Template Estimation in Ultra-Wideband Radio

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***Abstract*— The short duration of transmitted pulses in ultra-wideband (UWB) impulse radio enables fine time resolution in the receiver and therefore the opportunity to take great advantage of multipath diversity. However, unlike narrowband channels, many UWB channels are frequency selective even if the channel consists of only a single path, significantly altering the shape of the pulse. In this paper we present a least squares approach to finding a suitable correlation template in a UWB receiver based on our knowledge of the transmitted waveform and some properties of the channel. It is shown that the template increases energy capture in the receiver when compared to a template with the same shape as the transmitted pulse, and is more resistant to narrowband interference than a transmitted reference system.**

I. Introduction

Ultra-wideband (UWB) impulse radio uses short impulses on the order of a nanosecond to form a communications link. A typical indoor channel environment will be frequency selective to an ultra-wideband transmitted waveform, significantly altering the shape of the pulse. Even line-of-sight propagation without multipath can distort the waveform between transmitter and receiver due to the frequency dependent behaviour of the antennas and free space. Moreover, different environments, different antennas and even different look angles of a single antenna have different frequency responses, making it difficult to choose a receiver correlation template that is suited to all environments.

To solve this problem the receiver must determine for itself what template function to use. Suggested solutions to this problem are of the transmitted reference nature, where the receiver observes one or more unmodulated signals before communication begins to form a suitable template waveform [1][2][3]. An obvious potential problem with these systems is that they are more susceptible to corruption by noise and other outside interference.

In this paper we aim to mitigate this problem by first showing that the received pulse over any channel must lie in the space spanned by the transmitted waveform, all derivatives of the transmitted waveform, the Hilbert transform of the transmitted waveform and all derivatives of its Hilbert transform. By finding the least squares solution to the received waveform restricted to this space, or some subspace, we eliminate NBI and noise components that are orthogonal to that subspace. The proposed template design technique is compared to a fixed template and a template without a restricted basis.

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II. UWB Channel Model

We would like to model a multipath channel impulse response with the following equation

$$h(t) = \sum_{k=1}^{L} c_k h_k(t - \tau_k) \tag{1}$$

where $h_k(t)$ is the impulse response of the k^{th} propagation path, τ_k is the propagation delay of the k^{th} path and c_k is the gain factor. Hence the received waveform when $s(t)$ is transmitted is given by $r(t) = h(t) * s(t) + I(t) + n(t)$, where $*$ denotes convolution, $I(t)$ is interference and $n(t)$ is white Gaussian noise, or

$$r(t) = \sum_{k=1}^{L} c_k w_k(t - \tau_k) + I(t) + n(t). \tag{2}$$

Each waveform $w_k(t)$ can be written as $w_k(t) = h_k(t) * s(t)$, or in the frequency domain

$$W_k(\omega) = S(\omega) H_k(\omega) \tag{3}$$

In general, we have knowledge of $S(\omega)$ but not of $H_k(\omega)$, however, we do know some properties of $H_k(\omega)$. As we will see in some examples in Section IV the transfer functions of single transmission paths that involve interactions with objects in the environment are typically smooth and relatively slowly varying, hence it is natural to approximate them with a Taylor Series expansion. Because $h_k(t)$ is real we know that $H_k(\omega)$ is conjugate symmetric about 0, therefore if we write $H_k(\omega)$ in terms of its Taylor Series expansion about ω_0 for positive frequencies and $-\omega_0$ for negative frequencies we get

$$\begin{aligned} H(\omega) &= R(\omega) + jI(\omega) \\ &= \begin{cases} \displaystyle\sum_{n=0}^{\infty} (\omega - \omega_0)^n \\ \qquad \times \left(\frac{1}{n!} \frac{d^n R(\omega)}{d\omega^n} + j \frac{d^n I(\omega)}{d\omega^n} \right) \Big|_{\omega=\omega_0} & \omega \geq 0 \\ \displaystyle\sum_{n=0}^{\infty} (-1)^n (\omega + \omega_0)^n \\ \qquad \times \left(\frac{1}{n!} \frac{d^n R(\omega)}{d\omega^n} - j \frac{d^n I(\omega)}{d\omega^n} \right) \Big|_{\omega=\omega_0} & \omega < 0 \end{cases} \end{aligned} \tag{4}$$

