Revisiting the Noise Figure Design Metric for Digital Communication Receiver

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Abstract

Noise figure is a commonly used system parameter that quantifies the degradation in the signal-to-noise ratio (SNR) as the signal passes through a receiving system. Because of the difficulty in defining the SNR, NF depends on how the SNR is computed and the underlying assumptions that are made. Existing NF measures and their shortcomings are explained. A new NF suitable for digital communication receiver is proposed by redefining the SNR, so that the NF measures the degradation in the achievable performance caused by the receiving system. The proposed NF, which we refer to as the effective NF, can be readily determined based on the existing NF measurement techniques.

1. Introduction

Noise factor (or noise figure in dB) is an important system parameter that is closely related to the overall receiver performance or the bit error rate (BER). It is commonly used to characterize the ability of a receiving system to process the input signals, where the receiving system refers to the entire analog front-end as well as its individual components, such as the low-noise amplifier [1], the mixer [2], and the baseband and IF amplifiers.

The formal definition of NF has been introduced in the 1940's by Friis [3]-[5] as

$$F \equiv \frac{SNR_{in}}{SNR_{out}} \tag{1}$$

where SNR_{in} is the input signal-to-noise ratio (SNR) and SNR_{out} is the output SNR. As such, NF represents the degradation in the SNR as the signal passes through the receiving system. Although the meaning of NF is straightforward, measuring the NF can be problematic because of the difficulty in defining the SNR. Consequently, the NF depends on how the SNR is computed and the underlying assumptions that are made.

There are basically three different NF's that are reported in the literature: spot NF, average NF, and weighted NF. As described in the following section, however, these NF measures suffer from several shortcomings. For example, the spot NF is often not unique to a receiving system. The average and weighted NF, which are well defined unlike the spot NF, are not necessarily accurate measures of the overall receiver performance.

The goal of the analog front-end in digital receivers is to condition the received analog signal for digitiziation, so that the highest performance can be achieved after decoding in the digital domain. For the NF of a receiver to be a meaningful metric, the SNR at the input and output of the receiving system should measure the performance after the eventual digital decoding process, as it is ultimately the most relevant measure of performance. Since the eventual performance depends on the choice of the detection algorithm, which is system dependent and often difficult to quantify, the SNR is defined as the matched filter bound (MFB) [6]. The MFB, which represents an upper bound on the performance of data transmission systems with intersymbol interference (ISI), is obtained when a whitened matched filter is employed to receive a single transmitted pulse. By defining the SNR as the MFB, the NF measures the degree of degradation in the achievable receiver performance caused by the receiving system.

In this paper, a new NF suitable for digital communication receiver is proposed by redefining the SNR as the MFB. The proposed NF, which we subsequently refer to as the effective NF, can be readily determined based on the spot NF measurements.

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2. Existing Noise Figure Measures

2.1. Spot noise figure

The spot NF is determined by computing the NF given in (1) at an infinitesimal frequency band centered at a frequency f within the input signal band [4]:

$$F_s(f) \equiv \frac{S_s(f)/S_{n_i}(f)}{S_s(f)G(f)/(S_{n_i}(f)G(f) + S_{n_g}(f))} \tag{2}$$

$$= \frac{S_{n_i}(f)G(f) + S_{n_g}(f)}{S_{n_i}(f)G(f)} \tag{3}$$

where $S_s(f)$, $S_{n_i}(f)$, and $S_{n_g}(f)$ represent the input signal power spectral density (PSD), input noise PSD, and internally generated noise PSD, respectively. The input noise PSD $S_{n_i}(f)$ is commonly assumed to be white with magnitude corresponding to a noise temperature of 290K. $G(f)$ is the power gain of the receiving system. As shown in (3), the spot NF $F_s(f)$ is independent of $S_s(f)$; it is simply the ratio of the noise power output at the infinitesimal frequency band to that portion of the noise power output due to the noise at the input. The absence of the input and output signals makes the spot NF attractive as a basis for measurement. Consequently, most of the noise figures reported in the literature are spot noise figures.

