Optimal and Suboptimal Receivers for Ultra-wideband Transmitted Reference Systems

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Abstract— **The optimal receiver for an ultra-wideband transmitted reference (UWB TR) system in a single user multipath environment is derived, based on knowledge of channel properties. The performances of this optimal receiver and other crosscorrelation receivers are analyzed and compared. The usual crosscorrelation receiver which is often used in UWB TR systems is shown to be suboptimal. In addition, an UWB differential transmitted reference (UWB DTR) system is also proposed and its performance is evaluated.**

I. INTRODUCTION

Ultra-Wideband (UWB) impulse radio systems transmit data by modulation of subnanosecond pulses. These narrow pulses are distorted by the channel, but often can resolve many distinct propagation paths (multipath) because of their fine time-resolution capability [1]. However, a Rake receiver that implements tens or even hundreds of correlation operations may be required to take full advantage of the available signal energy [2]. On the other hand, a receiver using a single correlator matched to one transmission path may be operating at a 10 - 15dB signal energy disadvantage relative to a full Rake receiver.

Recently, Hoctor and Tomlinson proposed a UWB transmitted reference (TR) system with a simple receiver structure to capture all of the energy available in a UWB multipath channel [3]. In this TR system, a reference waveform is transmitted before each data-modulated waveform for the purpose of determining the current multipath channel response. Since the reference signal and data signal are transmitted within the coherence time of the channel, it is assumed that the channel responses to these two signals are the same. The proposed receiver correlates the data signal with the reference to use all the energy of the data signal without requiring additional channel estimation and Rake reception. This simple receiver structure has one major drawback, namely the transmitted reference signal used as a correlator template is noisy.

It is worth noting that the TR approach is not new, but dates back to the early days of communication theory [4],[5],[6]. Recently, a UWB crosscorrelation receiver which averages several reference signals to produce a template waveform was described, and its performance was analyzed for a UWB TR system using pulse position modulation (PPM) [7]. The issue in this approach is the cost of implementing the averaging process which requires something more than a simple delay line.

In this paper, the multipath channel is modeled as being specular, i.e., having discrete and resolvable paths. This simplifying assumption, made for analytical purposes, means the time difference between two adjacent paths is greater than a pulse width. Since the pulse width in UWB system is less than a nanosecond, this resolvable multipath assumption always applies to paths whose propagation lengths are greater than one foot.

A one-shot receiver for a UWB TR system will be described in this paper. Specifically, an optimal receiver for UWB TR system using binary antipodal modulation is derived using a generalized likelihood ratio (GLRT) test. This derivation uses knowledge of the response function of the channel and assumes ideal synchronization resolvable multipath in the process of optimization. The bit error probabilities (BEPs) of this GLRT receiver and two crosscorrelation receivers are computed and compared for a resolvable multipath channel model.

Generally TR systems waste communication resources, i.e., power and time, to transmit reference signals. In this paper, a UWB differential transmitted reference (UWB DTR) system using a prior data-bearing waveform as a reference is proposed. The receiver structure is still simple and implementable, and it saves resources for communication. The performance of this differential receiver is analyzed in Section III.

II. UWB TR SYSTEM

The transmitted signal of a UWB TR system with antipodal modulation is

$$s_{\text{tr}}(t) = \sum_{i=-\infty}^{\infty} g_{\text{tr}}(t - iT_{\text{f}}) + b_{\lfloor i/N_{\text{s}} \rfloor} g_{\text{tr}}(t - iT_{\text{f}} - T_{\text{d}}) \quad (1)$$

Here $g_{\text{tr}}(t)$ is a transmitted monocycle waveform that is nonzero only for $t \in (0, T_{\text{w}})$, and T_{f} is its repetition time (frame time). Each frame contains two monocycle waveforms. The first is a reference and the second, T_{d} seconds later, is a data-modulated waveform. The data bits $b_{\lfloor i/N_{\text{s}} \rfloor} \in \{1, -1\}$ with equal probability. The index $\lfloor i/N_{\text{s}} \rfloor$, i.e., the integer part of i/N_{s}, represents the index of the data bit modulating the data waveform in the ith frame. Hence each bit is transmitted in N_{s} successive frames to achieve an adequate bit energy in the receiver, and the channel is assumed invariant over this bit time.