Let $c_n^r = \frac{d^n R(\omega)}{d\omega^n}$ and $c_n^i = \frac{d^n I(\omega)}{d\omega^n}$ then it can be shown that

$$W_k(\omega) = H_k(\omega)S(\omega)$$
$$= \sum_{n=0}^{\infty} \frac{1}{n!} \begin{cases} \sum\limits_{\substack{p=0 \\ p \text{ even}}}^{n} \binom{n}{p} \omega^{(n-p)} \omega_0^p \left(c_n^r S(\omega) - c_n^i \hat{S}(\omega) \right) \\ -j \sum\limits_{\substack{p=1 \\ p \text{ odd}}}^{n} \binom{n}{p} \omega^{(n-p)} \omega_0^p \left(c_n^r \hat{S}(\omega) + c_n^i S(\omega) \right) \\ \hfill n \quad \text{even} \\ \sum\limits_{\substack{p=0 \\ p \text{ odd}}}^{n} \binom{n}{p} \omega^{(n-p)} \omega_0^p \left(c_n^i \hat{S}(\omega) - c_n^r S(\omega) \right) \\ +j \sum\limits_{\substack{p=1 \\ p \text{ even}}}^{n} \binom{n}{p} \omega^{(n-p)} \omega_0^p \left(c_n^r \hat{S}(\omega) + c_n^i S(\omega) \right) \\ \hfill n \quad \text{odd} \end{cases} \tag{5}$$

where $\hat{S}(\omega)$ is the Fourier transform of the Hilbert transform of $s(t)$. Hence

$$\begin{aligned} w_k(t) &= \sum_{n=0}^{\infty} a_{k,n} s^{(n)}(t) + b_{k,n} \hat{s}^{(n)}(t) \\ &\approx w_k^N(t) = \sum_{n=0}^{N} a_{k,n} s^{(n)}(t) + b_{k,n} \hat{s}^{(n)}(t) \end{aligned} \tag{6}$$

where $a_{k,n}$ and $b_{k,n}$ are unknown constants, $\hat{s}(t)$ is the Hilbert transform of $s(t)$ and $s^{(n)}(t)$ is the n^{th} derivative of $s(t)$. In the receiver our aim is to accurately estimate $w_k(t)$ given $r(t)$ and our knowledge of $s(t)$. Ideally we would like to jointly optimize over the parameters c_k, t_k, $a_{k,n}$ and $b_{k,n}$ where L and N are fixed sufficiently high to achieve an acceptable approximation.

Note that N and L are complementary in their effect on the accuracy of the approximation. Even if $L = 1$, i.e., we model the channel as a single "path", we can achieve an arbitrarily accurate estimate of $w(t)$ by making N large enough. It is appealing to use a value of L greater than 1 because it reflects the physical process of multiple discrete arrivals that have undergone a filtering process, the short duration of typical UWB pulses enables the receiver to distinguish these arrivals, and with accurate estimation of τ_k we reduce the required value of N. From another perspective, we know that in the frequency domain multipath generally introduces ripples into the channel transfer function, meaning that more Taylor Series components are required for a sufficiently accurate approximation, hence by considering separate estimates for each multipath component we reduce the required value of N.

III. Least-Squares Waveform Estimation

If we assume that c_k, t_k and L are known then we can find the least squares estimate of $w_k(t)$ for fixed N. Define $r_k(t-\tau_k)$ as the time segment of $r(t)$ corresponding to $w_k(t-\tau_k)$, assuming the $w_k(t-\tau_k)$'s don't overlap and have finite duration. Let

$$\mathbf{S_N} = \left[\begin{array}{cccccccc} \mathbf{s} & \mathbf{s^{(1)}} & \dots & \mathbf{s^{(N-1)}} & \hat{\mathbf{s}} & \hat{\mathbf{s}}^{(1)} & \dots & \hat{\mathbf{s}}^{(\mathbf{N-1})} \end{array} \right] \tag{7}$$

where the boldface version of any variable not otherwise defined represents a vector of samples of the original variable. Then

$$\begin{aligned} \check{\mathbf{w}}_\mathbf{k}^\mathbf{N} &= \mathbf{S_N} \left[\mathbf{S_N^\dagger S_N} \right]^{-1} \mathbf{S_N^\dagger r_k} \\ &= \mathbf{U_N r_k} \end{aligned} \tag{8}$$

where $\dagger$ indicates Hermitian transpose, is the least squares estimate of $\mathbf{w_k^N}$, which will be referred to as the N^{th} order least-squares estimate . Note that calculation of $\mathbf{U_N}$ requires knowledge only of the transmitted pulse and can be calculated offline. The number of online calculations required to estimate each template is NM, and with reasonable estimation of τ_k no more than M basis functions should be needed.