The main drawback of the spot NF is that it can be frequency dependent. If $S_{n_i}(f)$, $S_{n_g}(f)$, and $G(f)$ in (3) are not fixed over the frequency band of interest, $F_s(f)$ can become a function of the center frequency f. The reported NF of a receiving system is then not unique and would depend on the selection of f. Therefore, when reporting the NF performance of a receiving system using $F_s(f)$, the underlying assumption is that either $F_s(f)$ is fixed over the frequency band of interest or a single-tone signal is applied. This assumption is often violated in modern digital receivers.

2.2. Average noise figure

The average NF removes the frequency dependency of the spot NF by defining the signal and noise components in (1) as the total signal power and noise power over the frequency band of interest B [7]. The average NF is

$$F_a \equiv \frac{\int_B S_s(f)df / \int_B S_{n_i}(f)df}{\int_B S_s(f)G(f)df / \int_B (S_{n_i}(f)G(f) + S_{n_g}(f))df} \tag{4}$$

where all of the integrations in (4) are over B. If B is an infinitesimal frequency band centered at f_0, , the average NF F_a becomes the spot NF $F_s(f_0)$.

If $S_s(f)$ is assumed white over B, which is a reasonable assumption since the propagation channel is in general unknown at design time, the average NF in (4) becomes simply the total output noise power divided by the total input noise power referred to the output:

$$F_a = \frac{\int_B (S_{n_i}(f)G(f) + S_{n_g}(f))df}{\int_B S_{n_i}(f)G(f)df} \tag{5}$$

This is the average NF that is often cited in the literature.

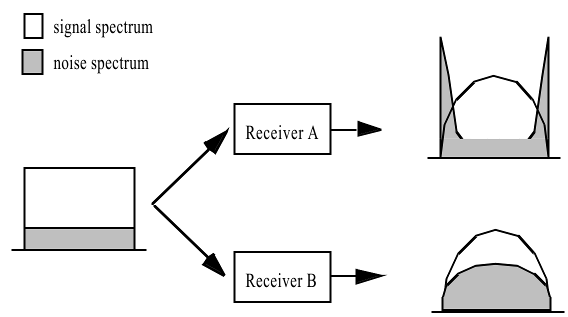

Figure 1 : Average noise figure.

Although the frequency dependency of the spot NF is removed, the main drawback of the average NF is that a lower average NF does not necessarily translate to a higher overall receiver performance. This point is best illustrated through an example shown in Figure 1. The input signal and noise, both of which are assumed white, pass through receivers A and B with different $G(f)$ and $S_{n_g}(f)$. In the resulting PSDs shown in Figure 1, the total output noise power of both receivers is assumed to be the same. Then, the average NF of the two receivers is also the same. However, receiver A clearly achieves a higher performance after the eventual digital decoding process, since the noise in receiver A is easily filtered out with little degradation on the overall signal spectrum. By contrast, the noise in receiver B is spread across the signal spectrum and cannot be selectively filtered out. Since the performance of the receiver after the digital decoding process does not depend on the total signal and noise power as this example illustrates, the average NF is in general not an accurate measure of the degradation in the overall SNR caused by the receiving system.

2.3. Weighted noise figure

Another NF employed in the literature is what we refer to as the weighted NF. The weighted NF is obtained by appropriately weighting the spot NF across the frequency band of interest B [4]:

$$F_w = \int_B F_s(f) W(f) df \tag{6}$$

where $W(f)$ is the weighting function that is constrained to be

$$\int_B W(f) df = 1 \tag{7}$$

The main difficulty in employing the weighted NF is in determing the weights $W(f)$. In the literature, $W(f)$ is often weighted uniformly or according to $G(f)$ without a sound technical basis. A more rigorous relationship between $W(f)$ and the overall receiver performance after the digital decoding process given $G(f)$ and $S_{n_g}(f)$ needs to be established. As shown in the following section, however, a meaningful NF that measures the degree of degradation in the overall receiver performance caused by the receiving system does not have a linear relationship with $F_s(f)$ as in the weighted NF.