In this TR system, T_{d} is greater than the multipath delay spread T_{mds} to assure that there is no interference between reference signal and data signal. The frame time is designed to be $T_{\text{f}} \geq 2T_{\text{d}} > 2T_{\text{mds}}$ so that no interframe interference exists. Because the single user case is considered here, the time-hopping

or direct sequence modulation which is used to reduce multiuser interference is eliminated for simplicity, but without loss of generality.

A. GLRT Optimal Receiver

We model the received TR signal $r(t)$ in a stationary channel over a bit time by

$$
\begin{aligned}
r(t) = & \sum_{i=0}^{N_s-1} \sum_{k=1}^{K} [\alpha_k g_{rx}(t - iT_f - \tau_k) \\
& + \alpha_k b_{\lfloor i/N_s \rfloor} g_{rx}(t - iT_f - T_d - \tau_k)] + n(u,t),
\end{aligned} \tag{2}
$$

where $n(u,t)$ represents white Gaussian receiver noise with two-sided power spectral density $\frac{N_0}{2}$. This model assumes the existence of K specular propagation paths, with the kth path's propagation delay and amplitude being denoted by τ_k and α_k. We assume that the received monocycle waveform $g_{rx}(t)$ arriving over a single path differs in shape from the transmitted waveform [8]-[9]. In the design and analysis of the GLRT optimal receiver, we assume that $g_{rx}(t)$ is known and can be used as a template signal in a correlator. This is an approximation because the waveform may vary with antenna orientations, obstacles along the propagation paths, reflection effects, temporal variations in the environment and propagation geometry, etc.

At this point we treat the channel parameter vectors $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_K]^t$, and $\boldsymbol{\tau} = [\tau_1, \tau_2, \cdots, \tau_K]^t$ as unknown. Assuming that the delay parameters are ordered with $\tau_j < \tau_k$ for $j < k$, and for simplicity that $\tau_1 = 0$, then $\tau_K + T_w = T_{mds}$, the delay spread of the channel, and hence the elimination of interpulse interference requires that $T_d > \tau_K + T_w$.

We now determine the optimal receiver for bit b_0, based on the observation $\tilde{r}$ of $r(t)$, $t \in (0, N_s T_f)$. Minimizing the bit error probability using a GLRT, the decision rule is of the form

$$
\frac{\max_{\boldsymbol{\alpha},\boldsymbol{\tau}} p(\tilde{r}|b_0 = 1, \boldsymbol{\alpha}, \boldsymbol{\tau})}{\max_{\boldsymbol{\alpha},\boldsymbol{\tau}} p(\tilde{r}|b_0 = -1, \boldsymbol{\alpha}, \boldsymbol{\tau})} \underset{-1}{\overset{1}{\gtrless}} 1 \tag{3}
$$

First, let's find the $\boldsymbol{\alpha}$ and $\boldsymbol{\tau}$ that maximize $p(\tilde{r}|b_0 = 1, \boldsymbol{\alpha}, \boldsymbol{\tau})$. It can be verified that

$$
\underset{\boldsymbol{\alpha},\boldsymbol{\tau}}{\text{argmax}} \; p(\tilde{r}|b_0 = 1, \boldsymbol{\alpha}, \boldsymbol{\tau}) = \underset{\boldsymbol{\alpha},\boldsymbol{\tau}}{\text{argmin}} \; S(\boldsymbol{\alpha}, \boldsymbol{\tau}) \tag{4}
$$

where

$$
S(\boldsymbol{\alpha}, \boldsymbol{\tau}) = 2N_s \boldsymbol{\alpha}^t \mathbf{R}(\boldsymbol{\tau}) \boldsymbol{\alpha} - 2\boldsymbol{\alpha}^t \mathbf{X}_1(\boldsymbol{\tau}). \tag{5}
$$