In the case that we want to find a single template $\check{w}(t)$ such that $\sum_{k=1}^{L} c_k \check{w}(t-\tau_k)$ is the least squares estimate of $r(t)$, the solution becomes

$$\check{\mathbf{W}}^\mathbf{N} = \frac{\mathbf{U_N r}}{\sum_{k=1}^{L} c_k^2} \tag{9}$$

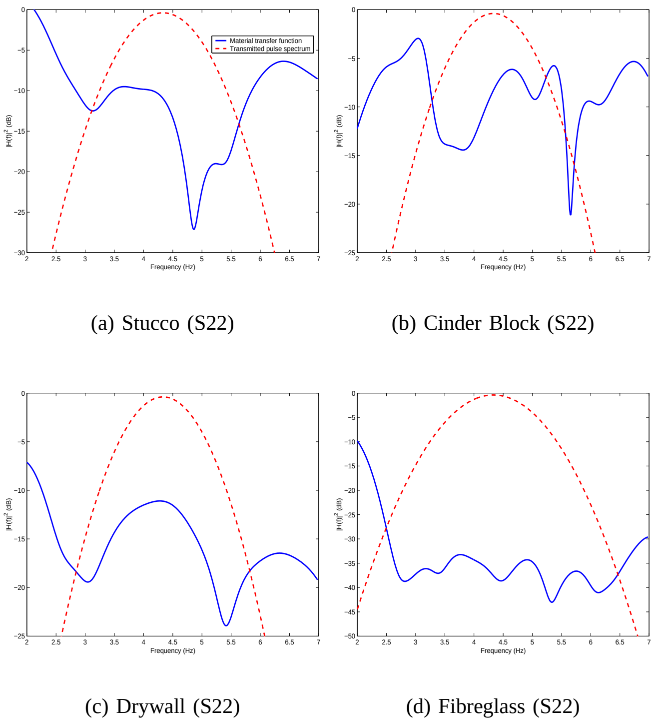
where $\mathbf{r} = \sum_{k=1}^{l} c_k \mathbf{r_k}$.

IV. Single Path Estimation

In this section we will apply the least-squares template estimation technique to some simulated single-path channels, in order to evaluate the frequency selective effects of common building materials in isolation, and the degree to which more accurate estimation of their transfer functions can achieve performance gains.

The transfer functions describing the filtering effects of the transmission through, or reflection from, a number of common building materials were measured in an anechoic chamber using an automatic network analyser. Four of the most interesting measurements are given in Fig. 1, with the normalised spectrum of an FCC mask [4] compliant pulse overlaid.

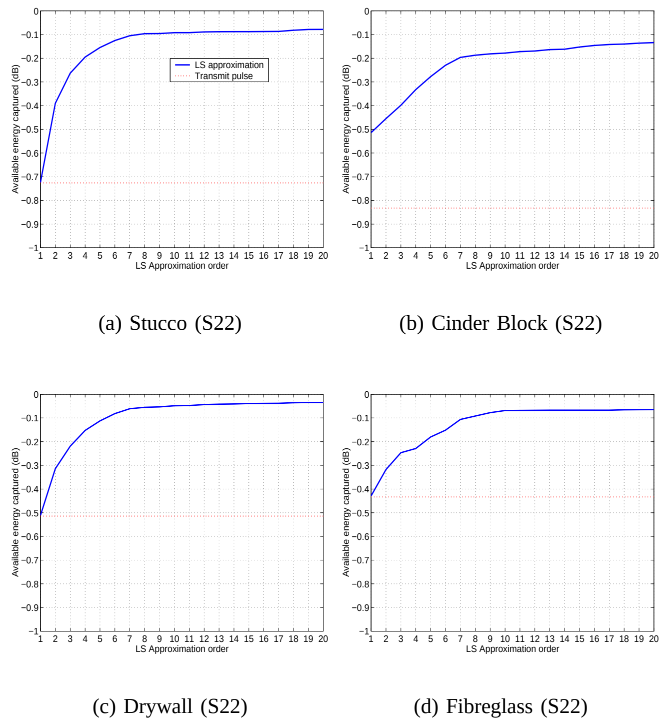
For each of these materials the transmission of the shown pulse over the channel defined in Fig. 1 was simulated by multiplication of pulse spectrum and material transfer function in the frequency domain. Given the simulated received pulse the peak cross-correlation between the received pulse and the transmitted pulse was compared to the cross-correlation between the received pulse and the function $\check{\mathbf{w}}_\mathbf{k}^\mathbf{N}$ as given in equation (8), for the same time-of-arrival. The results are shown for different values of N in Fig. 2. All pulses have been normalized to have unit energy, hence the peak cross-correlation for a template that matches the received waveform exactly would be 1.