3. The Effective Noise Figure

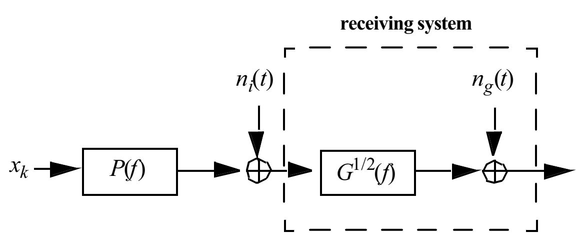

Figure 2 : General system model.

As stated earlier, the main difficulty in computing the NF is in defining the SNR. By defining the SNR as the MFB, the NF represents the degradation in the achievable SNR after the digital decoding process.

A general system model of a communication channel including the receiving system is shown in Figure 2. The kth transmit signal x_k is filtered by the equivalent pulse response then corrupted by the additive noise $n_i(t)$. The equivalent pulse response (whose frequency response is $P(f)$) represents the combination of both the transmit pulse and the propagation channel. The resulting corrupted signal is the input of the receiving system, which has a transfer function given by ($\sqrt{G(f)}$) and additive noise $n_g(t)$.

The MFB, also called "one-shot" bound, is an upper limit on the performance of data transmission systems with ISI. The MFB is determined by employing a whitened matched filter to receive a "one-shot" transmission pulse. Assuming unity transmit signal energy, i.e., $E\{x_k^2\} = 1$, the MFB at the input and output of the receiving system is [6]

$$SNR_{in} = \int \frac{|P(f)|^2}{S_{n_i}(f)} df \tag{8}$$

$$SNR_{out} = \int \frac{|P(f)|^2 G(f)}{S_{n_i}(f) G(f) + S_{n_g}(f)} df \tag{9}$$

Substituting (8) and (9) into (1), the effective NF of the receiving system is

$$F_{eff} = \frac{\int \frac{|P(f)|^2}{S_{n_i}(f)} df}{\int \frac{|P(f)|^2 G(f)}{S_{n_i}(f) G(f) + S_{n_g}(f)} df} \tag{10}$$

Assuming as is commonly done that the input noise $n_i(t)$ is white, the NF can then be written as a function of the spot NF $F_s(f)$:

$$F_{eff} = \frac{\int |P(f)|^2 df}{\int |P(f)|^2 \frac{S_{n_i}(f) G(f)}{S_{n_i}(f) G(f) + S_{n_g}(f)} df} \tag{11}$$

$$= \frac{\int |P(f)|^2 df}{\int |P(f)|^2 \frac{1}{F_s(f)} df} \tag{12}$$

$$= \frac{1}{\int \left(\frac{|P(f)|^2}{P_T} \right) \frac{1}{F_s(f)} df} \tag{13}$$

where $P_T = \int |P(f)|^2 df$. For a cascade of the multiple-stage receiving systems, the effective noise figure in (13) can be determined by the well-known Friis formula [3], i.e.,

$$F_s(f) = F_{s1}(f) + \frac{F_{s2}(f) - 1}{G_1(f)} + \dots + \frac{F_{sN}(f) - 1}{G_1(f) \dots G_{N-1}(f)} \quad , \tag{14}$$

where $F_{si}(f)$ and $G_i(f)$ denote the spot NF and gain of the ith cascaded receiving system.

