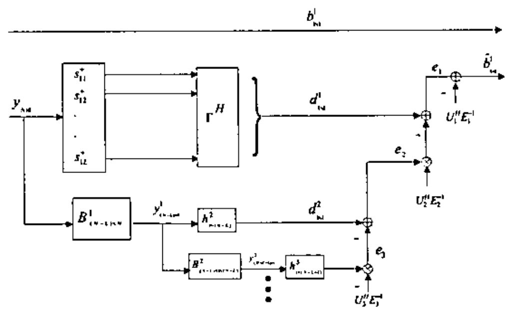

Fig. 2. The structure of the pre-combining MMSE/MOE filter, where $\mathbf{H_1}$ is combined with channel coefficients $A_1\Gamma_1$.

VI. Simulation Results

Simulations have been conducted to study the performance of the MS-MMSE-FB and the MS-MOE-FB. In our simulations, a short code CDMA (Gold code) with spreading gain $N = 31$ is adopted. The simulated channel for each user is a 3-path (i.e. $L = 3$) Rayleigh fading channel with $f_d T_s = 5 \times 10^{-3}$. All interfering users are assumed to have the same received power.

Fig. 3 shows the rank effect on the output SINRs of the reduced-rank MS-MMSE-FB and MS-MOE-FB. The desired user's channel information are assumed given. There are a total of 10 users in the asynchronous CDMA system, the desired user's Eb/N_0 is 20dB. The simulation result matches well with the analytical result derived in Proposition 2.

Fig. 4 shows the convergence of the output SINRs for the adaptive reduced-rank MS-MMSE and the MS-MOE filters of stage 6 ($D = 6$) with the same environment setup before. The convergence speeds for the MSWF (rank 6) and the MS-MOE (rank 8) are clearly better than their MS-MMSE-FB and MS-MOE-FB counterparts (both of rank 18). However, the steady state SINRs are the same for the MS-MMSE filters and the MS-MOE filters.

Fig. 5 shows the bit error rate (BER) vs. Eb/N_0 for the adaptive reduced-rank MS-MMSE-FB and the MS-MOE-FB at $D = 6$ without the knowledge of channel coefficients.

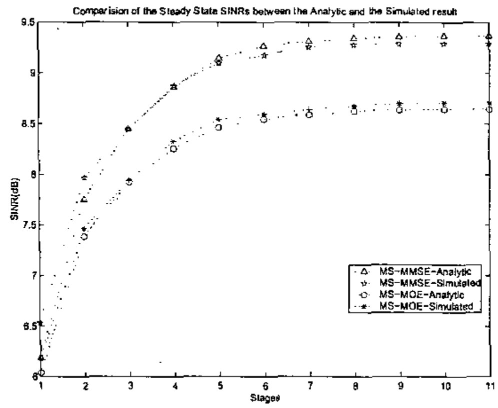

Fig. 3. Output SINRs vs. the number of stages for adaptive reduced-rank filters over a Rayleigh fading channel with $L = 3$, where stage 11 in the plot actually corresponds to the full rank SINRs.

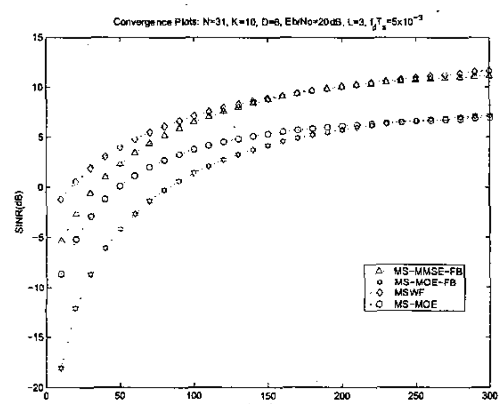

Fig. 4. Output SINRs vs. the number of iterations for adaptive reduced-rank filters over a Rayleigh fading channel with $L = 3$ and $D = 6$.

The transmitted data are modulated by DBPSK. The differential equal gain combining is employed for detection. We see from the simulation result that the performance of the proposed reduced rank algorithms is very close to the full rank implementations in most of the regime. The simulation result also indicates that the MS-MOE-FB has an excellent performance with differential equal gain combining. That is due to the fact the soft output $z(m-1)$ of the MS-MOE-FB provides an better estimate for $d_1(m-1)\hat{\Gamma}$ than that of the MS-MMSE-FB [9], and thus achieves better detection performance

$$\mathbf{z}(m-1)^H \mathbf{z}(m) = d_1(m-1)\hat{\Gamma}^H(m-1)\mathbf{z}(m) = \hat{b}(m).$$

VII. Conclusions

The implementation of adaptive multi-stage MMSE/MOE filter banks for interference suppression of DS-CDMA in multi-path fading channels were presented. The analytic expression for the output SINRs given the number of stages

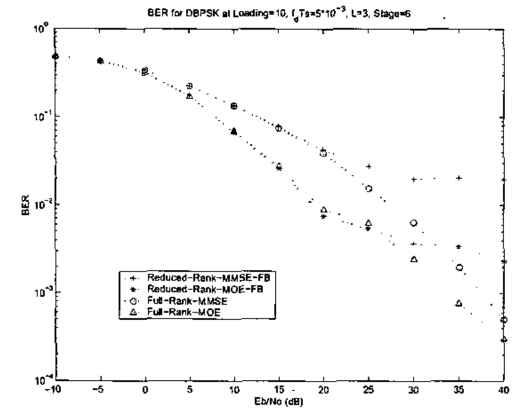

Fig. 5. BER vs. system loading for adaptive reduced-rank filters over the Rayleigh fading channel with $L = 3$.

is provided. It was shown that the SINRs does not scale with the rank. Therefore, an excellent convergence speed of the output SINR can be obtained with a lower rank implementation. However, the steady state SINR is still maintained. This is an important feature for receivers in non-stationary fading channels. It was confirmed by simulation results that the MS-MOE-FB has an excellent performance for differential detection in multi-path fading channels.

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