Here $\mathbf{R}(\boldsymbol{\tau})$ is a $K \times K$ matrix whose ij^{th} entry is

$$
[\mathbf{R}(\boldsymbol{\tau})]_{i,j} = R(\tau_i - \tau_j), \tag{6}
$$

$$
R(\tau) = \int_{-\infty}^{\infty} g_{rx}(t - \tau) g_{rx}(t) dt, \tag{7}
$$

and $\mathbf{X}_1(\boldsymbol{\tau})$ is given by

$$
\mathbf{X}_1(\boldsymbol{\tau}) = \mathbf{C}_R(\boldsymbol{\tau}) + \mathbf{C}_D(\boldsymbol{\tau}) \tag{8}
$$

where $\mathbf{C}_R(\boldsymbol{\tau}) = [C_R(\tau_1), C_R(\tau_2), \ldots, C_R(\tau_K)]^t$ with

$$
C_R(\tau_k) = \int_0^{N_s T_f} r(t) \sum_{j=0}^{N_s-1} g_{rx}(t - jT_f - \tau_k) dt, \tag{9}
$$

and $\mathbf{C}_D(\boldsymbol{\tau}) = [C_D(\tau_1), C_D(\tau_2), \ldots, C_D(\tau_K)]^t$ with

$$
C_D(\tau_k) = \int_0^{N_s T_f} r(t) \sum_{j=0}^{N_s-1} g_{rx}(t - jT_f - \tau_k - T_d) dt \tag{10}
$$

Because of the resolvable multipath assumption, $\mathbf{R}(\boldsymbol{\tau}) = 2R(0)\mathbf{I}_{K \times K}$ where $\mathbf{I}_{K \times K}$ is a K by K identity matrix. The choices for $\boldsymbol{\alpha}$ and $\boldsymbol{\tau}$ which minimize $S(\boldsymbol{\alpha}, \boldsymbol{\tau})$ in (4) are [8]

$$
\begin{aligned}
\hat{\boldsymbol{\tau}}^{(1)} &= \underset{\boldsymbol{\tau}}{\text{argmax}} \|\mathbf{X}_1(\boldsymbol{\tau})\|^2 \\
&= \underset{\boldsymbol{\tau}}{\text{argmax}} \sum_{k=1}^{K} |C_R(\tau_k) + C_D(\tau_k)|^2
\end{aligned} \tag{11}
$$

$$
\hat{\boldsymbol{\alpha}}^{(1)} = \frac{1}{2N_s R(0)} [\mathbf{C}_R(\hat{\boldsymbol{\tau}}^{(1)}) + \mathbf{C}_D(\hat{\boldsymbol{\tau}}^{(1)})] \tag{12}
$$

The computation of $\hat{\boldsymbol{\tau}}^{(2)}$ and $\hat{\boldsymbol{\alpha}}^{(2)}$ which minimizes $p(\tilde{r}|b_0 = -1, \boldsymbol{\alpha}, \boldsymbol{\tau})$ is similar to the computation in (4)-(12). Defining $\mathbf{X}_2(\boldsymbol{\tau}) = \mathbf{C}_R(\boldsymbol{\tau}) - \mathbf{C}_D(\boldsymbol{\tau})$ and completing the computation gives

$$
\begin{aligned}
\hat{\boldsymbol{\tau}}^{(2)} &= \underset{\boldsymbol{\tau}}{\text{argmax}} \|\mathbf{X}_2(\boldsymbol{\tau})\|^2 \\
&= \underset{\boldsymbol{\tau}}{\text{argmax}} \sum_{k=1}^{K} |C_R(\tau_k) - C_D(\tau_k)|^2
\end{aligned} \tag{13}
$$

$$
\hat{\boldsymbol{\alpha}}^{(2)} = \frac{1}{2N_s R(0)} [\mathbf{C}_R(\hat{\boldsymbol{\tau}}^{(2)}) - \mathbf{C}_D(\hat{\boldsymbol{\tau}}^{(2)})] \tag{14}
$$

Next, the GLRT decision rule of (3) can be reduced to

$$
\frac{\exp\left\{\frac{2}{N_0} \mathbf{X}_1^t(\hat{\boldsymbol{\tau}}^{(1)}) \hat{\boldsymbol{\alpha}}^{(1)}\right\}}{\exp\left\{\frac{2}{N_0} \mathbf{X}_2^t(\hat{\boldsymbol{\tau}}^{(2)}) \hat{\boldsymbol{\alpha}}^{(2)}\right\}} \underset{-1}{\overset{1}{\gtrless}} 1, \tag{15}
$$

which is equivalent to

$$
\frac{\left\|\mathbf{C}_R(\hat{\boldsymbol{\tau}}^{(1)}) + \mathbf{C}_D(\hat{\boldsymbol{\tau}}^{(1)})\right\|^2}{\left\|\mathbf{C}_R(\hat{\boldsymbol{\tau}}^{(2)}) - \mathbf{C}_D(\hat{\boldsymbol{\tau}}^{(2)})\right\|^2} \underset{-1}{\overset{1}{\gtrless}} 1 \tag{16}
$$