(a) Stucco (S22) (b) Cinder Block (S22)

(c) Drywall (S22) (d) Fibreglass (S22)

Fig. 1. Transfer functions of some common building materials



(a) Stucco (S22) (b) Cinder Block (S22)

(c) Drywall (S22) (d) Fibreglass (S22)

Fig. 2. Energy captured (normalised cross-correlation) for different orders of LS template estimation for signals reflected from common building materials.

It is clear from Fig. 2 that even for materials with significant variation in frequency response over the 10dB bandwidth of the transmitted pulse, there is less than 1dB to be gained by more accurate estimation of the received pulse shape. Also, a significant part of the gain is made using just a second or third order of the channel approximation. The results presented here are of course specific to the particular pulse shape used and one would expect the distortion suffered by a pulse with a flatter spectrum to be more significant in terms of energy capture at the receiver.

V. Multipath Channel Estimation

In the second scenario we consider more complex multipath channels to investigate how receiver complexity can be reduced, and more intuitive channel models constructed, by more accurate estimation of the received pulse shape.

Sixteen indoor multipath channel measurements were taken from the database[1] of measurements performed by researchers at Intel Corporation [5]. The channel measurements were performed at different locations in a bungalow type home, in each case with 5 metres separation between transmit and receive antennas and one intermediate wall. The mean delay spread was 11.7ns with mean RMS delay spread of 8.2ns, where delay spreads are defined in the usual way [6]. The observations had been post-processed to remove the known effects of transmit and receive filters in the measurement process, however the effects of the transmit and receive antennas were still present. Compensating for the effects of antennas in a multipath channel is a difficult problem because the response of the antennas at both ends of the link is typically a function of the look direction. Hence the pulse shaping can be different for each multipath component, increasing the need for waveform estimation at the receiver.

For each channel the frequency domain channel observations were multiplied by the spectrum of the candidate transmit pulse to give the spectrum of the received observation, which was then converted to the time domain by inverse discrete fourier transform. An iterative method is used to determine the times-of-arrival of the multipath components. An initial cross-correlation between the received waveform and the transmit pulse is taken and the location of the peak is taken as the arrival time of one component. The template waveform, which is either the transmitted pulse scaled to minimize the square error or the nth order least-squares estimate of equation (8), is then subtracted from the received waveform. The cross-correlation between the resulting waveform and the transmit pulse taken to determine the time-of-arrival of the next component and the process is repeated until some threshold maximum number of paths or minimum path energy is reached.

In Fig. 3 a sample stem plot of the estimated multipath channel is shown using both the transmitted pulse as a template and using a second order least-squares estimate. Both channel estimates capture approximately -1.75dB of the total energy

[1]The complete database of Intel measurements are available at the URL http://ultra.usc.edu/New_Site/database.html

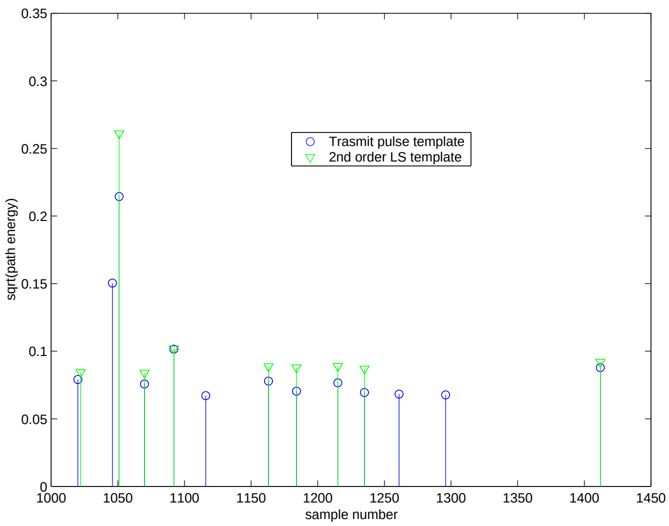


Fig. 3. Two discrete multipath channel models, one using a fixed template and one a template that is a LS fit to the observed waveform.