The relationship between spot NF and effective NF can be better understood by approximating (13) using finite summations. The effective NF is then

$$F_{eff} \approx \frac{1}{\sum_{i=0}^{N-1} \frac{1}{\alpha_i F_s(f_i)}} \tag{15}$$

where $\{f_0, f_1, \ldots, f_{N-1}\}$ represent the center frequencies for each of the spot NF measurements in the frequency band of interest, N is the total number of measured values, and

$$\alpha_i = \frac{\sum_{i=0}^{N-1} |P(f_i)|^2}{|P(f_i)|^2} \tag{16}$$

In (15), the effective NF equation is analogous to the effective resistance of N resistors with resistance $\alpha_i F_s(f_i)$ placed in parallel. The resistance is obtained by scaling the spot NF at frequency f_i by α_i, which is a function of the shape of $|P(f)|^2$ as shown in (16). If $P(f)$ is assumed fixed over the frequency band of interest because of the uncertainty in the propagation channel response, $\alpha_i = N$ for $i \in \{0, 1, \ldots, N-1\}$ and all the spot NF values are weighted equally.

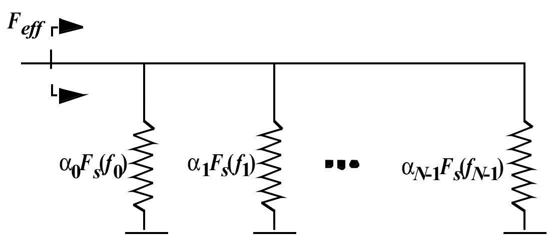

Figure 3 : Equivalent model of weighted NF.

The parallel resistor perspective, illustrated in Figure 3, suggests that having a few very high $\alpha_i F_s(f_i)$ values have little effect on the effectve NF, since the effective resistance of parallel resistors is dominated by the smaller resistors. This observation allows new design strategies such as significantly increasing the spot NF in some frequencies to achieve other implementation benefits while incurring minimal overall performance degradation.

3.1. Relationship to existing NF measures

As described above, the effective NF can be understood as a nonlinear average of the spot NF across the frequency band of interest. Also note that in (13) the effective NF becomes the spot NF when either $F_s(f)$ is fixed over the frequency band of interest or the transmited signal is a single tone. Unlike the spot NF, a simple mathematical relationship between the effective NF and either the average or the weighted NF is difficult to establish. The weighted NF in (6) assumes a linear relationship with the spot NF; whereas, the effective NF in (15) has a nonlinear relationship.

4. CONCLUSIONS

For the NF of a receiving system to be a meaningful metric, the SNR at the input and output of the receiving system should measure the performance after the eventual digital decoding process, as it is ultimately the most relevant measure of performance. By defining the SNR as the MFB, the effective NF measures the degree of degradation in the achievable receiver performance caused by the receiving system. The effective NF is shown to be analogous to the effective resistance of parallel resistors, where the resistance of each resistor is obtained by appropriately scaling the spot NF measured at different frequencies.

REFERENCES

[1] D. K. Shaeffer, T. H. Lee, "A 1.5-V, 1.5-Hz CMOS low noise amplifier," *IEEE J. Solid-State Circuits*, vol. 32, pp. 745-759, May 1997.

[2] C. D. Huller, R. G. Meyer, "A systematic approach to the analysis of noise in mixer," *IEEE Trans. Circuits Syst.*, vol. 40, pp. 909-919, Dec. 1993.

[3] H. T. Friis, "I. Noise figure of radio receivers," *Proc. IRE*, vol. 32, pp. 419-422, Jul. 1944.

[4] H. A. Haus, "IRE standards on methods of measuring noise in linear two ports, 1959," *Proc. IRE*, vol. 48, pp. 60-74, 1960.

[5] J. Randa, "Noise characterization of multiport amplifiers," *IEEE Trans. Microwave Theory Tech.*, vol. 49, pp. 1757-1763, Oct. 2001.

[6] J. G. Proakis, Digital Communications, Boston: McGraw-Hill, 2001

[7] H. A. Haus, "Noise figure definition valid from RF to optical frequencies," *IEEE J. selected topics in quantum electronics*, vol. 6, pp. 240-247, Mar. 2000.