Suppose the estimate of multipath arrival times is correct, i.e., $\hat{\boldsymbol{\tau}}^{(1)} = \hat{\boldsymbol{\tau}}^{(2)} \triangleq \boldsymbol{\tau}$. Under this condition, the decision rule in (16) reduces to

$$
\sum_{k=1}^{K} \underbrace{[C_R(\tau_k) C_D(\tau_k)]}_{\triangleq D_k(u)} \underset{-1}{\overset{1}{\gtrless}} 0. \tag{17}
$$

At this point we begin to show dependence on the sample space explicitly by indicating dependence on a point u in a sample space. This sets the stage for probability of bit error for this rule ideal synchronization conditions.

B. Performance of GLRT Receiver

Evaluation of the quantity $D_k(u)$ indicated in (17) is the key to determining the bit error probability of the GLRT receiver. Substituting the received signal into $D_k(u)$ yields

$$
D_k(u) = \left[N_{\mathrm{s}}R(0)\alpha_k + \sum_{j=0}^{N_{\mathrm{s}}-1} N_{\mathrm{R}kj}(u) \right] \\
\cdot \left[bN_{\mathrm{s}}R(0)\alpha_k + \sum_{j=0}^{N_{\mathrm{s}}-1} N_{\mathrm{D}kj}(u) \right], \quad (18)
$$

where, because $g_{\mathrm{rx}}(t - jT_{\mathrm{f}} - \tau_k)$ is non-zero only for $t \in jT_{\mathrm{f}}, (j+1)T_{\mathrm{f}})$,

$$
N_{\mathrm{R}kj}(u) \triangleq \int_{jT_{\mathrm{f}}}^{jT_{\mathrm{f}}+T_{\mathrm{mds}}} n(u,t)g_{\mathrm{rx}}(t - jT_{\mathrm{f}} - \tau_k)dt, \quad (19)
$$

$$
N_{\mathrm{D}kj}(u) \triangleq \int_{jT_{\mathrm{f}}+T_{\mathrm{d}}}^{jT_{\mathrm{f}}+T_{\mathrm{d}}+T_{\mathrm{mds}}} n(u,t)g_{\mathrm{rx}}(t - jT_{\mathrm{f}} - T_{\mathrm{d}} - \tau_k)dt. \quad (20)
$$

The test statistic in (17) then can be represented by

$$
\begin{aligned}
D(u) &\triangleq \sum_{k=1}^{K} D_k(u) \\
&= \sum_{k=1}^{K} \big[bN_{\mathrm{s}}^2 R^2(0)\alpha_k^2 + N_{\mathrm{R}k}(u)N_{\mathrm{D}k}(u) \\
&\quad + N_{\mathrm{s}}R(0)\alpha_k N_{\mathrm{D}k}(u) + bN_{\mathrm{s}}R(0)\alpha_k N_{\mathrm{R}k}(u) \big], \quad (21)
\end{aligned}
$$

where

$$
N_{\mathrm{R}k}(u) \triangleq \sum_{j=0}^{N_{\mathrm{s}}-1} N_{\mathrm{R}kj}(u), \quad N_{\mathrm{D}k}(u) \triangleq \sum_{j=0}^{N_{\mathrm{s}}-1} N_{\mathrm{D}kj}(u). \quad (22)
$$