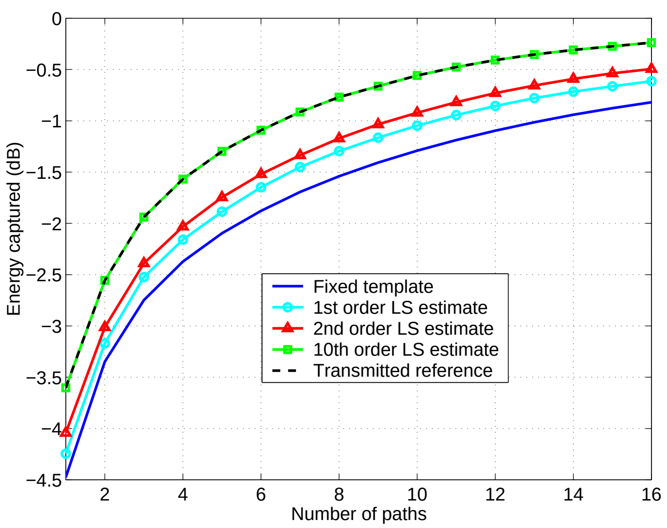


Fig. 4. Amount of energy captured of the total available energy with respect to the number of correlators used, for different fixed and adaptive templates.

in the channel, however note that the estimate using a LS estimate generally captures more energy per path and requires four fewer paths to capture the same total amount of energy. Also note that when using the transmit pulse as a template the strongest arrival appears to be modelled as two closely spaced arrivals, while when using the 2nd order LS template it is modelled as a single, higher energy, arrival. In Fig. 4 the mean energy captured over all channels, calculated in decibels with respect to the total amount of available energy, is plotted against the number of arrivals, which might correspond to the number of correlators used in a Rake receiver.

As the number of correlators is increased the amount of energy captured follows a law of diminishing returns, where less than 3 correlators are required to capture half the available energy for all templates, but up to an additional 10 are required to capture the next 2dB. Hence the benefit of using template estimation, in terms of correlator reduction, increases as more energy is squeezed from the multipath, for example there is almost no margin at -3dB energy capture, but 4 correlators can be saved by using 2nd order LS estimation at -1dB energy capture. The logical limit to increasing the channel approximation order is to use the observed waveform itself as the template, with no further processing, which is known as transmitted reference radio [2]. The energy capture curve for this system, when the template is restricted to the same duration as the other techniques, is shown as the upper bound in Fig. 4 and is met by the 10th order LS template estimate.

Note that these comparisons have been made in a noiseless environment. In general the more degrees of freedom a template has the more susceptible it is to corruption due to observation noise. The issue of template corruption is considered in the next section where the observation includes narrowband interference.

VI. Narrowband Interference Resistance

In the final scenario we show how least squares template estimation can be combined with transmitted reference systems to improve resistance to narrow band interference (NBI). It was shown in the previous section that the amount of energy captured can be increased, or alternatively the number of correlators reduced, by using template estimation in the receiver. It was also seen that the largest gain is made by simply using the observation itself as the template without doing any more processing. However, taking this approach makes the receiver susceptible to outside interference signals that could be reduced if our knowledge of the transmitted pulse was taken into account.

When finding the template according to equation (8) the choice of N effectively determines the order of the filter used to model the channel. While a higher order model can better approximate the channel response, it is also better able to replicate sudden spectral jumps in the observed signal due to narrowband interference, and hence admits more distortion of the template due to NBI than lower order models. Thus the choice of model order is a trade-off between template distortion due to inaccurate channel estimation and that due to narrowband interference.

The experiments of Section V were repeated with different levels of narrowband interference added to the observed signal, once with the frequency of the NBI at the 4GHz centre frequency of the transmitted pulse, and again at 5GHZ where the power spectral density of transmitted pulse is 20dB down.

In Fig. 5 the output signal-to-interference ratio (SIR) is plotted against the input SIR for the fixed (transmitted pulse) template, various orders of LS estimated templates, and the observed waveform template, where the NBI frequency is 5GHz. Input SIR is defined as

$$\text{Input SIR} = \frac{\varepsilon_{\mathrm{w}} \times R_{\mathrm{u}}}{A^2/2}$$