We assume in this paper that the power spectral density $S_n(f)$ of the noise $n(u,t)$ satisfies the white noise approximation, namely $S_n(f)|G_{\mathrm{rx}}(f)|^2 \approx \frac{N_0}{2}|G_{\mathrm{rx}}(f)|^2$, where $G_{\mathrm{rx}}(f)$ is the Fourier transform of $g_{\mathrm{rx}}(t)$. Then, as a result of the resolvable multipath assumption, the set $\{N_{\mathrm{R}kj}(u), N_{\mathrm{D}kj}(u) : 1 \leq k \leq K, 1 \leq j \leq N_{\mathrm{s}} - 1\}$ of elementary random variables in this representation of the test statistic $D(u)$ can be shown to be independent Gaussian random variables with zero mean and variance $\frac{N_0}{2}R(0)$. Hence the random variables $N_{\mathrm{R}k}(u)$ and $N_{\mathrm{D}k}(u)$ are independent Gaussian random variables. The term $\sum_{k=1}^{K} N_{Rk}(u)N_{Dk}(u)$ in (21) is generally non-Gaussian, and its probability density can be computed precisely (see [10], chapter 2, section 3). For our purposes here, we assume that the number of paths, K, (or we should say $N_{\mathrm{s}}^2 K$) is large enough to conclude that this sum of products of Gaussian random variables is itself approximately Gaussian by central-limit theorem arguments.

It follows from this discussion that $D(u)$ is an approximately Gaussian random variable given b, $\boldsymbol{\alpha}$, and $\boldsymbol{\tau}$. The mean and variance of $D(u)$ and the bit-error probability can be computed under these conditions.

$$
\mathbb{E}\{D(u)|b, \boldsymbol{\alpha}, \boldsymbol{\tau}\} = bN_{\mathrm{s}}^2 R^2(0) \sum_{k=1}^{K} \alpha_k^2 \quad (23)
$$

$$
\begin{aligned}
\mathbb{V}\mathrm{ar}\{D(u)|\boldsymbol{\alpha}, \boldsymbol{\tau}\} &= K\left[\frac{N_{\mathrm{s}}R(0)N_0}{2} \right]^2 \\
&\quad + N_{\mathrm{s}}^3 R^3(0)N_0 \sum_{k=1}^{K} \alpha_k^2 \quad (24)
\end{aligned}
$$

Under our assumptions that $D(u)$ is a Gaussian random variable and the error probability P_{bit} for the decision rule (17) is

$$
P_{\mathrm{bit}} = Q\left(\left[\frac{2}{N_{\mathrm{s}}}\left(\frac{N_0}{E_{\mathrm{f}}}\right) + \frac{K}{N_{\mathrm{s}}^2}\left(\frac{N_0}{E_{\mathrm{f}}}\right)^2 \right]^{-\frac{1}{2}} \right), \quad (25)
$$

where $E_{\mathrm{f}} = 2R(0)|\boldsymbol{\alpha}|^2$ is the received energy per frame at two pulses per frame, and $Q(x) \triangleq (2\pi)^{-\frac{1}{2}} \int_x^{\infty} e^{-x^2/2}dx$.

C. Performance of Crosscorrelation Receivers

When the observation interval is fixed to one bit duration, the crosscorrelation receiver may have two different structures. The easiest receiver to mechanize correlates each received data pulse with the reference pulse received T_{d} seconds earlier, and sums the N_{s} results over the N_{s} pulses that are affected by a single data bit [3]. We will call this the simple transmitted reference (STR) receiver.

The second crosscorrelation receiver averages the N_{s} received reference pulses to reduce the noise in the reference waveform (and in fact the same averaging process can be done on the received data-modulated pulses), and then data detection proceeds with the reduced noise reference. We will call this an averaged transmitted reference (ATR) receiver. The ATR receiver generally will have higher complexity and better performance than the STR receiver. In [7], the channel was assumed to be time invariant over the duration of two data bits, and the received reference waveforms in the first bit interval were averaged and used to correlate with the data modulated waveforms in the second bit interval. This mechanization condition is different from that discussed here, but has comparable performance.

Define $D_{\mathrm{s}}(u)$ as the output of the STR correlator for $b = b_0$, then

$$
\begin{aligned}
D_{\mathrm{s}}(u) &= \sum_{j=0}^{N_{\mathrm{s}}-1} \int_{jT_{\mathrm{f}}+T_{\mathrm{d}}}^{jT_{\mathrm{f}}+T_{\mathrm{d}}+T_{\mathrm{mds}}} r(t - T_{\mathrm{d}})r(t)dt \\
&= bN_{\mathrm{s}}R(0)|\boldsymbol{\alpha}|^2 + \sum_{k=1}^{K} \alpha_k N_{\mathrm{D}k}(u) \\
&\quad + \sum_{k=1}^{K} b\alpha_k N_{\mathrm{R}k}(u) + N(u) \quad (26)
\end{aligned}
$$

where $N(u)$ is the sum of N_{s} independent random variables, each of the form $\int_{jT_{\mathrm{f}}+T_{\mathrm{d}}}^{jT_{\mathrm{f}}+T_{\mathrm{d}}+T_{\mathrm{mds}}} n(u, t - T_{\mathrm{d}})n(u,t)dt$ for different values of j. Again, using central-limit theorem arguments, $N(u)$ can be modeled as a Gaussian random variable with zero mean and variance $\frac{N_{\mathrm{s}}N_0^2 T_{\mathrm{mds}}W}{2}$, where W is the one-sided noise bandwidth of the receiver. It is assumed in developing this result that $T_{\mathrm{mds}}W$ is large, as is the case for highly time-spread

channels. All the other random variables can be computed as in the GLRT receiver case.

Under the above development, the test statistic $D_\mathrm{s}(u)$ for the STR receiver is a Gaussian random variable under each channel realization with $\mathbb{E}\{D_\mathrm{s}(u)|b,\boldsymbol{\alpha},\boldsymbol{\tau}\} = bN_\mathrm{s}R(0)|\boldsymbol{\alpha}|^2$ and $\mathbb{V}\mathrm{ar}\{D_\mathrm{s}(u)|\boldsymbol{\alpha},\boldsymbol{\tau}\} = N_\mathrm{s}N_0R(0)|\boldsymbol{\alpha}|^2 + \frac{N_\mathrm{s}WT_\mathrm{mds}N_0^2}{2}$. The BEP of the STR receiver is

$$P_\mathrm{bit} = Q\left(\left[\frac{2}{N_\mathrm{s}}\left(\frac{N_0}{E_\mathrm{f}}\right) + \frac{2WT_\mathrm{mds}}{N_\mathrm{s}}\left(\frac{N_0}{E_\mathrm{f}}\right)^2\right]^{-\frac{1}{2}}\right). \quad (27)$$

In the ATR receiver, we define $D_\mathrm{a}(u)$ as the output of the crosscorrelator for $b = b_0$, where

$$\begin{aligned}
D_\mathrm{a}(u) &= \sum_{j=0}^{N_\mathrm{s}-1}\int_{jT_\mathrm{f}+T_\mathrm{d}}^{jT_\mathrm{f}+T_\mathrm{d}+T_\mathrm{mds}} r(t) \\
&\quad \cdot\left[\frac{1}{N_\mathrm{s}}\sum_{i=-j}^{N_\mathrm{s}-1-j} r(t+iT_\mathrm{f}-T_\mathrm{d})\right]dt. \quad (28)
\end{aligned}$$

The statistical computation is left to the readers and the BEP of the ATR receiver is given here.

$$P_\mathrm{bit} = Q\left(\left[\frac{2}{N_\mathrm{s}}\left(\frac{N_0}{E_\mathrm{f}}\right) + \frac{2WT_\mathrm{mds}}{N_\mathrm{s}^2}\left(\frac{N_0}{E_\mathrm{f}}\right)^2\right]^{-\frac{1}{2}}\right). \quad (29)$$

III. A UWB DTR SYSTEM

A. System Structure

It is possible to construct a UWB DTR system in which no references are transmitted, but instead, the data signal in the previous frame is used as reference. A block diagram of a UWB DTR system is plotted in Figure 1. Here the differentially modulated UWB transmitted signal is

$$s_\mathrm{tr}(t) = \sum_{i=-\infty}^{\infty} m_i g_\mathrm{tr}(t-iT_\mathrm{f}) \quad (30)$$

where $m_i = m_{i-1}b_{\lfloor i/N_\mathrm{s}\rfloor}$. All the other parameters are the same as defined in the TR system. Because there is no extra reference signal imbedded in each frame, the requirement for no interframe interference is simply that $T_\mathrm{f} > T_\mathrm{mds}$. This transmitted signal can be modified for multiuser applications by adding distinctive time-hopping or direct sequence modulations.

The modulation and demodulation techniques of this DTR system are similar to those used in DBPSK. It is not easy for this differential receiver to average several signals to produce a cleaner reference because the data is transmitted as the difference of two consecutive signals. To do so would require a decision directed approach. So, as described here, the differential UWB system is suitable for situations in which a simple receiver is required.