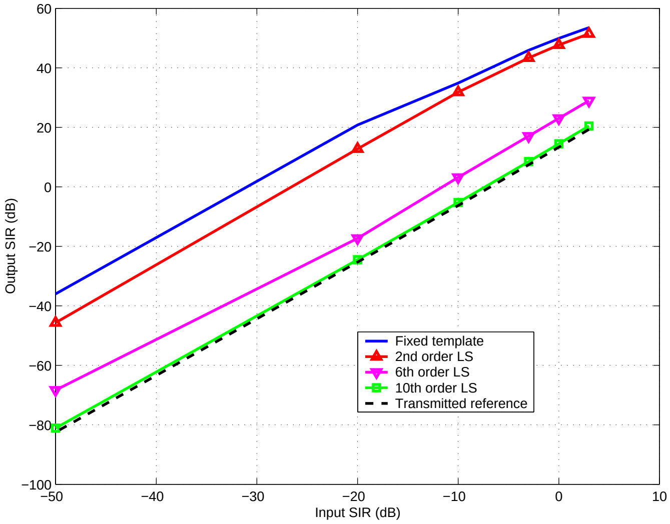


Fig. 5. Output SIR against input SIR for sinusoidal interference at 5GHz.

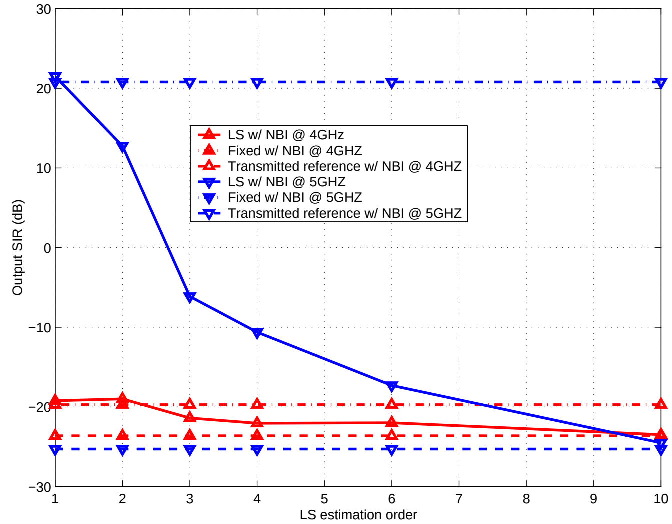


Fig. 6. Output SIR against LS template estimation order for narrowband interference at 4GHZ and 5GHZ.

where ε_{w} is the total energy of the received UWB signal over the multipath channel, R_{u} is the pulse repetition frequency of the UWB pulse, which is chosen as 10MHz, and the narrowband interference signal is given by

$$I(t,u) = A\sin(2\pi f t + \psi(u)) \tag{10}$$

where $\psi(u)$ is the random phase uniformly distributed over $(0, 2\pi]$.

Output SIR is defined as

$$\text{Output SIR} = \frac{\mathbb{E}\left[\left(\int v(t,u)w(t,u)dt\right)^2\right]}{\mathbb{E}\left[\left(\int v(t,u)I(t,u)dt\right)^2\right]} \tag{11}$$

where $v(t,u)$ is the template, $w(t,u)$ is the received signal due to the UWB transmission (the desired part of the observation), and dependence on u indicates a random variable.

For each level of input SIR a first observed waveform is used to generate the template waveform and a second observed waveform used to calculate the output SIR, each with narrowband interference of independent phase, and the results are averaged over 30 experiments per multipath channel and over all 16 channels.

In Fig. 6 the output SIR is plotted against the order of the channel Taylor series approximation of the least-squares estimated template, with the output SIRs of the receivers using the transmitted pulse and unmodified observed waveform as templates plotted for reference. As expected, as the basis of the LS template goes to higher orders its performance approaches that of the observed waveform template. When the interference is strongly correlated with the transmitted pulse performance can still be improved by using an estimated template if the channel is modelled by one or two orders of its expansion. For NBI that is not strongly correlated with the transmitted pulse the effect of the interference is much weaker for low order estimates, but performance degrades more rapidly and is asymptotically worse at high orders.

VII. Conclusions

A technique for template estimation in UWB radio based on a Taylor series expansion of the channel has been proposed. It was shown, for a specific pulse shape, that for some simple single path channels that interact with common building materials most of the available energy can be captured by using only the first few orders of the expansion. Subsequently it was shown that by applying this technique to multipath channels such channels can be modelled with fewer paths than are necessary when using a fixed template, while maintaining the same degree of accuracy. Such a reduced path model is useful in Rake receivers by reducing the number of correlators required. Finally, it was shown that although using higher order expansions captures more channel energy it also makes the template susceptible to narrowband interference, a complication that also applies to transmitted reference as the logical limit of large order channel approximations, and that using a 1 or 2 order expansion might provide a good compromise between correlator reduction and narrowband interference resistance.

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