B. Performance of a DTR receiver

The received signal of this differential system is

$$r(t) = \sum_{i=-\infty}^{\infty}\sum_{k=1}^{K} \alpha_k m_{i-1} b_{\lfloor i/N_\mathrm{s}\rfloor} g_\mathrm{rx}(t-iT_\mathrm{f}-\tau_k)+n(u,t) \quad (31)$$
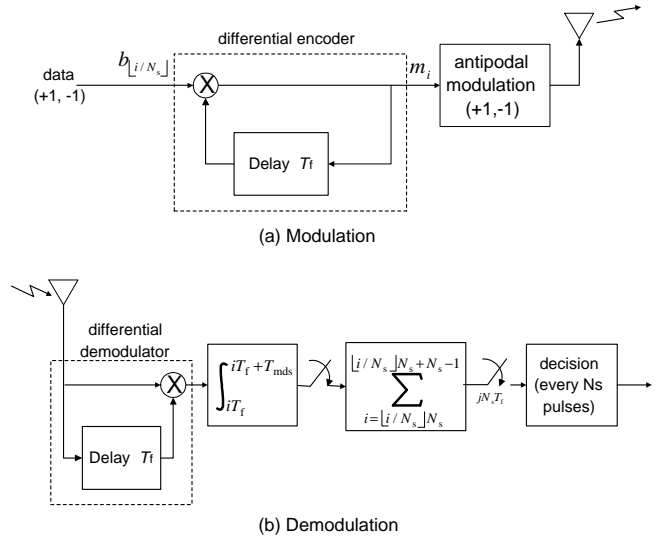

Fig. 1. Block diagram of the modulator and demodulator in UWB DTR system.

Again, we will assume perfect synchronization. Without loss of generality, assume $b = b_0$ is the bit to be determined. The differential correlator's output is

$$\begin{aligned}
D_\mathrm{d}(u) &= \sum_{j=0}^{N_\mathrm{s}-1}\int_{jT_\mathrm{f}}^{jT_\mathrm{f}+T_\mathrm{mds}} r(t-T_\mathrm{f})r(t)dt \\
&= \sum_{j=0}^{N_\mathrm{s}-1}(|\boldsymbol{\alpha}|^2bR(0)+N_j(u)) \quad (32) \\
&\quad +\sum_{k=1}^{K}\alpha_k m_{j-1}[N_{\mathrm{R}kj}(u)+bN_{\mathrm{R}k(j-1)}(u)])
\end{aligned}$$

where $N_{\mathrm{R}kj}(u)$ is defined in (19), and

$$N_j(u) = \int_{jT_\mathrm{f}}^{jT_\mathrm{f}+T_\mathrm{mds}} n(u,t)n(u,t-T_\mathrm{f})dt. \quad (33)$$

Under the white noise approximation and the resolvable multipath assumption, all of the random variables on the right side of (32) are uncorrelated and mean-zero. As before, $\mathbb{E}\{[N_{\mathrm{R}kj}(u)]^2\} = \frac{N_0}{2}R(0)$ and $\mathbb{E}\{[N_{\mathrm{R}kj}(u)]^2\} = \frac{N_0^2T_\mathrm{mds}W}{2}$, and Gaussian statistics apply to $D_\mathrm{d}(u)$, given $\boldsymbol{\alpha}$, $\boldsymbol{\tau}$, and b.

With this development, it can be verified that the BEP of the differential system is

$$P_\mathrm{bit} = Q\left(\left[\frac{2N_\mathrm{s}-1}{N_\mathrm{s}^2}\left(\frac{N_0}{E_\mathrm{p}}\right) + \frac{WT_\mathrm{mds}}{2N_\mathrm{s}}\left(\frac{N_0}{E_\mathrm{p}}\right)^2\right]^{-\frac{1}{2}}\right) \quad (34)$$

where $E_\mathrm{p} = R(0)|\boldsymbol{\alpha}|^2$ is the received energy per pulse at one pulse per frame.

IV. RESULT

The bit error probability in (34) is defined as a function of the received energy per pulse. Since each frame contains only

one pulse in the UWB DTR system, the received energy per pulse is equal to the received energy per frame. In order to compare the performance of four receiver structures, $\frac{N_0}{E_p}$ in (34) can be replaced by $\frac{N_0}{E_f}$ without confusion. In (25), (27), (29), and (34), it can be seen that all the bit error probabilities are a function of both $\frac{E_f}{N_0}$ and $\left(\frac{E_f}{N_0}\right)^2$. The second power term comes from the reference channel noise. It can also be seen that the coefficient of this term is proportional to the number K of multipaths, or receiver bandwidth W. Because the number of multipaths and receiver bandwidth are large in UWB system, this term dominates the performance in the interested range of SNR. For the GLRT receiver and ATR receiver, this term decays with N_s^2, it is obvious these two receivers should perform better.

Under the resolvable dense multipath assumption, WT_{mds} can be approximately equal to the number of paths, K. If the multipath is not dense, K should be less than WT_{mds}. Therefore the number of paths can be upper bounded by WT_{mds}, i.e., $K \leq \frac{T_{\mathrm{mds}}}{T_p} \cong T_{\mathrm{mds}}W$. Substitute $K = WT_{\mathrm{mds}}$ in (27), (29), and (34) can give the lower bounds of BEPs of STR, ATR, and DTR receivers under the resolvable multipath assumption.

The BEP versus $\frac{E_f}{N_0}$ curves of the four receiver structures described in this paper are plotted in Figure 2 assuming that $K = WT_{\mathrm{mds}}$ in (27), (29), and (34). In other words, the performance curves in Figure 2 represent the resolvable dense multipath case. In Figure 2, K and N_s are set to be 200 and 20, respectively. Over one-bit observation time, GLRT receiver outperforms all other receivers. It should be keep in mind that the GLRT receiver must estimate multipath arrival times and implement a large number of correlators to catch all the energy in received signals. Although the ATR receiver is not optimal, it is very competitive if the trade-off between receiver complexity and performance is concerned. The DTR receiver and STR receiver have comparable complexity but the former has better BEP performance and may have higher data rate.

The performance curves in Figure 2 represent the dense multipath case, i.e., $K = WT_{\mathrm{mds}}$. This situation usually happens in a complex indoor environment. In outdoor and simple indoor environments, the multipath is generally not dense, i.e., $K < WT_{\mathrm{mds}}$. Therefore the relative gain of GLRT receiver over other three receivers should be greater than what we have seen in Figure 2. Note also that it is in every receiver that T_{mds} is known. If this is not the case, the GLRT receiver may have an increased advantage because of excess noise or reduced received energy in the other receivers.

V. CONCLUSION

Of the four receiver structures studied here, the ATR receiver and DTR receiver are the better choices for moderate and simple receiver complexities, respectively. The GLRT receiver has to estimate the arrival time of each path, and implement a large number of correlators. But the required $\frac{E_f}{N_0}$ of GLRT receiver in our example is just 1.1dB less than the ATR receiver at BER= 10^{-5} in dense resolvable multipath environment. Because the idea of using a UWB TR system is to simplify the receiver structure, GLRT receiver serves primarily as a performance benchmark.
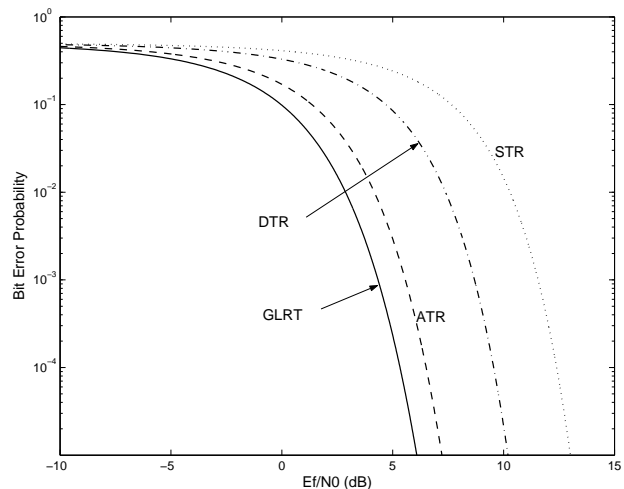

Fig. 2. BEP of four receiver structures in a dense resolvable multipath environment with $K = 200$ and $N_s = 20$.

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